Combinatorics After CC-Conjecture

— Notions and Achievements

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Abstract: As a powerful technique for holding relations in things, combinatorics has experienced rapidly development in the past century, particularly, enumeration of configurations, combinatorial design and graph theory. However, the main objective for mathematics is to bring about a quantitative analysis for other sciences, which implies a natural question on combinatorics. Thus, *how combinatorics can contributes to other mathematical sciences, not just in discrete mathematics, but metric mathematics and physics?* After a long time speculation, I brought the CC conjecture for advancing mathematics by combinatorics, i.e., *any mathematical science can be reconstructed from or made by combinatorialization* in my postdoctoral report for Chinese Academy of Sciences in 2005, and then reported it at a few conferences of China. Clearly, CC conjecture is in fact a combinatorial notion and holds by a philosophical law, i.e., all things are inherently related, not isolated. The main purpose of this report is to survey the roles of CC conjecture in developing mathematical sciences with notions, such as those of its contribution to algebra, topology, Euclidean geometry and differential geometry, non-solvable differential equations or classical mathematical systems with contradictions to mathematics, quantum fields after it appeared 10 years ago. All of these show the importance of combinatorics to mathematical sciences in the past and future.

Key Words: CC conjecture, Smarandache system, G^L-system, non-solvable system of equations, combinatorial manifold, geometry, quantum field.

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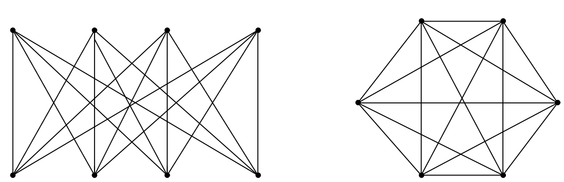
§1. Introduction

There are many techniques in combinatorics, particularly, the enumeration and counting with graph, a visible, also an abstract model on relations of things in the world. A graph G is a 3-tuple (V, E, I) with finite sets V, E and a mapping $I: E \rightarrow V \times V$, and *simple* if it is without loops and multiple edges, denoted by $(V; E)$ for convenience. All elements v in V, e in E are said respectively *vertices* and *edges*.

A graph with given properties are particularly interested. For example, a *path* P_n in a graph G is an alternating sequence of vertices and edges $u_1, e_1, u_2, e_2, \cdots, e_n, u_{n_1}$, $e_i = (u_i, u_{i+1})$ with distinct vertices for an integer $n \geq 1$, and if $u_1 = u_{n+1}$, it is called a *circuit* or *cycle* C_n. For example, $v_1v_2v_3v_4$ and $v_1v_2v_3v_4v_1$ are respective path and circuit in Fig.1. A graph G is *connected* if for $u, v \in V(G)$, there are paths with end vertices u and v in G.

A *complete graph* $K_n = (V_c, E_c; I_c)$ is a simple graph with $V_c = \{v_1, v_2, \cdots, v_n\}$, $E_c = \{e_{ij}, 1 \leq i, j \leq n, i \neq j\}$ and $I_c(e_{ij}) = (v_i, v_j)$, or simply by a pair (V, E) with $V = \{v_1, v_2, \cdots, v_n\}$ and $E = \{v_iv_j, 1 \leq i, j \leq n, i \neq j\}$.

A simple graph $G = (V, E)$ is *r-partite* for an integer $r \geq 1$ if it is possible to partition V into r subsets $V_1, V_2, \cdots, V_r$ such that for $\forall e(u, v) \in E$, there are integers $i \neq j, 1 \leq i, j \leq r$ such that $u \in V_i$ and $v \in V_j$. If there is an edge $e_{ij} \in E$ for $\forall v_i \in V_i$, $\forall v_j \in V_j$, where $1 \leq i, j \leq r, i \neq j$, then, G is called a *complete r-partite graph*, denoted by $G = K(|V_1|, |V_2|, \cdots, |V_r|)$. Thus a complete graph is nothing else but a complete 1-partite graph. For example, the bipartite graph $K(4, 4)$ and the complete graph K_6 are shown in Fig.1.



$K(4,4)$ $\qquad\qquad$ K_6

Fig.1

Notice that a few edges in Fig.1 have intersections besides end vertices. Contrast to this case, a *planar graph* can be realized on a Euclidean plane $\mathbb{R}^2$ by letting points $p(v) \in \mathbb{R}^2$ for vertices $v \in V$ with $p(v_i) \neq p(v_j)$ if $v_i \neq v_j$, and letting curve $C(v_i, v_j) \subset \mathbb{R}^2$ connecting points $p(v_i)$ and $p(v_j)$ for edges $(v_i, v_j) \in E(G)$, such as those shown in Fig.2.

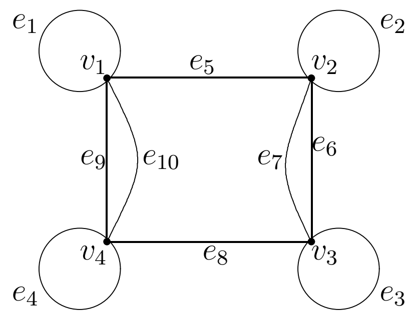


Fig.2

Generally, let $\mathscr{E}$ be a topological space. A graph G is said to be embeddable into $\mathscr{E}$ ([32]) if there is a $1-1$ continuous mapping $f : G \rightarrow \mathscr{E}$ with $f(p) \neq f(q)$ if $p \neq q$ for $\forall p, q \in G$, i.e., edges only intersect at vertices in $\mathscr{E}$. Such embedded graphs are called *topological graphs*.

There is a well-known result on embedding of graphs without loops and multiple edges in $\mathbb{R}^n$ for $n \geq 3$ ([32]), i.e., *there always exist such an embedding of G that all edges are straight segments in $\mathbb{R}^n$*, which enables us turn to characterize embeddings of graphs on $\mathbb{R}^2$ and its generalization, 2-*manifolds* or *surfaces* ([3]).

However, all these embeddings of G are established on a assumption that *each vertex of G is mapped exactly into one point of $\mathscr{E}$* in combinatorics for simplicity. If we put off this assumption, *what will happens*? *Are these resultants important for understanding the world*? The answer is certainly YES because this will enables us to pullback more characters of things, characterize more precisely and then hold the truly faces of things in the world.

All of us know an objective law in philosophy, namely, *the integral always consists of its parts and all of them are inherently related, not isolated*. This idea implies that every thing in the world is nothing else but a union of sub-things underlying a graph embedded in space of the world.

Formally, we introduce some conceptions following.

Definition 1.1([30]-[31], [12]) *Let* $(\Sigma_1; \mathcal{R}_1)$, $(\Sigma_2; \mathcal{R}_2)$, $\cdots$, $(\Sigma_m; \mathcal{R}_m)$ *be* m *math-*

ematical system, different two by two. A Smarandache multi-system $\widetilde{\Sigma}$ is a union $\bigcup\limits_{i=1}^{m} \Sigma_i$ with rules $\widetilde{\mathcal{R}} = \bigcup\limits_{i=1}^{m} \mathcal{R}_i$ on $\widetilde{\Sigma}$, denoted by $\left(\widetilde{\Sigma}; \widetilde{\mathcal{R}}\right)$.

Definition 1.2([11]-[13]) *For any integer $m \geq 1$, let $\left(\widetilde{\Sigma}; \widetilde{\mathcal{R}}\right)$ be a Smarandache multi-system consisting of m mathematical systems $(\Sigma_1; \mathcal{R}_1)$, $(\Sigma_2; \mathcal{R}_2)$, $\cdots$, $(\Sigma_m; \mathcal{R}_m)$. An inherited topological structure $G^L\left[\widetilde{\Sigma}; \widetilde{\mathcal{R}}\right]$ of $\left(\widetilde{\Sigma}; \widetilde{\mathcal{R}}\right)$ is a topological vertex-edge labeled graph defined following:*

$$V\left(G^L\left[\widetilde{\Sigma}; \widetilde{\mathcal{R}}\right]\right) = \{\Sigma_1, \Sigma_2, \cdots, \Sigma_m\},$$

$$E\left(G^L\left[\widetilde{\Sigma}; \widetilde{\mathcal{R}}\right]\right) = \{(\Sigma_i, \Sigma_j) | \Sigma_i \bigcap \Sigma_j \neq \emptyset,\ 1 \leq i \neq j \leq m\} \text{ with labeling}$$

$$L:\ \Sigma_i \to L(\Sigma_i) = \Sigma_i \quad \text{and} \quad L:\ (\Sigma_i, \Sigma_j) \to L(\Sigma_i, \Sigma_j) = \Sigma_i \bigcap \Sigma_j$$

for integers $1 \leq i \neq j \leq m$, also denoted by $G^L\left[\widetilde{\Sigma}; \widetilde{\mathcal{R}}\right]$ for $\left(\widetilde{\Sigma}; \widetilde{\mathcal{R}}\right)$.

For example, let $\Sigma_1 = \{a, b, c\}$, $\Sigma_2 = \{c, d, e\}$, $\Sigma_3 = \{a, c, e\}$, $\Sigma_4 = \{d, e, f\}$ and $\mathcal{R}_i = \emptyset$ for integers $1 \leq i \leq 4$, i.e., all these system are sets. Then the multi-space $\left(\widetilde{\Sigma}; \widetilde{\mathcal{R}}\right)$ with $\widetilde{\Sigma} = \bigcup\limits_{i=1}^{4} \Sigma_i = \{a, b, c, d, e, f\}$ and $\widetilde{\mathscr{R}} = \emptyset$ underlying a topological graph $G^L\left[\widetilde{\Sigma}; \widetilde{\mathcal{R}}\right]$ shown in Fig.3.

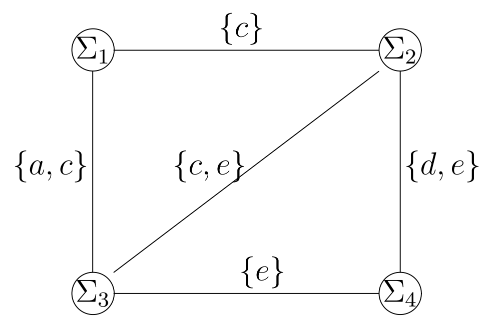


Fig.3

Combinatorially, the Smarandache multi-systems can be classified by their inherited topological structures, i.e., isomorphic labeled graphs following.

Definition 1.3 ([13]) *Let*

$$G_1{}^{L_1} = \left(\bigcup_{i=1}^{m} \Sigma_i^{(1)};\ \bigcup_{i=1}^{m} \mathcal{R}_i^{(1)}\right) \quad \text{and} \quad G_2{}^{L_2} = \left(\bigcup_{i=1}^{n} \Sigma_i^{(2)};\ \bigcup_{i=1}^{n} \mathcal{R}_i^{(2)}\right).$$

be two Smarandache multi-systems underlying topological graphs G_1 and G_2, re-

spectively. They are isomorphic if there is a bijection $\varpi : {G_1}^{L_1} \to {G_2}^{L_2}$ *with* $\varpi : \bigcup_{i=1}^{m} \Sigma_i^{(1)} \to \bigcup_{i=1}^{n} \Sigma_i^{(2)}$ *and* $\varpi : \bigcup_{i=1}^{m} \mathcal{R}_i^{(1)} \to \bigcup_{i=1}^{n} \mathcal{R}_i^{(2)}$ *such that*

$$\varpi|_{\Sigma_i}\left(a\mathcal{R}_i^{(1)}b\right) = \varpi|_{\Sigma_i}(a)\varpi|_{\Sigma_i}\left(\mathcal{R}_i^{(1)}\right)\varpi|_{\Sigma_i}(b)$$

for $\forall a, b \in \Sigma_i^{(1)}$, $1 \leq i \leq m$, *where* $\varpi|_{\Sigma_i}$ *denotes the constraint of* ϖ *on* $(\Sigma_i, \mathcal{R}_i)$.

Consequently, the previous discussion implies that

Every thing in the world is nothing else but a topological graph G^L *in space of the world, and two things are similar if they are isomorphic.*

After speculation over a long time, I presented the CC conjecture on mathematical sciences in the final chapter of my post-doctoral report for *Chines Academy of Sciences* in 2005 ([9],[10]), and then reported at *The 2nd Conference on Combinatorics and Graph Theory of China* in 2006, which is in fact an inverse of the understand of things in the world.

CC Conjecture([9-10],[14]) *Any mathematical science can be reconstructed from or made by combinatorization.*

Certainly, this conjecture is true in philosophy. It is in fact a *combinatorial notion* for developing mathematical sciences following.

Notion 1.1 *Finds the combinatorial structure, particularly, selects finite combinatorial rulers to reconstruct or make a generalizations of a classical mathematical science.*

This notion appeared even in classical mathematics. For examples, Hilbert axiom system for Euclidean geometry, complexes in algebraic topology, particularly, 2-cell embeddings of graphs on surface are essentially the combinatorization for Euclidean geometry, topological spaces and surfaces, respectively.

Notion 1.2 *Combine different mathematical sciences and establish new enveloping theory on topological graphs, with classical theory being a special one, and this combinatorial process will never end until it has been done for all mathematical sciences.*

A few fields can be also found in classical mathematics on this notion, for instance the topological groups, which is in fact a multi-space of topological space with groups, and similarly, the Lie groups, a multi-space of manifold with that of diffeomorphisms.

Even in the developing process of physics, the trace of Notions 1.1 and 1.2 can be also found. For examples, the many-world interpretation [2] on quantum mechanics by Everett in 1957 is essentially a multi-space formulation of quantum state (See also [35] for details), and the unifying of the four known forces, i.e., gravity, electro-magnetism, the strong and weak nuclear force into one super force by many researchers, i.e., establish the *unified field theory* is nothing else but also a following of combinatorial notion by let Lagrangian $\mathscr{L}$ of the unified field being a combination of its subfields.

Even so, the CC conjecture includes more deeply thoughts for developing mathematics by combinatorics i.e., *mathematical combinatorics* which extends the field of all existent mathematical sciences. After it was presented, more methods were suggested for developing mathematics in last decade. The main purpose of this report is to survey its contribution to algebra, topology and geometry, mathematical analysis, particularly, non-solvable algebraic and differential equations, theoretical physics with its producing notions in developing mathematical sciences.

All terminologies and notations used in this paper are standard. For those not mentioned here, we follow reference [5] and [32] for topology, [3] for topological graphs, [1] for algebraic systems, [4], [34] for differential equations and [12], [30]-[31] for Smarandache systems.

§2. Algebraic Combinatorics

Algebraic systems, such as those of groups, rings, fields and modules are combinatorial themselves. However, the CC conjecture also produces notions for their development following.

Notion 2.1 *For an algebraic system $(\mathscr{A};\mathcal{O})$, determine its underlying topological structure $G^L[\mathscr{A},\mathcal{O}]$ on subsystems, and then classify by graph isomorphism.*

Notion 2.2 *For an integer $m \geq 1$, let $(\Sigma_1;\mathcal{R}_1)$, $(\Sigma_2;\mathcal{R}_2)$, $\cdots$, $(\Sigma_m;\mathcal{R}_m)$ all be algebraic systems in Definition* 1.2 *and $\left(\widetilde{\mathscr{G}};\mathcal{O}\right)$ underlying $G^L\left[\widetilde{\mathscr{G}};\mathcal{O}\right]$ with $\widetilde{\mathscr{G}}=\bigcup\limits_{i=1}^{m}\Sigma_i$ and $\mathcal{O}=\bigcup\limits_{i=1}^{m}\mathcal{R}_i$, i.e., an algebraic multisystem. Characterize $\left(\widetilde{G};\mathcal{O}\right)$ and establish algebraic theory, i.e., combinatorial algebra on $\left(\widetilde{G};\mathcal{O}\right)$.*

For example, let

$$\langle \mathscr{G}_1; \circ_1\rangle = \left\langle a, b | a \circ_1 b = b \circ_1 a, a^2 = b^n = 1\right\rangle$$
$$\langle \mathscr{G}_2; \circ_2\rangle = \left\langle b, c | b \circ_2 c = c \circ_2 b, c^5 = b^n = 1\right\rangle$$
$$\langle \mathscr{G}_3; \circ_3\rangle = \left\langle c, d | c \circ_3 d = d \circ_3 c, d^2 = c^5 = 1\right\rangle$$

be groups with respective operations $\circ_1$, $\circ_2$ and $\circ_3$. Then the set $(\widetilde{\mathscr{G}}; \{\circ_1, \circ_2, \circ_3\})$ is an algebraic multisyatem with $\widetilde{\mathscr{G}} = \bigcup\limits_{i=1}^{3} \mathscr{G}_i$.

2.1 K_2^L-Systems

A K_2^L-system is such a multi-system consisting of exactly 2 algebraic systems underlying a topological graph K_2^L, including bigroups, birings, bifields and bimodules, etc.. For example, an algebraic field $(R; +, \cdot)$ is a K_2^L-system. Clearly, $(R; +, \cdot)$ consists of groups $(R; +)$ and $(R \setminus \{0\}; \cdot)$ underlying K_2^L such as those shown in Fig.4, where $L: \ V\left(K_2^L\right) \to \{(R; +), (R \setminus \{0\}; \cdot)\}$ and $L: \ E\left(K_2^L\right) \to \{R \setminus \{0\}\}$.

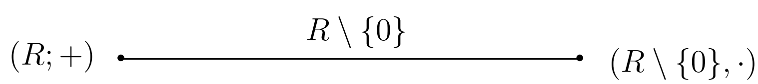


Fig.4

A generalization of field is replace $R\setminus\{0\}$ by any subset $H \leq R$ in Fig.4. Then a bigroup comes into being, which was introduced by Maggu [8] for industrial systems in 1994, and then Vasantha Kandasmy [33] further generalizes it to bialgebraic structures.

Definition 2.3 *A bigroup (biring, bifield, bimodule, $\cdots$) is a 2-system $(\mathscr{G}; \circ, \cdot)$ such that*

(1) $\mathscr{G} = \mathscr{G}_1 \bigcup \mathscr{G}_2$;

(2) $(\mathscr{G}_1; \circ)$ *and* $(\mathscr{G}_2; \cdot)$ *both are groups (rings, fields, modules,$\cdots$).*

For example, let $\widetilde{\mathscr{P}}$ be a permutation multigroup action on $\widetilde{\Omega}$ with

$$\widetilde{\mathscr{P}} = \mathscr{P}_1 \bigcup \mathscr{P}_2 \ \text{ and } \ \widetilde{\Omega} = \{1, 2, 3, 4, 5, 6, 7, 8\} \bigcup \{1, 2, 5, 6, 9, 10, 11, 12\},$$

where $\mathscr{P}_1 = \langle (1, 2, 3, 4), (5, 6, 7, 8)\rangle$ and $\mathscr{P}_2 = \langle (1, 5, 9, 10), (2, 6, 11, 12)\rangle$. Clearly, $\widetilde{\mathscr{P}}$ is a permutation bigroup.

Let $(\mathscr{G}_1;\circ_1,\cdot_1)$ and $((\mathscr{G}_2;\circ_2,\cdot_2))$ be bigroups. A mapping pair (ϕ,ι) with $\phi:\mathscr{G}_1\rightarrow\mathscr{G}_2$ and $\iota:\{\circ_1,\cdot_1\}\rightarrow\{\circ_2,\cdot_2\}$ is a *homomorphism* if

$$\phi(a\bullet b)=\phi(a)\iota(\bullet)\phi(b)$$

for $\forall a,b\in\mathscr{G}_1$ and $\bullet\in\{\circ_1,\cdot_1\}$ provided $a\bullet b$ existing in $(\mathscr{G}_1;\circ_1,\cdot_1)$. Define the *image* $\mathrm{Im}(\phi,\iota)$ and *kernel* $\mathrm{Ker}(\phi,\iota)$ respectively by

$$\begin{aligned}\mathrm{Im}(\phi,\iota) &= \{\ \phi(g)\mid g\in\mathscr{G}_1\ \},\\ \mathrm{Ker}(\phi,\iota) &= \{\ g\in\mathscr{G}_1|\ \phi(g)=1_\bullet,\forall\bullet\in\{\circ_2,\cdot_2\}\},\end{aligned}$$

where $1_\bullet$ denotes the unit of $(\mathscr{G}_\bullet;\bullet)$ with $\mathscr{G}_\bullet$ a maximal closed subset of $\mathscr{G}$ on operation $\bullet$.

For subsets $\widetilde{H}\subset\widetilde{G}$, $O\subset\mathcal{O}$, define $(\widetilde{H};O)$ to be a submultisystem of $\left(\widetilde{G};\mathcal{O}\right)$ if $(\widetilde{H};O)$ is multisystem itself, denoted by $\left(\widetilde{H};O\right)\leq\left(\widetilde{G};\mathcal{O}\right)$, and a subbigroup $(\mathscr{H};\circ,\cdot)$ of $(\mathscr{G};\circ,\cdot)$ is *normal*, denoted by $\mathscr{H}\lhd\mathscr{G}$ if for $\forall g\in\mathscr{G}$,

$$g\bullet\mathscr{H}=\mathscr{H}\bullet g,$$

where $g\bullet\mathscr{H}=\{g\bullet h|h\in\mathscr{H}$ provided $g\bullet h$ existing$\}$ and $\mathscr{H}\bullet g=\{h\bullet g|h\in\mathscr{H}$ provided $h\bullet g$ existing$\}$ for $\forall\bullet\in\{\circ,\cdot\}$. The next result is a generalization of isomorphism theorem of group in [33].

Theorem 2.4([11]) *Let $(\phi,\iota):(\mathscr{G}_1;\{\circ_1,\cdot_1\})\rightarrow(\mathscr{G}_2;\{\circ_2,\cdot_2\})$ be a homomorphism. Then*

$$G_1/\mathrm{Ker}(\phi,\iota)\simeq\mathrm{Im}(\phi,\iota).$$

Particularly, if $(\mathscr{G}_2;\{\circ_2,\cdot_2\})$ is a group $(\mathscr{A};\circ)$, we know the corollary following.

Corollary 2.5 *Let $(\phi,\iota):(\mathscr{G};\{\circ,\cdot\})\rightarrow(\mathscr{A};\circ)$ be an epimorphism. Then*

$$\mathscr{G}_1/\mathrm{Ker}(\phi,\iota)\simeq(\mathscr{A};\circ).$$

Similarly, a bigroup $(\mathscr{G};\circ,\cdot)$ is *distributive* if

$$a\cdot(b\circ c)=a\cdot b\circ a\cdot c$$

hold for all $a,b,c\in\mathscr{G}$. Then, we know the following result.

Theorem 2.6([11]) *Let $(\mathscr{G};\circ,\cdot)$ be a distributive bigroup of order≥ 2 with $\mathscr{G}=\mathscr{A}_1\cup\mathscr{A}_2$ such that $(\mathscr{A}_1;\circ)$ and $(\mathscr{A}_2;\cdot)$ are groups. Then there must be $\mathscr{A}_1\neq\mathscr{A}_2$. consequently, if $(\mathscr{G};\circ)$ it a non-trivial group, there are no operations $\cdot\neq\circ$ on $\mathscr{G}$ such that $(\mathscr{G};\circ,\cdot)$ is a distributive bigroup.*

2.2 G^L-Systems

Definition 2.2 is easily generalized also to multigroups, i.e., consisting of m groups underlying a topological graph G^L, and similarly, define conceptions of homomorphism, submultigroup and normal submultigroup, $\cdots$ of a multigroup without any difficult.

For example, a normal submultigroup of $(\widetilde{\mathscr{G}};\widetilde{O})$ is such submutigroup $(\widetilde{\mathscr{H}};O)$ that holds

$$g\circ\widetilde{\mathscr{H}}=\widetilde{\mathscr{H}}\circ g$$

for $\forall g\in\widetilde{\mathscr{G}}$, $\forall\circ\in O$, and generalize Theorem 2.3 to the following.

Theorem 2.7([16]) *Let $(\phi,\iota):(\widetilde{\mathscr{G}}_1;\widetilde{O}_1)\to(\widetilde{\mathscr{G}}_2;\widetilde{O}_2)$ be a homomorphism. Then*

$$\widetilde{\mathscr{G}}_1/\mathrm{Ker}(\phi,\iota)\simeq\mathrm{Im}(\phi,\iota).$$

Particularly, for the transitive of multigroup action on a set $\widetilde{\Omega}$, let $\widetilde{\mathscr{P}}$ be a permutation multigroup action on $\widetilde{\Omega}$ with $\widetilde{\mathscr{P}}=\bigcup\limits_{i=1}^{m}\mathscr{P}_i,\widetilde{\Omega}=\bigcup\limits_{i=1}^{m}\Omega_i$ and for each integer $i,1\leq i\leq m$, the permutation group $\mathscr{P}_i$ acts on Ω_i, which is *globally k-transitive* for an integer $k\geq 1$ if for any two k-tuples $x_1,x_2,\cdots,x_k\in\Omega_i$ and $y_1,y_2,\cdots,y_k\in\Omega_j$, where $1\leq i,j\leq m$, there are permutations $\pi_1,\pi_2,\cdots,\pi_n$ such that

$$x_1^{\pi_1\pi_2\cdots\pi_n}=y_1,\ x_2^{\pi_1\pi_2\cdots\pi_n}=y_i,\ \cdots,\ x_k^{\pi_1\pi_2\cdots\pi_n}=y_k$$

and abbreviate the globally 1-transitive to that *globally transitive* of a permutation multigroup. The following result characterizes transitive multigroup.

Theorem 2.8([17]) *Let $\widetilde{\mathscr{P}}$ be a permutation multigroup action on $\widetilde{\Omega}$ with*

$$\widetilde{\mathscr{P}}=\bigcup_{i=1}^{m}\mathscr{P}_i\ \ \textit{and}\ \ \widetilde{\Omega}=\bigcup_{i=1}^{m}\Omega_i,$$

where, each permutation group $\mathscr{P}_i$ transitively acts on Ω_i for each integers $1\leq i\leq m$. Then $\widetilde{\mathscr{P}}$ is globally transitive on $\widetilde{\Omega}$ if and only if the graph $G^L\left[\widetilde{\Omega}\right]$ is connected.

Similarly, let $\widetilde{R} = \bigcup\limits_{i=1}^{m} R_i$ be a completed multisystem with a double operation set $\mathcal{O}\left(\widetilde{R}\right) = \mathcal{O}_1 \bigcup \mathcal{O}_2$, where $\mathcal{O}_1 = \{\ \cdot_i, 1 \leq i \leq m\}$, $\mathcal{O}_2 = \{+_i, 1 \leq i \leq m\}$. If for any integers $i, 1 \leq i \leq m$, $(R_i; +_i, \cdot_i)$ is a ring, then $\widetilde{R}$ is called a multiring, denoted by $\left(\widetilde{R}; \mathcal{O}_1 \hookrightarrow \mathcal{O}_2\right)$ and $(+_i, \cdot_i)$ a double operation for any integer i, which is *integral* if for $\forall a, b \in \widetilde{R}$ and an integer i, $1 \leq i \leq m$, $a \cdot_i b = b \cdot_i a$, $1_{\cdot_i} \neq 0_{+i}$ and $a \cdot_i b = 0_{+_i}$ implies that $a = 0_{+_i}$ or $b = 0_{+_i}$. Such a multiring $\left(\widetilde{R}; \mathcal{O}_1 \hookrightarrow \mathcal{O}_2\right)$ is called a *skew multifield* or a *multifield* if each $(R; +_i, \cdot_i)$ is a skew field or a field for integers $1 \leq i \leq m$. The next result is a generalization of finitely integral ring.

Theorem 2.9([16]) *A finitely integral multiring is a multifield.*

For multimodule, let $\mathcal{O} = \{\ +_i \mid 1 \leq i \leq m\}$, $\mathcal{O}_1 = \{\cdot_i | 1 \leq i \leq m\}$ and $\mathcal{O}_2 = \{\dot{+}_i | 1 \leq i \leq m\}$ be operation sets, $(\mathscr{M}; \mathcal{O})$ a commutative multigroup with units 0_{+_i} and $(\mathscr{R}; \mathcal{O}_1 \hookrightarrow \mathcal{O}_2)$ a multiring with a unit $1_\cdot$ for $\forall \cdot \in \mathcal{O}_1$. A pair $(\mathscr{M}; \mathcal{O})$ is said to be a *multimodule over* $(\mathscr{R}; \mathcal{O}_1 \hookrightarrow \mathcal{O}_2)$ if for any integer i, $1 \leq i \leq m$, a binary operation $\times_i : \mathscr{R} \times \mathscr{M} \to \mathscr{M}$ is defined by $a \times_i x$ for $a \in \mathscr{R}$, $x \in \mathscr{M}$ such that the conditions following

(1) $a \times_i (x +_i y) = a \times_i x +_i a \times_i y$;

(2) $(a \dot{+}_i b) \times_i x = a \times_i x +_i b \times_i x$;

(3) $(a \cdot_i b) \times_i x = a \times_i (b \times_i x)$;

(4) $1_{\cdot_i} \times_i x = x$.

hold for $\forall a, b \in \mathscr{R}$, $\forall x, y \in \mathscr{M}$, denoted by $\mathbf{Mod}(\mathscr{M}(\mathcal{O}) : \mathscr{R}(\mathcal{O}_1 \hookrightarrow \mathcal{O}_2))$. Then we know the following result for finitely multimodules.

Theorem 2.10([16]) *Let* $\mathbf{Mod}(\mathscr{M}(\mathcal{O}) : \mathscr{R}(\mathcal{O}_1 \hookrightarrow \mathcal{O}_2)) = \left\langle \widehat{S} | \mathscr{R} \right\rangle$ *be a finitely generated multimodule with* $\widehat{S} = \{u_1, u_2, \cdots, u_n\}$. *Then*

$$\mathbf{Mod}(\mathscr{M}(\mathcal{O}) : \mathscr{R}(\mathcal{O}_1 \hookrightarrow \mathcal{O}_2)) \cong \mathbf{Mod}(\mathscr{R}^{(n)} : \mathscr{R}(\mathcal{O}_1 \hookrightarrow \mathcal{O}_2)),$$

where $\mathbf{Mod}(\mathscr{R}^{(n)} : \mathscr{R}(\mathcal{O}_1 \hookrightarrow \mathcal{O}_2))$ *is a multimodule on* $\mathscr{R}^{(n)} = \{(x_1, x_2, \cdots, x_n) \mid x_i \in \mathscr{R}, 1 \leq i \leq n\}$ *with*

$$(x_1, x_2, \cdots, x_n) +_i (y_1, y_2, \cdots, y_n) = (x_1 \dot{+}_i y_1, x_2 \dot{+}_i y_2, \cdots, x_n \dot{+}_i y_n),$$
$$a \times_i (x_1, x_2, \cdots, x_n) = (a \cdot_i x_1, a \cdot_i x_2, \cdots, a \cdot_i x_n)$$

for $\forall a \in \mathscr{R}$, *integers* $1 \leq i \leq m$. *Particularly, a finitely module over a commutative ring* $(\mathscr{R}; +, \cdot)$ *generated by* n *elements is isomorphic to the module* $\mathscr{R}^n$ *over* $(\mathscr{R}; +, \cdot)$.

§3. Geometrical Combinatorics

Classical geometry, such as those of Euclidean or non-Euclidean geometry, or projective geometry are not combinatorial. Whence, the CC conjecture produces combinatorial notions for their development further, for instance the topological space shown in Fig.5 following.

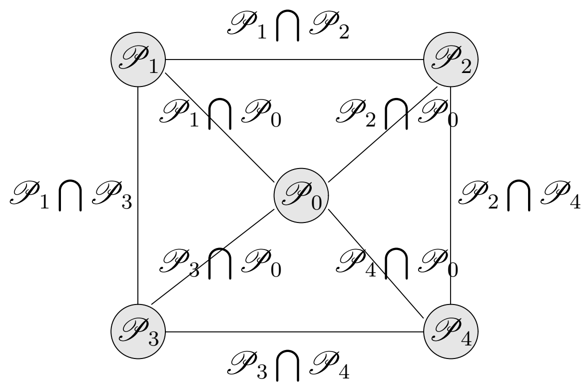


Fig.5

Notion 3.1 *For a geometrical space* $\mathscr{P}$, *determine its underlying topological structure* $G^L[\mathscr{A}, \mathcal{O}]$ *on subspaces, for instance,* n*-manifolds and classify them by graph isomorphisms.*

Notion 3.2 *For an integer* $m \geq 1$, *let* $\mathscr{P}_1, \mathscr{P}_2, \cdots, \mathscr{P}_m$ *all be geometrical spaces in Definition* 1.2 *and* $\widetilde{\mathscr{P}}$ *underlying* $G^L\left[\widetilde{\mathscr{P}}\right]$ *with* $\widetilde{\mathscr{P}} = \bigcup\limits_{i=1}^{m} \mathscr{P}_i$, *i.e., a geometrical multispace. Characterize* $\widetilde{\mathscr{P}}$ *and establish geometrical theory, i.e., combinatorial geometry on* $\widetilde{\mathscr{P}}$.

3.1 Euclidean Spaces

Let $\overline{\epsilon}_1 = (1, 0, \cdots, 0)$, $\overline{\epsilon}_2 = (0, 1, 0 \cdots, 0)$, $\cdots$, $\overline{\epsilon}_n = (0, \cdots, 0, 1)$ be the normal basis of a Euclidean space $\mathbb{R}^n$ in a general position, i.e., for two Euclidean spaces $\mathbb{R}^{n_\mu}, \mathbb{R}^{n_\nu}$, $\mathbb{R}^{n_\mu} \cap \mathbb{R}^{n_\nu} \neq \mathbb{R}^{\min\{n_\mu, n_\nu\}}$. In this case, let $\mathcal{X}_{v_\mu}$ be the set of orthogonal orientations in $\mathbb{R}^{n_{v_\mu}}$, $\mu \in \Lambda$. Then $\mathbb{R}^{n_\mu} \cap \mathbb{R}^{n_\nu} = \mathcal{X}_{v_\mu} \cap \mathcal{X}_{v_\nu}$, which enables us to construct topological spaces by the combination.

For an index set Λ, a *combinatorial Euclidean space* $\mathscr{E}_{G^L}(n_\nu; \nu \in \Lambda)$ underlying a connected graph G^L is a topological spaces consisting of Euclidean spaces $\mathbb{R}^{n_\nu}$, $\nu \in \Lambda$ such that

$V\left(G^L\right) = \{ \ \mathbb{R}^{n_\nu} \ | \ \nu \in \Lambda \ \}$;

$E\left(G^L\right) = \{ \ (\mathbb{R}^{n_\mu}, \mathbb{R}^{n_\nu}) \ | \ \mathbb{R}^{n_\mu} \cap \mathbb{R}^{n_\nu} \neq \emptyset, \mu, \nu \in \Lambda \ \}$ and labeling

$$L : \mathbb{R}^{n_\nu} \to \mathbb{R}^{n_\nu} \text{ and } L : (\mathbb{R}^{n_\mu}, \mathbb{R}^{n_\nu}) \to \mathbb{R}^{n_\mu} \bigcap \mathbb{R}^{n_\nu}$$

for $(\mathbb{R}^{n_\mu}, \mathbb{R}^{n_\nu}) \in E\left(G^L\right)$, $\nu, \mu \in \Lambda$.

Clearly, for any graph G, we are easily construct a combinatorial Euclidean space underlying G, which induces a problem following.

Problem 3.3 *Determine the dimension of a combinatorial Euclidean space consisting of m Euclidean spaces $\mathbb{R}^{n_1}, \mathbb{R}^{n_2}, \cdots, \mathbb{R}^{n_m}$.*

Generally, the combinatorial Euclidean spaces $\mathscr{E}_{G^L}(n_1, n_2, \cdots, n_m)$ are not unique and to determine $\dim\mathscr{E}_{G^L}(n_1, n_2, \cdots, n_m)$ converts to calculate the cardinality of $|X_{n_1} \cup X_{n_2} \cup \cdots \cup X_{n_m}|$, where X_{n_i} is the set of orthogonal orientations in $\mathbb{R}^{n_i}$ for integers $1 \leq i \leq m$, which can be determined by the inclusion-exclusion principle, particularly, the maximum dimension following.

Theorem 3.4([21]) $\dim\mathscr{E}_{G^L}(n_1, \cdots, n_m) \leq 1 - m + \sum\limits_{i=1}^{m} n_i$ *and with the equality holds if and only if* $\dim\left(\mathbf{R}^{n_i} \cap \mathbf{R}^{n_j}\right) = 1$ *for* $\forall\left(\mathbb{R}^{n_i}, \mathbb{R}^{n_j}\right) \in E\left(G^L\right), 1 \leq i, j \leq m$.

To determine the minimum $\dim\mathscr{E}_{G^L}(n_1, \cdots, n_m)$ is still open. However, we know this number for $G = K_m$ and $n_i = r$ for integers $1 \leq i \leq m$, i.e., $\mathscr{E}_{K_m}(r)$ by following results.

Theorem 3.5([21]) *For any integer $r \geq 2$, let $\mathscr{E}_{K_m}(r)$ be a combinatorial Euclidean space of $\underbrace{\mathbf{R}^r, \cdots, \mathbf{R}^r}_{m}$, and there exists an integer s, $0 \leq s \leq r - 1$ such that*

$$\binom{r + s - 1}{r} < m \leq \binom{r + s}{r}.$$

Then

$$\dim_{min}\mathscr{E}_{K_m}(r) = r + s.$$

Particularly,

$$\dim_{min}\mathscr{E}_{K_m}(3)=\begin{cases}3, & \text{if} \quad m=1,\\ 4, & \text{if} \quad 2\leq m\leq 4,\\ 5, & \text{if} \quad 5\leq m\leq 10,\\ 2+\lceil\sqrt{m}\rceil, & \text{if} \quad m\geq 11.\end{cases}$$

3.2 Manifolds

An n-manifold is a second countable Hausdorff space of locally Euclidean n-space without boundary, which is in fact a combinatorial Euclidean space $\mathscr{E}_{G^L}(n)$. Thus, we can further replace these Euclidean spaces by manifolds and to get topological spaces underlying a graph, such as those shown in Fig.6.

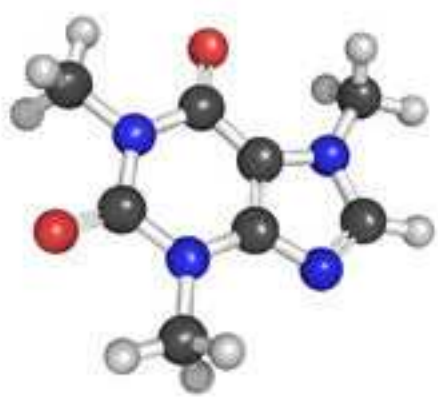


Fig.6

Definition 3.6([22]) *Let $\widetilde{M}$ be a topological space consisting of finite manifolds M_μ, $\mu\in\Lambda$. An inherent graph $G^{in}\left[\widetilde{M}\right]$ of $\widetilde{M}$ is such a graph with*

$$V\left(G^{in}\left[\widetilde{M}\right]\right)=\{M_\mu,\ \mu\in\Lambda\};$$

$$E\left(G^{in}\left[\widetilde{M}\right]\right)=\{(M_\mu,M_\nu)_i\,,1\leq i\leq\kappa_{\mu\nu}+1|M_\mu\cap M_\nu\neq\emptyset,\ \mu,\nu\in\Lambda\},$$

where $\kappa_{\mu\nu}+1$ is the number of arcwise connected components in $M_\mu\cap M_\nu$ for $\mu,\nu\in\Lambda$.

Notice that $G^{in}\left[\widetilde{M}\right]$ is a multiple graph. If replace all multiple edges $(M_\mu,M_\nu)_i$, $1\leq i\leq\kappa_{\mu\nu}+1$ by (M_μ,M_ν), such a graph is denoted by $G[\widetilde{M}]$, which is also an underlying graph of $\widetilde{M}$.

Clearly, if $m = 1$, then $\widetilde{M}(n_i,\ i \in \Lambda)$ is nothing else but exactly an n_1-manifold by definition. Even so, Notion 3.1 enables us characterizing manifolds by graphs. The following result shows that every manifold is in fact homeomorphic to combinatorial Euclidean space.

Theorem 3.7([22]) *Any locally compact n-manifold M with an alta $\mathscr{A} = \{\ (U_\lambda; \varphi_\lambda) | \lambda \in \Lambda\}$ is a combinatorial manifold $\widetilde{M}$ homeomorphic to a combinatorial Euclidean space $\mathscr{E}_{G^L}(n, \lambda \in \Lambda)$ with countable graphs $G^{in}[M] \cong G$.*

Topologically, a Euclidean space $\mathbb{R}^n$ is homeomorphic to an opened ball $\mathbb{B}^n(R) = \{(x_1, x_2, \cdots, x_n) | x_1^2 + x_2^2 + \cdots + x_n^2 < R\}$. Thus, we can view a combinatorial Euclidean space $\mathscr{E}_G(n, \lambda \in \Lambda)$ as a graph with vertices and edges replaced by ball $\mathbb{B}^n(R)$ in space, such as those shown in Fig.6, a 3-dimensional graph.

Definition 3.8 *An n-dimensional graph $\widetilde{M}^n[G]$ is a combinatorial ball space $\widetilde{B}$ of $B^n,\ \mu \in \Lambda$ underlying a combinatorial structure G such that*

(1) *$V(G)$ is discrete consisting of B^n, i.e., $\forall v \in V(G)$ is an open ball B_v^n;*

(2) *$\widetilde{M}^n[G] \setminus V(\widetilde{M}^n[G])$ is a disjoint union of open subsets $e_1, e_2, \cdots, e_m$, each of which is homeomorphic to an open ball B^n;*

(3) *the boundary $\overline{e}_i - e_i$ of e_i consists of one or two B^n and each pair $(\overline{e}_i, e_i)$ is homeomorphic to the pair $(\overline{B}^n, B^n)$;*

(4) *a subset $A \subset \widetilde{M}^n[G]$ is open if and only if $A \cap \overline{e}_i$ is open for $1 \le i \le m$.*

Particularly, a topological graph $\mathscr{T}[G]$ of a graph G embedded in a topological space $\mathscr{P}$ is 1-dimensional graph.

According to Theorem 3.7, an n-manifold is homeomorphic to a combinatorial Euclidean space, i.e., n-dimensional graph. This enables us knowing a result following on manifolds.

Theorem 3.9([22]) *Let $\mathscr{A}[M] = \{\ (U_\lambda; \varphi_\lambda) \mid \lambda \in \Lambda\}$ be a atlas of a locally compact n-manifold M. Then the labeled graph $G^L_{|\Lambda|}$ of M is a topological invariant on $|\Lambda|$, i.e., if $H^{L_1}_{|\Lambda|}$ and $G^{L_2}_{|\Lambda|}$ are two labeled n-dimensional graphs of M, then there exists a self-homeomorphism $h : M \to M$ such that $h : H^{L_1}_{|\Lambda|} \to G^{L_2}_{|\Lambda|}$ naturally induces an isomorphism of graph.*

Theorem 3.9 enables us listing manifolds by two parameters, the dimensions and inherited graph. For example, let $|\Lambda| = 2$ and then $\mathscr{A}_{min}[M] = \{(U_1;\varphi_1),\ (U_2;\varphi_2)\}$, i.e., M is double covered underlying a graphs $D^L_{0,\kappa_{12}+1,0}$ shown in Fig.7,

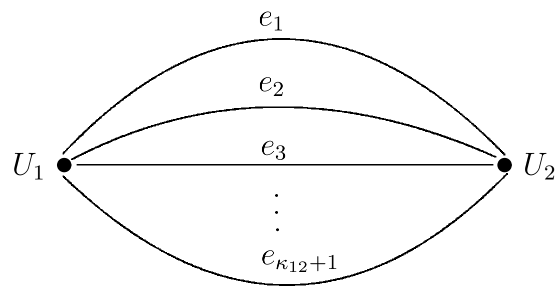


Fig.7

For example, let $U_1 = \mathbb{R}^2$, $\varphi_1 = z$, $U_2 = (\mathbb{R}^2 \setminus \{(0,0)\} \cup \{\infty\}$, $\varphi_2 = 1/z$ and $\kappa_{12} = 0$. Then the 2-manifold is nothing else but the Riemannian sphere.

The G^L-structure on combinatorial manifold $\widetilde{M}$ can be also applied for characterizing a few topological invariants, such as those fundamental groups, for instance the conclusion following.

Theorem 3.10([23]) *For* $\forall(M_1, M_2) \in E\left(G^L\left[\widetilde{M}\right]\right)$, *if* $M_1 \cap M_2$ *is simply connected, then*

$$\pi_1\left(\widetilde{M}\right) \cong \left(\bigotimes_{M \in V\left(G\left[\widetilde{M}\right]\right)} \pi_1(M)\right) \bigotimes \pi_1\left(G^{in}\left[\widetilde{M}\right]\right).$$

Particularly, for a compact n-manifold M with charts $\{(U_\lambda, \varphi_\lambda)|\ \varphi_\lambda : U_\lambda \to \mathbf{R}^n, \lambda \in \Lambda\}$, if $U_\mu \cap U_\nu$ is simply connected for $\forall \mu, \nu \in \Lambda$, then

$$\pi_1(M) \cong \pi_1\left(G^{in}[M]\right).$$

3.3 Algebraic Geometry

The topological group, particularly, Lie group is a typical example of K_2^L-systems that of algebra with geometry. Generally, let

$$AX = (b_1, b_2, \cdots, b_m)^T \qquad (LEq)$$

be a linear equation system with

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} \quad \text{and} \quad X = \begin{bmatrix} x_1 \\ x_2 \\ \cdots \\ x_n \end{bmatrix}$$

for integers $m,\ n \geq 1$, and all equations in (LEq) are non-trivial, i.e., there are no numbers λ such that $(a_{i1}, a_{i2}, \cdots, a_{in}, b_i) = \lambda(a_{j1}, a_{j2}, \cdots, a_{jn}, b_j)$ for any integers $1 \leq i, j \leq m$.

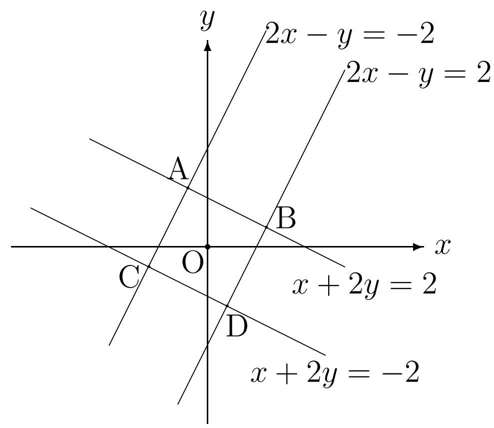


Fig.8

It should be noted that the geometry of a linear equation in n variables is a plane in $\mathbb{R}^n$. Whence, a linear system (LEA) is non-solvable or not dependent on their intersection is empty or not. For example, the linear system shown in Fig.8 is non-solvable because their intersection is empty.

Definition 3.11 *For any integers $1 \leq i, j \leq m,\ i \neq j$, the linear equations*

$$a_{i1}x_1 + a_{i2}x_2 + \cdots a_{in}x_n = b_i,$$
$$a_{j1}x_1 + a_{j2}x_2 + \cdots a_{jn}x_n = b_j$$

are called parallel if there no solution $x_1, x_2, \cdots, x_n$ hold both with the 2 equations.

Define a graph $G^L[LEq]$ on linear system (LEq) following:

$V\left(G^L[LEq]\right) = \{$ the solution space S_i of ith equation $|1 \leq i \leq m\}$,

$E\left(G^L[Eq]\right) = \{\ (S_i, S_j) \mid S_i \bigcap S_j \neq \emptyset,\ 1 \leq i, j \leq m\}$ and with labels

$$L : S_i \to S_i \text{ and } L : (S_i, S_j) \to S_i \bigcap S_j$$

for $\forall S_i \in V\left(G^L[LEq]\right)$, $(S_i, S_j) \in E\left(G^L[LEq]\right)$. For example, the system of equations shown in Fig.8 is

$$\begin{cases} x + 2y &= & 2 \\ x + 2y &= & -2 \\ 2x - y &= & -2 \\ 2x - y &= & 2 \end{cases}$$

and C_4^L is its underlying graph $G^L[LEq]$ shown in Fig.9.

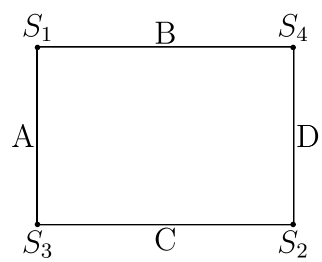


Fig.9

Let L_i be the ith linear equation. By definition we divide these equations $L_i,\ 1 \leq i \leq m$ into *parallel families*

$$\mathscr{C}_1, \mathscr{C}_2, \cdots, \mathscr{C}_s$$

by the property that all equations in a family $\mathscr{C}_i$ are parallel and there are no other equations parallel to lines in $\mathscr{C}_i$ for integers $1 \leq i \leq s$. Denoted by $|\mathscr{C}_i| = n_i,\ 1 \leq i \leq s$. Then, we can characterize $G^L[LEq]$ following.

Theorem 3.12([24]) *Let (LEq) be a linear equation system for integers $m, n \geq 1$. Then*

$$G^L[LEq] \simeq K^L_{n_1, n_2, \cdots, n_s}$$

with $n_1 + n + 2 + \cdots + n_s = m$, where $\mathscr{C}_i$ is the parallel family with $n_i = |\mathscr{C}_i|$ for integers $1 \leq i \leq s$ in (LEq) and (LEq) is non-solvable if $s \geq 2$.

Notice that this result is not sufficient, i.e., even if $G^L[LEq] \simeq K_{n_1, n_2, \cdots, n_s}$, we can not claim that (LEq) is solvable or not. How ever, if $n = 2$, we can get a necessary and sufficient condition on non-solvable linear equations.

Let H be a planar graph with each edge a straight segment on $\mathbb{R}^2$. Its *c-line graph* $L_C(H)$ is defined by

$V(L_C(H)) = \{\text{straight lines } L = e_1e_2\cdots e_l, s \geq 1 \text{ in } H\}$;

$E(L_C(H)) = \{(L_1, L_2)|\text{ if } e_i^1 \text{ and } e_j^2 \text{ are adjacent in } H\ for\ L_1 = e_1^1e_2^1\cdots e_l^1,\ L_2 = e_1^2e_2^2\cdots e_s^2,\ l, s \geq 1\}$.

Theorem 3.13([24]) *A linear equation system $(LEq2)$ is non-solvable if and only if $G^L[LEq2] \simeq L_C(H))$, where H is a planar graph of order $|H| \geq 2$ on $\mathbb{R}^2$ with each edge a straight segment*

Similarly, let

$$P_1(\overline{x}), P_2(\overline{x}), \cdots, P_m(\overline{x}) \qquad (ES_m^{n+1})$$

be m homogeneous polynomials in $n+1$ variables with coefficients in $\mathbf{C}$ and each equation $P_i(\overline{x}) = 0$ determine a hypersurface $M_i,\ 1 \leq i \leq m$ in $\mathbb{R}^{n+1}$, particularly, a curve C_i if $n = 2$. We introduce the parallel property following.

Definition 3.14 *Let $P(\overline{x}), Q(\overline{x})$ be two complex homogenous polynomials of degree d in $n+1$ variables and $I(P,Q)$ the set of intersection points of $P(\overline{x})$ with $Q(\overline{x})$. They are said to be parallel, denoted by $P \parallel Q$ if $d \geq 1$ and there are constants $a, b, \cdots, c$ (not all zero) such that for $\forall \overline{x} \in I(P,Q)$, $ax_1 + bx_2 + \cdots + cx_{n+1} = 0$, i.e., all intersections of $P(\overline{x})$ with $Q(\overline{x})$ appear at a hyperplane on $\mathbb{P}^n\mathbf{C}$, or $d = 1$ with all intersections at the infinite $x_{n+1} = 0$. Otherwise, $P(\overline{x})$ are not parallel to $Q(\overline{x})$, denoted by $P \nparallel Q$.*

Define a topological graph $G^L\left[ES_m^{n+1}\right]$ in $\mathbb{C}^{n+1}$ by

$V\left(G^L\left[ES_m^{n+1}\right]\right) = \{P_1(\overline{x}), P_2(\overline{x}), \cdots, P_m(\overline{x})\}$;

$E\left(G^L\left[ES_m^{n+1}\right]\right) = \{(P_i(\overline{x}), P_j(\overline{x}))|P_i \nparallel P_j,\ 1 \leq i, j \leq m\}$

with a labeling

$$L:\ P_i(\overline{x}) \to P_i(\overline{x}), \qquad (P_i(\overline{x}), P_j(\overline{x})) \to I(P_i, P_j),$$

where $1 \leq i \neq j \leq m$, and the topological graph of $G^L\left[ES_m^{n+1}\right]$ without labels is denoted by $G\left[ES_m^{n+1}\right]$. The following result generalizes Theorem 3.12 to homogenous polynomials.

Theorem 3.15([26]) *Let $n \geq 2$ be an integer. For a system (ES_m^{n+1}) of homogenous* *polynomials with a parallel maximal classification* $\mathscr{C}_1, \mathscr{C}_2, \cdots, \mathscr{C}_l$,

$$G[ES_m^{n+1}] \leq K(\mathscr{C}_1, \mathscr{C}_2, \cdots, \mathscr{C}_l)$$

and with equality holds if and only if $P_i \parallel P_j$ *and* $P_s \nparallel P_i$ *implies that* $P_s \nparallel P_j$*, where* $K(\mathscr{C}_1, \mathscr{C}_2, \cdots, \mathscr{C}_l)$ *denotes a complete* l*-partite graphs*

Conversely, for any subgraph $G \leq K(\mathscr{C}_1, \mathscr{C}_2, \cdots, \mathscr{C}_l)$*, there are systems* (ES_m^{n+1}) *of homogenous polynomials with a parallel maximal classification* $\mathscr{C}_1, \mathscr{C}_2, \cdots, \mathscr{C}_l$ *such that*

$$G \simeq G[ES_m^{n+1}].$$

Particularly, if $n = 2$, i.e., an (ES_m^3) system, we get the following necessary and sufficient condition.

Theorem 3.16([26]) *Let* G^L *be a topological graph labeled with* $I(e)$ *for* $\forall e \in E\left(G^L\right)$*. Then there is a system* (ES_m^3) *of homogenous polynomials such that* $G^L\left[ES_m^3\right] \simeq G^L$ *if and only if there are homogenous polynomials* $P_{v_i}(x, y, z), 1 \leq i \leq \rho(v)$ *for* $\forall v \in V\left(G^L\right)$ *such that*

$$I(e) = I\left(\prod_{i=1}^{\rho(u)} P_{u_i}, \prod_{i=1}^{\rho(v)} P_{v_i}\right)$$

for $e = (u, v) \in E\left(G^L\right)$*, where* $\rho(v)$ *denotes the valency of vertex* v *in* G^L.

These G^L-system of homogenous polynomials enables us to get combinatorial manifolds, for instance, the following result appeared in [26].

Theorem 3.17 *Let* (ES_m^{n+1}) *be a* G^L*-system consisting of homogenous polynomials* $P(\overline{x}_1), P(\overline{x}_2), \cdots, P(\overline{x}_m)$ *in* $n+1$ *variables with respectively hypersurfaces* $S_1, S_2, \cdots, S_m$*. Then there is a combinatorial manifold* $\widetilde{M}$ *in* $\mathbb{C}^{n+1}$ *such that* $\pi : \widetilde{M} \rightarrow \widetilde{S}$ *is* $1-1$ *with* $G^L\left[\widetilde{M}\right] \simeq G^L\left[\widetilde{S}\right]$*, where,* $\widetilde{S} = \bigcup\limits_{i=1}^{m} S_i$.

Particularly, if $n = 2$, we can further determine the genus of surface $g\left(\widetilde{S}\right)$ by closed formula as follows.

Theorem 3.18([26]) *Let* $C_1, C_2, \cdots, C_m$ *be complex curves determined by homogenous polynomials* $P_1(x, y, z), P_2(x, y, z), \cdots, P_m(x, y, z)$ *without common component,* *and let*

$$R_{P_i,P_j} = \prod_{k=1}^{\deg(P_i)\deg(P_j)} \left(c_k^{ij} z - b_k^{ij} y\right)^{e_k^{ij}}, \qquad \omega_{i,j} = \sum_{k=1}^{\deg(P_i)\deg(P_j)} \sum_{e_k^{ij}\neq 0} 1$$

be the resultant of $P_i(x,y,z), P_j(x,y,z)$ *for* $1 \leq i \neq j \leq m$. *Then there is an orientable surface* $\widetilde{S}$ *in* $\mathbb{R}^3$ *of genus*

$$\begin{aligned} g\left(\widetilde{S}\right) &= \beta\left(G\left\langle \widetilde{C}\right\rangle\right) + \sum_{i=1}^{m}\left(\frac{(\deg(P_i)-1)(\deg(P_i)-2)}{2} - \sum_{p^i\in \mathrm{Sing}(C_i)} \delta(p^i)\right) \\ &\quad + \sum_{1\leq i\neq j\leq m}(\omega_{i,j}-1) + \sum_{i\geq 3}(-1)^i \sum_{C_{k_1}\bigcap\cdots\bigcap C_{k_i}\neq\emptyset}\left[c\left(C_{k_1}\bigcap\cdots\bigcap C_{k_i}\right)-1\right] \end{aligned}$$

with a homeomorphism $\varphi:\ \widetilde{S} \to \widetilde{C} = \bigcup\limits_{i=1}^{m} C_i$. *Furthermore, if* $C_1, C_2, \cdots, C_m$ *are non-singular, then*

$$\begin{aligned} g\left(\widetilde{S}\right) &= \beta\left(G\left\langle \widetilde{C}\right\rangle\right) + \sum_{i=1}^{m}\frac{(\deg(P_i)-1)(\deg(P_i)-2)}{2} \\ &\quad + \sum_{1\leq i\neq j\leq m}(\omega_{i,j}-1) + \sum_{i\geq 3}(-1)^i \sum_{C_{k_1}\bigcap\cdots\bigcap C_{k_i}\neq\emptyset}\left[c\left(C_{k_1}\bigcap\cdots\bigcap C_{k_i}\right)-1\right], \end{aligned}$$

where

$$\delta(p^i) = \frac{1}{2}\left(I_{p^i}\left(P_i, \frac{\partial P_i}{\partial y}\right) - \nu_\phi(p^i) + |\pi^{-1}(p^i)|\right)$$

is a positive integer with a ramification index $\nu_\phi(p^i)$ *for* $p^i \in \mathrm{Sing}(C_i), 1 \leq i \leq m$.

Theorem 3.17 enables us to find interesting results in projective geometry, for instance the following result.

Corollary 3.19 *Let* $C_1, C_2, \cdots, C_m$ *be complex non-singular curves determined by homogenous polynomials* $P_1(x,y,z), P_2(x,y,z), \cdots, P_m(x,y,z)$ *without common component and* $C_i\bigcap C_j = \bigcap\limits_{i=1}^{m} C_i$ *with* $\left|\bigcap\limits_{i=1}^{m} C_i\right| = \kappa > 0$ *for integers* $1 \leq i \neq j \leq m$. *Then the genus of normalization* $\widetilde{S}$ *of curves* $C_1, C_2, \cdots, C_m$ *is*

$$g(\widetilde{S}) = g(\widetilde{S}) = (\kappa-1)(m-1) + \sum_{i=1}^{m}\frac{(\deg(P_i)-1)(\deg(P_i)-2)}{2}.$$

Particularly, if $C_1, C_2, \cdots, C_m$ *are distinct lines in* $\mathbb{P}^2\mathbf{C}$ *with respective normalizations of spheres* $S_1, S_2, \cdots, S_m$. *Then there is a normalization of surface* $\widetilde{S}$ *of*

$C_1, C_2, \cdots, C_m$ with genus $\beta\left(G\left\langle\widetilde{L}\right\rangle\right)$. Furthermore, if $G\left\langle\widetilde{L}\right\rangle)$ is a tree, then $\widetilde{S}$ is homeomorphic to a sphere.

3.4 Combinatorial Geometry

Furthermore, we can establish combinatorial geometry by Notion 3.2. For example, we have 3 classical geometries, i.e., *Euclidean, hyperbolic geometry* and *Riemannian geometries* for describing behaviors of objects in spaces with different axioms following:

Euclid Geometry:

(A1) *There is a straight line between any two points.*

(A2) *A finite straight line can produce a infinite straight line continuously.*

(A3) *Any point and a distance can describe a circle.*

(A4) *All right angles are equal to one another.*

(A5) *If a straight line falling on two straight lines make the interior angles on the same side less than two right angles, then the two straight lines, if produced indefinitely, meet on that side on which are the angles less than the two right angles.*

Hyperbolic Geometry:

Axioms $(A1)-(A4)$ and the axiom $(L5)$ following:

(L5) *there are infinitely many lines parallel to a given line passing through an exterior point.*

Riemannian Geometry:

Axioms $(A1)-(A4)$ and the axiom $(R5)$ following:

there is no parallel to a given line passing through an exterior point.

Then whether there is a geometry established by combining the 3 geometries, i.e., partially Euclidean and partially hyperbolic or Riemannian. Today, we have know such theory really exists, called *Smarandache geometry* defined following.

Definition 3.20([12]) *An axiom is said to be Smarandachely denied if the axiom behaves in at least two different ways within the same space, i.e., validated and invalided, or only invalided but in multiple distinct ways.*

A Smarandache geometry is a geometry which has at least one Smarandachely denied axiom (1969).

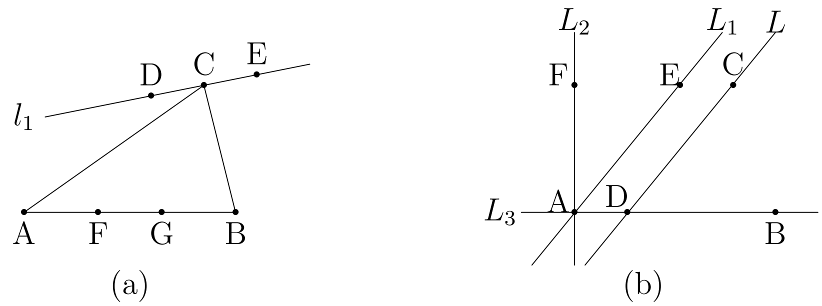


Fig.10

For example, let us consider a Euclidean plane $\mathbb{R}^2$ and three non-collinear points A, B and C shown in Fig.10. Define s-points as all usual Euclidean points on $\mathbb{R}^2$ and s-lines any Euclidean line that passes through one and only one of points A, B and C. Then such a geometry is a Smarandache geometry by the following observations.

Observation 1. The axiom (E1) that through any two distinct points there exist one line passing through them is now replaced by: *one s-line* and *no s-line*. Notice that through any two distinct s-points D, E collinear with one of A, B and C, there is one s-line passing through them and through any two distinct s-points F, G lying on AB or non-collinear with one of A, B and C, there is no s-line passing through them such as those shown in Fig.10(a).

Observation 2. The axiom (E5) that through a point exterior to a given line there is only one parallel passing through it is now replaced by two statements: *one parallel* and *no parallel*. Let L be an s-line passes through C and D on AE, and AE is parallel to CD in the Euclidean sense. Then there is one and only one line passing through E which is parallel to L, but passing a point not on AE, for instance, point F there are no lines parallel to L such as those shown in Fig.10(b).

Generally, we can construct a Smarandache geometry on smoothly combinatorial manifolds $\widetilde{M}$, i.e., combinatorial geometry because it is homeomorphic to combinatorial Euclidean space $\mathscr{E}_{G^L}(n_1, n_2, \cdots, n_m)$ by Definition 3.6 and Theorem 3.7. Such a theory is founded on the results for basis of tangent and cotangent vectors following.

Theorem 3.21([15]) *For any point $p \in \widetilde{M}(n_1, n_2, \cdots, n_m)$ with a local chart $(U_p; [\varphi_p])$, the dimension of $T_p\widetilde{M}(n_1, n_2, \cdots, n_m)$ is*

$$\dim T_p\widetilde{M}(n_1, n_2, \cdots, n_m) = \widehat{s}(p) + \sum_{i=1}^{s(p)}(n_i - \widehat{s}(p))$$

with a basis matrix $\left[\frac{\partial}{\partial \overline{x}}\right]_{s(p)\times n_{s(p)}} =$

$$\left[\begin{array}{cccccccc}
\frac{1}{s(p)}\frac{\partial}{\partial x^{11}} & \cdots & \frac{1}{s(p)}\frac{\partial}{\partial x^{1\widehat{s}(p)}} & \frac{\partial}{\partial x^{1(\widehat{s}(p)+1)}} & \cdots & \frac{\partial}{\partial x^{1n_1}} & \cdots & 0 \\
\frac{1}{s(p)}\frac{\partial}{\partial x^{21}} & \cdots & \frac{1}{s(p)}\frac{\partial}{\partial x^{2\widehat{s}(p)}} & \frac{\partial}{\partial x^{2(\widehat{s}(p)+1)}} & \cdots & \frac{\partial}{\partial x^{2n_2}} & \cdots & 0 \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & & \\
\frac{1}{s(p)}\frac{\partial}{\partial x^{s(p)1}} & \cdots & \frac{1}{s(p)}\frac{\partial}{\partial x^{s(p)\widehat{s}(p)}} & \frac{\partial}{\partial x^{s(p)(\widehat{s}(p)+1)}} & \cdots & \cdots & \frac{\partial}{\partial x^{s(p)(n_{s(p)}-1)}} & \frac{\partial}{\partial x^{s(p)n_{s(p)}}}
\end{array}\right]$$

where $x^{il} = x^{jl}$ for $1 \leq i, j \leq s(p), 1 \leq l \leq \widehat{s}(p)$, namely there is a smoothly functional matrix $[v_{ij}]_{s(p)\times n_{s(p)}}$ such that for any tangent vector $\overline{v}$ at a point p of $\widetilde{M}(n_1, n_2, \cdots, n_m)$,

$$\overline{v} = \left\langle [v_{ij}]_{s(p)\times n_{s(p)}}, [\frac{\partial}{\partial \overline{x}}]_{s(p)\times n_{s(p)}} \right\rangle,$$

where $\langle [a_{ij}]_{k\times l}, [b_{ts}]_{k\times l}\rangle = \sum_{i=1}^{k}\sum_{j=1}^{l} a_{ij}b_{ij}$, the inner product on matrixes.

Theorem 3.22([15]) *For $\forall p \in (\widetilde{M}(n_1, n_2, \cdots, n_m); \widetilde{\mathcal{A}})$ with a local chart $(U_p; [\varphi_p])$, the dimension of $T_p^*\widetilde{M}(n_1, n_2, \cdots, n_m)$ is*

$$\dim T_p^*\widetilde{M}(n_1, n_2, \cdots, n_m) = \widehat{s}(p) + \sum_{i=1}^{s(p)}(n_i - \widehat{s}(p))$$

with a basis matrix $[d\overline{x}]_{s(p)\times n_{s(p)}} =$

$$\left[\begin{array}{cccccccc}
\frac{dx^{11}}{s(p)} & \cdots & \frac{dx^{1\widehat{s}(p)}}{s(p)} & dx^{1(\widehat{s}(p)+1)} & \cdots & dx^{1n_1} & \cdots & 0 \\
\frac{dx^{21}}{s(p)} & \cdots & \frac{dx^{2\widehat{s}(p)}}{s(p)} & dx^{2(\widehat{s}(p)+1)} & \cdots & dx^{2n_2} & \cdots & 0 \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & & \\
\frac{dx^{s(p)1}}{s(p)} & \cdots & \frac{dx^{s(p)\widehat{s}(p)}}{s(p)} & dx^{s(p)(\widehat{s}(p)+1)} & \cdots & \cdots & dx^{s(p)n_{s(p)}-1} & dx^{s(p)n_{s(p)}}
\end{array}\right]$$

where $x^{il} = x^{jl}$ for $1 \leq i, j \leq s(p), 1 \leq l \leq \widehat{s}(p)$, namely for any co-tangent vector d at a point p of $\widetilde{M}(n_1, n_2, \cdots, n_m)$, there is a smoothly functional matrix $[u_{ij}]_{s(p)\times s(p)}$ such that,

$$d = \left\langle [u_{ij}]_{s(p)\times n_{s(p)}}, [d\overline{x}]_{s(p)\times n_{s(p)}} \right\rangle.$$

Then we can establish tensor theory with connections on smoothly combinatorial manifolds ([15]). For example, we can establish the curvatures on smoothly combinatorial manifolds, and get the curvature $\widetilde{R}$ formula following.

Theorem 3.23([18]) *Let $\widetilde{M}$ be a finite combinatorial manifold, $\widetilde{R} : \mathscr{X}(\widetilde{M}) \times \mathscr{X}(\widetilde{M}) \times \mathscr{X}(\widetilde{M}) \times \mathscr{X}(\widetilde{M}) \to C^\infty(\widetilde{M})$ a curvature on $\widetilde{M}$. Then for $\forall p \in \widetilde{M}$ with a local chart $(U_p; [\varphi_p])$,*

$$\widetilde{R} = \widetilde{R}_{(\sigma\varsigma)(\eta\theta)(\mu\nu)(\kappa\lambda)} dx^{\sigma\varsigma} \otimes dx^{\eta\theta} \otimes dx^{\mu\nu} \otimes dx^{\kappa\lambda},$$

where

$$\begin{aligned}\widetilde{R}_{(\sigma\varsigma)(\eta\theta)(\mu\nu)(\kappa\lambda)} &= \frac{1}{2}\left(\frac{\partial^2 g_{(\mu\nu)(\sigma\varsigma)}}{\partial x^{\kappa\lambda}\partial x^{\eta\theta}} + \frac{\partial^2 g_{(\kappa\lambda)(\eta\theta)}}{\partial x^{\mu\nu\nu}\partial x^{\sigma\varsigma}} - \frac{\partial^2 g_{(\mu\nu)(\eta\theta)}}{\partial x^{\kappa\lambda}\partial x^{\sigma\varsigma}} - \frac{\partial^2 g_{(\kappa\lambda)(\sigma\varsigma)}}{\partial x^{\mu\nu}\partial x^{\eta\theta}}\right) \\ &+ \Gamma^{\vartheta\iota}_{(\mu\nu)(\sigma\varsigma)}\Gamma^{\xi o}_{(\kappa\lambda)(\eta\theta)} g_{(\xi o)(\vartheta\iota)} - \Gamma^{\xi o}_{(\mu\nu)(\eta\theta)}\Gamma_{(\kappa\lambda)(\sigma\varsigma)}{}^{\vartheta\iota} g_{(\xi o)(\vartheta\iota)},\end{aligned}$$

and $g_{(\mu\nu)(\kappa\lambda)} = g(\frac{\partial}{\partial x^{\mu\nu}}, \frac{\partial}{\partial x^{\kappa\lambda}})$.

This enables us to characterize the combination of classical fields, such as the Einstein's gravitational fields and other fields on combinatorial spacetimes and hold their behaviors (See [19]-[20] for details).

§4. Differential Equation's Combinatorics

Let

$$(Eq_m) \quad \begin{cases} f_1(x_1, x_2, \cdots, x_{n+1}) = 0 \\ f_2(x_1, x_2, \cdots, x_{n+1}) = 0 \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ f_m(x_1, x_2, \cdots, x_{n+1}) = 0 \end{cases}$$

be a system of equations. It should be noted that the classical theory on equations is not combinatorics. However, the solutions of an equation usually form a manifold in the view of geometry. Thus, the CC conjecture bring us combinatorial notions for developing equation theory similar to that of geometry further.

Notion 4.1 *For a system (Eq_m) of equations, solvable or non-solvable, determine its underlying topological structure $G^L[Eq_m]$ on each solution manifold and classify them by graph isomorphisms and transformations.*

Notion 4.2 *For an integer $m \geq 1$, let $\mathscr{D}_1$, $\mathscr{D}_2$, $\cdots$, $\mathscr{D}_m$ be the solution manifolds of an equation system (Eq_m) in Definition 1.2 and $\widetilde{\mathscr{D}}$ underlying $G^L\left[\widetilde{\mathscr{D}}\right]$ with $\widetilde{\mathscr{D}} = \bigcup\limits_{i=1}^{m} \mathscr{D}_i$, i.e., a combinatorial solution manifold. Characterize the system (Eq_m) and establish an equation theory, i.e., equation's combinatorics on (Eq_m).*

Geometrically, let

$$S_{f_i} = \{(x_1, x_2, \cdots, x_{n+1})|f_i(x_1, x_2, \cdots, x_{n+1}) = 0\} \subset \mathbb{R}^{n+1}$$

the solution-manifold in $\mathbb{R}^{n+1}$ for integers $1 \leq i \leq m$, where f_i is a function hold with conditions of the implicit function theorem for $1 \leq i \leq m$. Then we are easily finding criterions on the solubility of system (ES_m), i.e., it is solvable or not dependent on

$$\bigcap_{i=1}^{m} S_{f_i} \neq \emptyset \quad \text{or} \quad = \emptyset.$$

Whence, if the intersection is empty, i.e., (ES_m) is non-solvable, there are no meanings in classical theory on equations, but it is important for hold the global behaviors of a complex thing. For such an objective, Notions 4.1 and 4.2 are helpful.

Let us begin at a linear differential equations system such as those of

$$\dot{X} = A_1X, \cdots, \dot{X} = A_kX, \cdots, \dot{X} = A_mX \qquad (LDES_m^1)$$

or

$$\begin{cases} x^{(n)} + a_{11}^{[0]}x^{(n-1)} + \cdots + a_{1n}^{[0]}x = 0 \\ x^{(n)} + a_{21}^{[0]}x^{(n-1)} + \cdots + a_{2n}^{[0]}x = 0 \\ \cdots\cdots\cdots\cdots \\ x^{(n)} + a_{m1}^{[0]}x^{(n-1)} + \cdots + a_{mn}^{[0]}x = 0 \end{cases} \qquad (LDE_m^n)$$

with

$$A_k = \begin{bmatrix} a_{11}^{[k]} & a_{12}^{[k]} & \cdots & a_{1n}^{[k]} \\ a_{21}^{[k]} & a_{22}^{[k]} & \cdots & a_{2n}^{[k]} \\ \cdots & \cdots & \cdots & \cdots \\ a_{n1}^{[k]} & a_{n2}^{[k]} & \cdots & a_{nn}^{[k]} \end{bmatrix} \quad \text{and} \quad X = \begin{bmatrix} x_1(t) \\ x_2(t) \\ \cdots \\ x_n(t) \end{bmatrix}$$

where each $a_{ij}^{[k]}$ is a real number for integers $0 \leq k \leq m,\ 1 \leq i, j \leq n$.

For example, let (LDE_6^2) be the following linear homogeneous differential equation system

$$\begin{cases} \ddot{x}+3\dot{x}+2x=0 & (1)\\ \ddot{x}+5\dot{x}+6x=0 & (2)\\ \ddot{x}+7\dot{x}+12x=0 & (3)\\ \ddot{x}+9\dot{x}+20x=0 & (4)\\ \ddot{x}+11\dot{x}+30x=0 & (5)\\ \ddot{x}+7\dot{x}+6x=0 & (6) \end{cases}$$

Certainly, it is non-solvable. However, we can easily solve equations (1)-(6) one by one and get their solution spaces as follows:

$$S_1=\left\langle e^{-t},\ e^{-2t}\right\rangle=\{C_1e^{-t}+\ C_2e^{-2t}|C_1,C_2\in\mathbb{R}\}=\{x|\ddot{x}+3\dot{x}+2x=0\}$$
$$S_2=\left\langle e^{-2t},e^{-3t}\right\rangle=\{C_1e^{-2t}+C_2e^{-3t}|C_1,C_2\in\mathbb{R}\}=\{x|\ddot{x}+5\dot{x}+6x=0\}$$
$$S_3=\left\langle e^{-3t},e^{-4t}\right\rangle=\{C_1e^{-3t}+C_2e^{-4t}|C_1,C_2\in\mathbb{R}\}=\{x|\ddot{x}+7\dot{x}+12x=0\}$$
$$S_4=\left\langle e^{-4t},e^{-5t}\right\rangle=\{C_1e^{-4t}+C_2e^{-5t}|C_1,C_2\in\mathbb{R}\}=\{x|\ddot{x}+9\dot{x}+20x=0\}$$
$$S_5=\left\langle e^{-5t},e^{-6t}\right\rangle=\{C_1e^{-5t}+C_2e^{-6t}|C_1,C_2\in\mathbb{R}\}=\{x|\ddot{x}+11\dot{x}+30x=0\}$$
$$S_6=\left\langle e^{-6t},\ e^{-t}\right\rangle=\{C_1e^{-6t}+\ C_2e^{-t}|C_1,C_2\in\mathbb{R}\}=\{x|\ddot{x}+7\dot{x}+6x=0\}$$

Replacing each Σ_i by solution space S_i in Definition 1.2, we get a topological graph $G^L[LDE_6^2]$ shown in Fig.11 on the linear homogeneous differential equation system (LDE_6^2). Thus we can solve a system of linear homogeneous differential equations on its underlying graph G^L, no matter it is solvable or not in the classical sense.

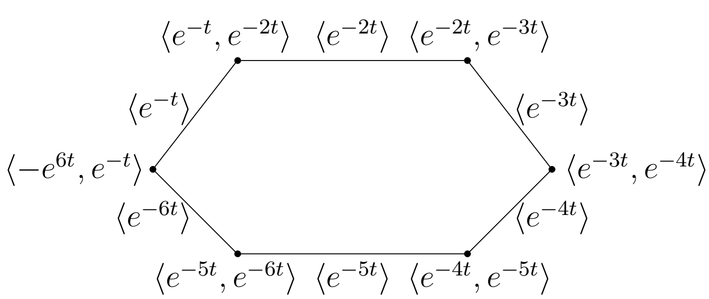


Fig.11

Generally, we know a result on G^L-solutions of linear homogenous differential equations following.

Theorem 4.3([25]) *Every linear homogeneous differential equation system* $(LDES_m^1)$ *(or* (LDE_m^n)*) has a unique* G^L*-solution, and for every* H^L *labeled with linear spaces* $\langle \overline{\beta}_i(t)e^{\alpha_i t}, 1 \le i \le n\rangle$ *on vertices such that*

$$\langle \overline{\beta}_i(t)e^{\alpha_i t}, 1 \le i \le n\rangle \bigcap \langle \overline{\beta}_j(t)e^{\alpha_j t}, 1 \le j \le n\rangle \ne \emptyset$$

if and only if there is an edge whose end vertices labeled by $\langle \overline{\beta}_i(t)e^{\alpha_i t}, 1 \le i \le n\rangle$ *and* $\langle \overline{\beta}_j(t)e^{\alpha_j t}, 1 \le j \le n\rangle$ *respectively, then there is a unique linear homogeneous differential equation system* $(LDES_m^1)$ *(or* (LDE_m^n)*) with* G^L*-solution* H^L*, where* α_i *is a* k_i*-fold zero of the characteristic equation,* $k_1 + k_2 + \cdots + k_s = n$ *and* $\overline{\beta}_i(t)$ *is a polynomial in* t *with degree*$\le k_i - 1$*.*

Applying G^L-solution of $(LDES_m^1)$ (or (LDE_m^n)), we can classify such systems by graph isomorphisms of graphs .

Definition 4.4 *A vertex-edge labeling* $\theta: G \to \mathbb{Z}^+$ *is said to be integral if* $\theta(uv) \le \min\{\theta(u), \theta(v)\}$ *for* $\forall uv \in E(G)$*, denoted by* G^{I_θ}*, and two integral labeled graphs* $G_1^{I_\theta}$ *and* $G_2^{I_\tau}$ *are called identical if* $G_1 \overset{\varphi}{\simeq} G_2$ *and* $\theta(x) = \tau(\varphi(x))$ *for any graph isomorphism* φ *and* $\forall x \in V(G_1) \bigcup E(G_1)$*, denoted by* $G_1^{I_\theta} = G_2^{I_\tau}$*.*

For example, $G_1^{I_\theta} = G_2^{I_\tau}$ but $G_1^{I_\theta} \ne G_3^{I_\sigma}$ for integral graphs shown in Fig.12.

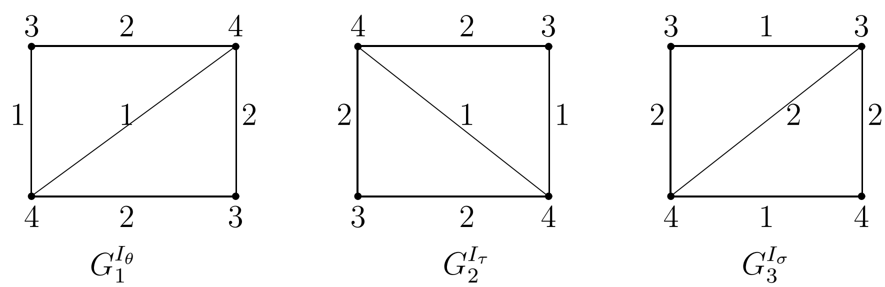


Fig.12

The following result classifies the systems $(LDES_m^1)$ and (LDE_m^n) by graphs.

Theorem 4.5([25]) *Let* $(LDES_m^1)$*,* $(LDES_m^1)'$ *(or* (LDE_m^n)*,* $(LDE_m^n)'$*) be two linear homogeneous differential equation systems with integral labeled graphs* H*,* H'*. Then* $(LDES_m^1) \overset{\varphi}{\simeq} (LDES_m^1)'$ *(or* $(LDE_m^n) \overset{\varphi}{\simeq} (LDE_m^n)'$*) if and only if* $H = H'$*.*

For partial differential equations, let

$$\begin{cases} F_1(x_1, x_2, \cdots, x_n, u, u_{x_1}, \cdots, u_{x_n}) = 0 \\ F_2(x_1, x_2, \cdots, x_n, u, u_{x_1}, \cdots, u_{x_n}) = 0 \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ F_m(x_1, x_2, \cdots, x_n, u, u_{x_1}, \cdots, u_{x_n}) = 0 \end{cases} \qquad (PDES_m)$$

be such a system of first order on a function $u(x_1, \cdots, x_n, t)$ with continuous $F_i : \mathbf{R}^n \to \mathbf{R}^n$ such that $F_i(\overline{0}) = \overline{0}$.

Definition 4.6 *The symbol of $(PDES_m)$ is determined by*

$$\begin{cases} F_1(x_1, x_2, \cdots, x_n, u, p_1, \cdots, p_n) = 0 \\ F_2(x_1, x_2, \cdots, x_n, u, p_1, \cdots, p_n) = 0 \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ F_m(x_1, x_2, \cdots, x_n, u, p_1, \cdots, p_n) = 0, \end{cases}$$

i.e., substitutes $u_{x_1}, u_{x_2}, \cdots, u_{x_n}$ by $p_1, p_2, \cdots, p_n$ in $(PDES_m)$, and it is algebraically contradictory if its symbol is non-solvable. Otherwise, differentially contradictory.

For example, the system of partial differential equations following

$$\begin{cases} (z-y)u_x + (x-z)u_y + (y-x)u_z = 0 \\ zu_x + xu_y + yu_z = x^2 + y^2 + z^2 + 1 \\ yu_x + zu_y + xu_z = x^2 + y^2 + z^2 + 4 \end{cases}$$

is algebraically contradictory because its symbol

$$\begin{cases} (z-y)p_1 + (x-z)p_2 + (y-x)p_3 = 0 \\ zp_1 + xp_2 + yp_3 = x^2 + y^2 + z^2 + 1 \\ yp_1 + zp_2 + xp_3 = x^2 + y^2 + z^2 + 4 \end{cases}$$

is contradictory. Generally, we know a result for Cauchy problem on non-solvable systems of partial differential equations of first order following.

Theorem 4.7([28]) *A Cauchy problem on systems*

$$\begin{cases} F_1(x_1, x_2, \cdots, x_n, u, p_1, p_2, \cdots, p_n) = 0 \\ F_2(x_1, x_2, \cdots, x_n, u, p_1, p_2, \cdots, p_n) = 0 \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ F_m(x_1, x_2, \cdots, x_n, u, p_1, p_2, \cdots, p_n) = 0 \end{cases}$$

of partial differential equations of first order is non-solvable with initial values

$$\begin{cases} x_i|_{x_n=x_n^0} = x_i^0(s_1, s_2, \cdots, s_{n-1}) \\ u|_{x_n=x_n^0} = u_0(s_1, s_2, \cdots, s_{n-1}) \\ p_i|_{x_n=x_n^0} = p_i^0(s_1, s_2, \cdots, s_{n-1}), \quad i = 1, 2, \cdots, n \end{cases}$$

if and only if the system

$$F_k(x_1, x_2, \cdots, x_n, u, p_1, p_2, \cdots, p_n) = 0, \ 1 \leq k \leq m$$

is algebraically contradictory, in this case, there must be an integer $k_0, \ 1 \leq k_0 \leq m$ *such that*

$$F_{k_0}(x_1^0, x_2^0, \cdots, x_{n-1}^0, x_n^0, u_0, p_1^0, p_2^0, \cdots, p_n^0) \neq 0$$

or it is differentially contradictory itself, i.e., there is an integer $j_0, \ 1 \leq j_0 \leq n-1$ *such that*

$$\frac{\partial u_0}{\partial s_{j_0}} - \sum_{i=0}^{n-1} p_i^0 \frac{\partial x_i^0}{\partial s_{j_0}} \neq 0.$$

According to Theorem 4.7, we know conditions for uniquely G^L-solution of Cauchy problem on system of partial differential equations of first order.

Theorem 4.8([28]) *A Cauchy problem on system* $(PDES_m)$ *of partial differential equations of first order with initial values* $x_i^{[k0]}, u_0^{[k]}, p_i^{[k0]}, \ 1 \leq i \leq n$ *for the* k*th equation in* $(PDES_m)$*,* $1 \leq k \leq m$ *such that*

$$\frac{\partial u_0^{[k]}}{\partial s_j} - \sum_{i=0}^{n} p_i^{[k0]} \frac{\partial x_i^{[k0]}}{\partial s_j} = 0$$

is uniquely G^L*-solvable, i.e.,* $G^L[PDES]$ *is uniquely determined.*

Applying the G^L-solution of a G^L-system (DES_m) of differential equations, the global stability, i.e, sum-stable or prod-stable of (DES_m) can be introduced. For example, the sum-stability of (DES_m) is defined following.

Definition 4.9 *Let* (DES_m^C) *be a Cauchy problem on a system of differential equations in* $\mathbb{R}^n$*,* $H^L \leq G^L\left[DES_m^C\right]$ *a spanning subgraph, and* $u^{[v]}$ *the solution of the* v*th equation with initial value* $u_0^{[v]}$*,* $v \in V\left(H^L\right)$*. It is sum-stable on the subgraph* H^L *if* *for any number $\varepsilon > 0$ there exists, $\delta_v > 0,\ v \in V\left(H^L\right)$ such that each $G^L(t)$-solution with*

$$\left|{u'}_0^{[v]} - u_0^{[v]}\right| < \delta_v, \quad \forall v \in V\left(H^L\right)$$

exists for all $t \geq 0$ and with the inequality

$$\left|\sum_{v \in V(H^L)} {u'}^{[v]} - \sum_{v \in V(H^L)} u^{[v]}\right| < \varepsilon$$

holds, denoted by $G^L[t] \overset{H}{\sim} G^L[0]$ and $G^L[t] \overset{\Sigma}{\sim} G^L[0]$ if $H^L = G^L\left[DES_m^C\right]$. Furthermore, if there exists a number $\beta_v > 0,\ v \in V\left(H^L\right)$ such that every $G^{L'}[t]$-solution with

$$\left|{u'}_0^{[v]} - u_0^{[v]}\right| < \beta_v, \quad \forall v \in V\left(H^L\right)$$

satisfies

$$\lim_{t \to \infty}\left|\sum_{v \in V(H)} {u'}^{[v]} - \sum_{v \in V(H^L)} u^{[v]}\right| = 0,$$

then the $G^L[t]$-solution is called asymptotically stable, denoted by $G^L[t] \overset{H}{\to} G^L[0]$ and $G^L[t] \overset{\Sigma}{\to} G^L[0]$ if $H^L = G^L\left[DES_m^C\right]$.

For example, let the system $(SDES_m^C)$ be

$$\left.\begin{array}{l} \dfrac{\partial u}{\partial t} = H_i(t, x_1, \cdots, x_{n-1}, p_1, \cdots, p_{n-1}) \\ u|_{t=t_0} = u_0^{[i]}(x_1, x_2, \cdots, x_{n-1}) \end{array}\right\} \quad 1 \leq i \leq m \qquad \left(SDES_m^C\right)$$

and a point $X_0^{[i]} = (t_0, x_{10}^{[i]}, \cdots, x_{(n-1)0}^{[i]})$ with $H_i(t_0, x_{10}^{[i]}, \cdots, x_{(n-1)0}^{[i]}) = 0$ for an integer $1 \leq i \leq m$ is *equilibrium* of the ith equation in $(SDES_m^C)$. A result on the sum-stability of $(SDES_m^C)$ is obtained in [30] following.

Theorem 4.10([28]) *Let $X_0^{[i]}$ be an equilibrium point of the ith equation in $(SDES_m^C)$ for each integer $1 \leq i \leq m$. If*

$$\sum_{i=1}^{m} H_i(X) > 0 \ \ and \ \ \sum_{i=1}^{m} \frac{\partial H_i}{\partial t} \leq 0$$

for $X \neq \sum\limits_{i=1}^{m} X_0^{[i]}$, then the system $(SDES_m^C)$ is sum-stability, i.e., $G^L[t] \overset{\Sigma}{\sim} G^L[0]$.

Furthermore, if

$$\sum_{i=1}^{m} \frac{\partial H_i}{\partial t} < 0$$

for $X \neq \sum\limits_{i=1}^{m} X_0^{[i]}$, *then* $G^L[t] \xrightarrow{\Sigma} G^L[0]$.

§5. Field's Combinatorics

The modern physics characterizes particles by fields, such as those of scalar field, Maxwell field, Weyl field, Dirac field, Yang-Mills field, Einstein gravitational field, $\cdots$, etc., which are in fact spacetime in geometry, isolated but non-combinatorics. Whence, the CC conjecture can bring us a combinatorial notion for developing field theory further, which enables us understanding the world and discussed extensively in the first edition of [13] in 2009, and references [18]-[20].

Notion 5.1 *Characterize the geometrical structure, particularly, determine its underlying topological* $G^L[\mathscr{D}]$ *of spacetime* $\mathscr{D}$ *on all fields appeared in theoretical physics.*

Notice that the essence of Notion 5.1 is to characterize the geometrical spaces of particles. Whence, it is in fact equivalent to Notion 3.1.

Notion 5.2 *For an integer* $m \geq 1$, *let* $\mathscr{D}_1, \mathscr{D}_2, \cdots, \mathscr{D}_m$ *be spacetimes in Definition* 1.2 *and* $\widetilde{\mathscr{L}}$ *underlying* $G^L\left[\widetilde{\mathscr{D}}\right]$ *with* $\widetilde{\mathscr{D}} = \bigcup\limits_{i=1}^{m} \mathscr{D}_i$, *i.e., a combinatorial spacetime. Select suitable Lagrangian or Hamiltonian density* $\widetilde{\mathscr{L}}$ *to determine field equation of* $\widetilde{\mathscr{D}}$, *hold with the principle of covariance and characterize its global behaviors.*

There are indeed such fields, for instance the gravitational waves in Fig.13.

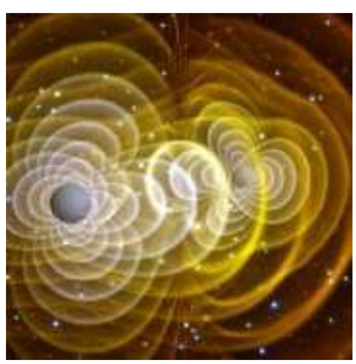


Fig.13

A combinatorial field $\widetilde{\mathscr{D}}$ is a combination of fields underlying a topological graph G^L with actions between fields. For this objective, a natural way is to characterize each field C_i, $1 \leq i \leq n$ of them by itself reference frame $\{\overline{x}\}$. Whence, the principles following are indispensable.

Action Principle of Fields. *There are always exist an action $\overrightarrow{A}$ between two fields C_1 and C_2 of a combinatorial field if* $\dim(C_1 \cap C_2) \geq 1$, *which can be found at any point on a spatial direction in their intersection.*

Thus, a combinatorial field $\widetilde{\mathscr{D}}$ depends its underlying graph $G^L\left[\widetilde{\mathscr{D}}\right]$, such as those shown in Fig.14.

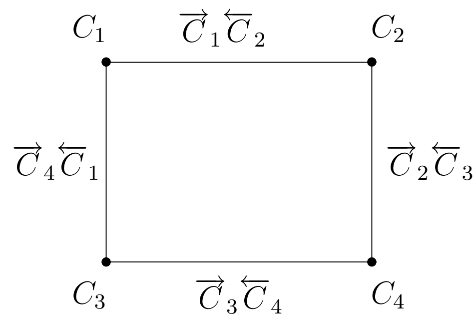


Fig.14

For understanding the world by combinatorial fields, the *anthropic principle*, i.e., *the born of human beings is not accidental but inevitable in the world* will applicable, which implies the generalized principle of covariance following.

Generalized Principle of Covariance([20]) *A physics law in a combinatorial field is invariant under all transformations on its coordinates, and all projections on its a subfield.*

Then, we can construct the Lagrangian density $\widetilde{\mathscr{L}}$ and find the field equations of combinatorial field $\widetilde{\mathscr{D}}$, which are divided into two cases ([13], first edition).

Case 1. *Linear*

In this case, the expression of the Lagrange density $\mathscr{L}_{G^L[\widetilde{\mathscr{D}}]}$ is

$$\mathscr{L}_{G^L[\widetilde{\mathscr{D}}]} = \sum_{i=1}^{n} a_i \mathscr{L}_{\mathscr{D}_i} + \sum_{(\mathscr{D}_i,\mathscr{D}_j)\in E\left(G^L[\widetilde{\mathscr{D}}]\right)} b_{ij}\mathscr{T}_{ij} ,$$

where $a_i,\ b_{ij}$ are coupling constants determined only by experiments.

Case 2. *Non-Linear*

In this case, the Lagrange density $\mathscr{L}_{G^L\left[\widetilde{\mathscr{D}}\right]}$ is a non-linear function on $\mathscr{L}_{\mathscr{D}_i}$ and $\mathscr{T}_{ij}$ for $1 \leq i,j \leq n$. Let the minimum and maximum indexes j for $(M_i, M_j) \in E\left(G^L\left[\widetilde{\mathscr{D}}\right]\right)$ are i^l and i^u, respectively. Denote by

$$\overline{x} = (x_1, x_2, \cdots) = (\mathscr{L}_{\mathscr{D}_1}, \mathscr{L}_{\mathscr{D}_2}, \cdots, \mathscr{L}_{\mathscr{D}_n}, \mathscr{T}_{11^l}, \cdots, \mathscr{T}_{11^u}, \cdots, \mathscr{T}_{22^l}, \cdots,).$$

If $\mathscr{L}_{G^L\left[\widetilde{\mathscr{D}}\right]}$ is $k+1$ differentiable, $k \geq 0$ by Taylor's formula we know that

$$\begin{aligned}
\mathscr{L}_{G^L\left[\widetilde{\mathscr{D}}\right]} &= \mathscr{L}_{G^L\left[\widetilde{\mathscr{D}}\right]}(\overline{0}) + \sum_{i=1}^{n}\left[\frac{\partial \mathscr{L}_{G^L\left[\widetilde{\mathscr{D}}\right]}}{\partial x_i}\right]_{x_i=0} x_i + \frac{1}{2!}\sum_{i,j=1}^{n}\left[\frac{\partial^2 \mathscr{L}_{G^L\left[\widetilde{\mathscr{D}}\right]}}{\partial x_i \partial x_j}\right]_{x_i,x_j=0} x_i x_j \\
&+ \cdots + \frac{1}{k!}\sum_{i_1,i_2,\cdots,i_k=1}^{n}\left[\frac{\partial^k \mathscr{L}_{G^L\left[\widetilde{\mathscr{D}}\right]}}{\partial x_{i_1}\partial x_{i_2}\cdots\partial x_{i_k}}\right]_{x_{i_j}=0, 1\leq j\leq k} x_{i_1}x_{i_2}\cdots x_{i_k} \\
&+ R(x_1, x_2, \cdots),
\end{aligned}$$

where

$$\lim_{\|\overline{x}\|\to 0}\frac{R(x_1, x_2, \cdots)}{\|\overline{x}\|} = 0,$$

and choose the first s terms

$$\begin{aligned}
&\mathscr{L}_{G^L\left[\widetilde{\mathscr{D}}\right]}(\overline{0}) + \sum_{i=1}^{n}\left[\frac{\partial \mathscr{L}_{G^L\left[\widetilde{\mathscr{D}}\right]}}{\partial x_i}\right]_{x_i=0} x_i + \frac{1}{2!}\sum_{i,j=1}^{n}\left[\frac{\partial^2 \mathscr{L}_{G^L\left[\widetilde{\mathscr{D}}\right]}}{\partial x_i \partial x_j}\right]_{x_i,x_j=0} x_i x_j \\
&+ \cdots + \frac{1}{k!}\sum_{i_1,i_2,\cdots,i_k=1}^{n}\left[\frac{\partial^k \mathscr{L}_{G^L\left[\widetilde{\mathscr{D}}\right]}}{\partial x_{i_1}\partial x_{i_2}\cdots\partial x_{i_k}}\right]_{x_{i_j}=0, 1\leq j\leq k} x_{i_1}x_{i_2}\cdots x_{i_k}
\end{aligned}$$

to be the asymptotic value of Lagrange density $\mathscr{L}_{G^L\left[\widetilde{\mathscr{D}}\right]}$, particularly, the linear parts

$$\mathscr{L}_{G^L\left[\widetilde{\mathscr{D}}\right]}(\overline{0}) + \sum_{i=1}^{n}\left[\frac{\partial \mathscr{L}_{G^L\left[\widetilde{\mathscr{D}}\right]}}{\partial \mathscr{L}_{\mathscr{D}_i}}\right]_{\mathscr{L}_{\mathscr{D}_i}=0} \mathscr{L}_{M_i} + \sum_{(M_i,M_j)\in E(G^L[\widetilde{M}])}\left[\frac{\partial \mathscr{L}_{G^L\left[\widetilde{\mathscr{D}}\right]}}{\partial \mathscr{T}_{ij}}\right]_{\mathscr{T}_{ij}=0} \mathscr{T}_{ij}.$$

Notice that such a Lagrange density maybe intersects. We need to consider Lagrange density without intersections. For example,

$$\mathscr{L}_{G^L\left[\widetilde{\mathscr{D}}\right]} = \sum_{i=1}^{4}\mathscr{L}^2_{C_i} - \sum_{i=1}^{4}\mathscr{L}_{\overrightarrow{C}_i\overleftarrow{C}_{i+1}}$$

for the combinatorial field shown in Fig.14.

Then, apply the Euler-Lagrange equations, i.e.,

$$\partial_\mu \frac{\partial \mathscr{L}_{G^L[\widetilde{\mathscr{D}}]}}{\partial \partial_\mu \phi_{\widetilde{\mathscr{D}}}} - \frac{\partial \mathscr{L}_{G^L[\widetilde{\mathscr{D}}]}}{\partial \phi_{\widetilde{\mathscr{D}}}} = 0,$$

where $\phi_{\widetilde{\mathscr{D}}}$ is the wave function of combinatorial field $\widetilde{\mathscr{D}}(t)$, we are easily find the equations of combinatorial field $\widetilde{\mathscr{D}}$.

For example, for a combinatorial scalar field $\phi_{\widetilde{\mathscr{D}}}$, without loss of generality let

$$\phi_{\widetilde{\mathscr{D}}} = \sum_{i=1}^{n} c_i \phi_{\mathscr{D}_i}$$

$$\mathscr{L}_{G^L[\widetilde{\mathscr{D}}]} = \frac{1}{2}\sum_{i=1}^{n}(\partial_{\mu_i}\phi_{\mathscr{D}_i}\partial^{\mu_i}\phi_{\mathscr{D}_i} - m_i^2\phi_{\mathscr{D}_i}^2) + \sum_{(\mathscr{D}_i,\mathscr{D}_j)\in E\left(G^L[\widetilde{\mathscr{D}}]\right)} b_{ij}\phi_{\mathscr{D}_i}\phi_{\mathscr{D}_j},$$

i.e., linear case

$$\mathscr{L}_{G^L[\widetilde{\mathscr{D}}]} = \sum_{i=1}^{n} \mathscr{L}_{\mathscr{D}_i} + \sum_{(\mathscr{D}_i,\mathscr{D}_j)\in E\left(G^L[\widetilde{\mathscr{D}}]\right)} b_{ij}\mathscr{T}_{ij}$$

with $\mathscr{L}_{\mathscr{D}_i} = \frac{1}{2}(\partial_{\mu_i}\phi_{\mathscr{D}_i}\partial^{\mu_i}\phi_{\mathscr{D}_i} - m_i^2\phi_{\mathscr{D}_i}^2)$, $\mathscr{T}_{ij} = \phi_{\mathscr{D}_i}\phi_{\mathscr{D}_j}$, $\mu_i = \mu_{\mathscr{D}_i}$ and constants b_{ij}, m_i, c_i for integers $1 \leq i, j \leq n$. Then the equation of combinatorial scalar field is

$$\sum_{i=1}^{n} \frac{1}{c_i}(\partial_\mu \partial^{\mu_i} + m_i^2)\phi_{M_i} - \sum_{(M_i,M_j)\in E(G^L[\widetilde{M}])} b_{ij}\left(\frac{\phi_{M_j}}{c_i} + \frac{\phi_{M_i}}{c_j}\right) = 0.$$

Similarly, we can determine the equations on combinatorial Maxwell field, Weyl field, Dirac field, Yang-Mills field and Einstein gravitational field in theory. For more such conclusions, the reader is refers to references [13], [18]-[20] in details.

Notice that the *string theory* even if arguing endlessly by physicist, it is in fact a combinatorial field $\mathbb{R}^4 \times \mathbb{R}^7$ under supersymmetries. Recently, there is a claim on the *Theory of Everything* with equations ([6], [7]) by letting the Lagrangian density

$$\mathscr{L} = \mathscr{L}_{EH} + \mathscr{L}_{EM} + \mathscr{L}_{W} + \mathscr{L}_{QCD},$$

where $\mathscr{L}_{EH}, \mathscr{L}_{EM}, \mathscr{L}_{W}$ and $\mathscr{L}_{QCD}$ are respectively the gravitational field, electromagnetic field, weak and strong nuclear fields. However, it is a little ambiguous because $\mathscr{L}_{EH}$ is on gravitation, a field on real bodies, but $\mathscr{L}_{EM}, \mathscr{L}_{W}$ and $\mathscr{L}_{QCD}$

on state field of particles, not real ones. Even so, Notions 5.1 and 5.2 produce developing space for physics, merely with examining by experiment.

§6. **Conclusions**

The role of CC conjecture to mathematical sciences has been shown in previous sections by examples of results. Actually, it is a mathematical machinery of philosophical notion: *there always exist universal connection between things* $\mathscr{T}$ with a disguise $G^L[\mathscr{T}]$ on connections, which enables us converting a mathematical system with contradictions to a compatible one ([27]), and opens thoroughly new ways for developing mathematical sciences.

However, *is a topological graph an element of a mathematical system with measures, not only viewed as a geometrical figure*? The answer is YES! Recently, the author introduces $\overrightarrow{G}$-flow in [29], i.e., an oriented graph $\overrightarrow{G}$ embedded in a topological space $\mathscr{S}$ associated with an injective mappings $L : (u,v) \rightarrow L(u,v) \in \mathscr{V}$ such that $L(u,v) = -L(v,u)$ for $\forall (u,v) \in X\left(\overrightarrow{G}\right)$ holding with conservation laws

$$\sum_{u \in N_G(v)} L(v,u) = \mathbf{0} \ \text{ for } \ \forall v \in V\left(\overrightarrow{G}\right),$$

where V is a Banach space over a field $\mathscr{F}$ and showed all these $\overrightarrow{G}$-flows $\overrightarrow{G}^{\mathscr{V}}$ form a Banach space by defining

$$\left\|\overrightarrow{G}^L\right\| = \sum_{(u,v)\in X\left(\overrightarrow{G}\right)} \|L(u,v)\|$$

for $\forall \overrightarrow{G}^L \in \overrightarrow{G}^{\mathscr{V}}$, and furthermore, Hilbert space by introducing inner product similarly, where $\|L(u,v)\|$ denotes the norm of $F(u^v)$ in $\mathscr{V}$, which enables us to get $\overrightarrow{G}$-flow solutions, i.e., combinatorial solutions on differential equations.

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