ON THE SMARANDACHE FUNCTION AND THE FIXED - POINT THEORY OF NUMBERS

by

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This brief note points out several basic connections between the Smarandache function, fixed-point theory [1] and prime-number theory. First recall that fixed-point theory in function spaces provides elegent, if not short, proofs of the existence of solutions to many kinds of differential equations, integral equations, optimization problems and game-theoretic problems. Further, fixed-point theory in the ring of rational integers and fixed-lattice-point theory provide many results on the existence of solutions in diophantine theory. Here are four fundamental examples of fixed-point theory in number theory. (1) There is the well-known basic result that for p>4, p is prime iff S(p) = p. (2) Recall that the present author defined [2] the number-theoretic function $\Psi(n)$ as the *product* of the primes alone in the *mosaic* of n, where the mosaic of n is obtained from n by recursively applying the unique factorization theorem/fundamental theorem of arithmetic to itself! Now the asymptotic density of fixed points of $\Psi(n)$ is $7/\pi^2$, just as the asymptotic density of square-free numbers is $6/\pi^2$. Indeed, (3) the theory of *perfect* numbers is also connected to fixed-point theory, since if one puts $f(n) = \delta(n) - n$, where $\delta(n)$ is the sum of the divisors on n, then n is perfect iff f(n) = n. Finally, (4) the present author defined [2] the number-theoretic function $\Psi^*(n)$ as the *sum* of the primes alone in the *mosaic* of n. Here we have a striking similarity to the Smarandache function itself (see example (1) above), since $\Psi^*(n) = n$ iff n = 4 or n = p for some prime p; i.e., if > 4, n is prime iff $\Psi^*(n) = n$. Thus, the distribution function for the fixed points of S(n) or of $\Psi^*(n)$ is essentially the distribution function for the primes, $\Pi(n)$.

Problems

(1) Put $S^2(n) = S(S(n))$ and define $S^m(n)$ recursively, where S(n) is the Smarandache function. (*Note*: This approach aligns Smarandache function theory more closely with recursive function theory/computer theory.) For each n, determine the *least* m for which $S^m(n)$ is prime.

(2) Prove that S(n) = S(n+3) for only finitely many n.

(3) Prove that S(n) = S(n+2) for only finitely many n.

(4) Prove that S(n) = S(n+1) for no n.

References

[1] D.R.Smart, Fixed Point Theorems, Cambridge Univ. Press (1974) 93 pp.

[2] A.A.Mullin, Models of the Fundamental Theorem of Arithmetic, *Proc. National Acad. Sciences U.S.A.* 50 (1963), 604-606.

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