Four conjectures on the Smarandache prime partial digital sequence

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Abstract. In this paper I make the following four conjectures on the *Smarandache prime-partial-digital sequence* defined as the sequence of prime numbers which admit a deconcatenation into a set of primes: (I) there exist an infinity of primes p obtained concatenating two primes m and n, both of the form 6*k + 1, such that n = m*h – h + 1 , where h positive integer; (II) there exist an infinity of primes p obtained concatenating two primes m and n, both of the form 6*k - 1, such that n = m*h + h - 1 , where h positive integer; (III) there exist an infinity of primes p obtained concatenating two primes m and n, both of the form 6*k + 1, such that n + m - 1 is prime or power of prime; (IV) there exist an infinity of primes p obtained concatenating two primes m and n, both of the form 6*k - 1, such that n - m + 1 is prime or power of prime. Note that almost all from the first 65 primes obtained concatenating two primes of the form 6k + 1 (exceptions: 3779, 4373, 6173, 6719, 6779), and all the first 65 primes obtained concatenating two primes of the form 6k - 1, belong to one of the four sequences considered by the conjectures above.

The *Smarandache prime-partial-digital sequence* (see A019549 in OEIS):
: 23, 37, 53, 73, 113, 137, 173, 193, 197, 211, 223, 227, 229, 233, 241, 257, 271, 277, 283, 293, 311, 313, 317, 331, 337, 347, 353, 359, 367, 373, 379, 383, 389, 397, 433, 523, 541, 547, 557, 571, 577, 593, 613, 617, 673, 677, 719, 727, 733, 743, 757, 761, 773, 797, 977 (...)

Conjecture 1:

There exist an infinity of primes p obtained concatenating two primes m and n, both of the form 6*k + 1, such that n = m*h – h + 1 , where h positive integer.

Note that all primes n larger than 7 of the form 6*k + 1 can be written as 7*h – h + 1, where h positive integer, so all the primes obtained concatenating a prime of the form 6*k + 1 with 7 is term of this sequence.

The sequence of primes p:

: 137, 197, 317, 617, 677, 719, 743, 761, 773, 797, 977, 1097, 1277, 1361 (61 = 13*5 – 5 + 1), 1373 (73

= 13*6 – 6 + 1), 1637, 1973 (73 = 19*4 – 4 + 1), 1997, 2237, 2297, 2417, 2777, 2837, 3167, 3677, 3719 (37 = 19*2 – 2 + 1), 3797, 4217, 4337, 4637, 5237, 5477, 5717, 5897, 6113 (61 = 13*5 – 5 + 1), 6131 (61 = 31*2 – 2 + 1), 6197, 6317, 6917, 7151, 7229, 7283, 7331, 7349 (...)

Example of larger p:

: p = 499943 where 4999 = 43*119 – 119 + 1.

Conjecture 2:

There exist an infinity of primes p obtained concatenating two primes m and n, both of the form 6*k - 1, such that n = m*h + h - 1 , where h positive integer.

Note that all primes n larger than 5 of the form 6*k - 1 can be written as 5*h + h - 1, where h positive integer, so all the primes obtained concatenating a prime of the form 6*k - 1 with 5 is term of this sequence.

The sequence of primes p:

: 541, 547, 571, 1123 (23 = 11*2 + 2 – 1), 1171 (71 = 11*6 + 6 – 1), 1753 (53 = 17*3 + 3 – 1), 1789 (89 = 17*5 + 5 – 1), 2311 (23 = 11*2 + 2 – 1), 2371 (71 = 23*3 + 3 – 1), 4723 (47 = 23*2 + 2 – 1), 5101, 5107, 5113, 5167, 5179, 5197, 5227, 5233, 5347, 5419, 5431, 5443, 5449, 5479, 5503, 5521, 5557, 5641, 5647, 5653, 5659, 5683, 5701, 5743, 5821, 5827, 5839, 5857, 5881, 5953 (...)

Conjecture 3:

There exist an infinity of primes p obtained concatenating two primes m and n, both of the form 6*k + 1, such that n + m - 1 is prime or power of prime.

The sequence of primes p:

: 137 (13 + 7 + 1 = 19), 197 (19 + 7 - 1 = 25 = 5^2), 317 (31 + 7 – 1 = 37), 617 (61 + 7 – 1 = 67), 677 (67 + 7 – 1 = 73), 719 (71 + 9 – 1 = 79), 743 (7 + 43 – 1 = 49 = 7^2), 761 (7 + 67 – 1 = 73), 773 (7 + 73 – 1 = 79), 797 (7 + 97 – 1 = 103), 977 (97 + 7 – 1 = 103), 1319 (13 + 9 – 1 = 31), 1361 (13 + 61 - 1 = 73), 1367 (13 + 67 – 1 = 79), 1637 (163 + 7 – 1 = 169 = 13^2), 1913 (19 + 13 – 1 = 31), 1931 (19 + 31 – 1 = 49 = 7^2), 1979 (19 + 79 – 1 = 97), 2237 (223 + 7 – 1 = 229), 2777 (277 + 7 – 1 = 283), 2837 (283

+ 7 – 1 = 289 = 17^2), 3119 (31 + 19 – 1 = 49 = 7^2), 3167 (31 + 67 – 1 = 97), 3677 (367 + 7 – 1 = 373), 3761 (37 + 61 – 1 = 97), 3767 (37 + 67 – 1 = 103), 4397 (43 + 97 – 1 = 139), 5237 (523 + 7 – 1 = 529 = 23^2), 5717 (571 + 7 – 1 = 577), 6113 (61 + 13 – 1 = 73), 6143 (61 + 43 – 1 = 103), 6197 (61 + 97 – 1 = 157), 6737 (67 + 37 – 1 = 103), 6761 (67 + 61 – 1 = 127), 7283 (7 + 283 – 1 = 289 = 17^2), 7331 (73 + 31 – 1 = 103 or 7 + 331 – 1 = 337), 7349 (349 – 7 + 1 = 343 = 7^3) (...)

Example of larger p:

: p = 499979 where 4999 + 79 – 1 = 5077, prime.

Conjecture 4:

There exist an infinity of primes p obtained concatenating two primes m and n, both of the form 6*k - 1, such that n - m + 1 is prime or power of prime.

The sequence of primes p:

: 541 (41 – 5 + 1 = 37), 547 (47 – 5 + 1 = 43), 571 (71 – 5 + 1 = 67), 1117 (17 – 11 + 1 = 7), 1123 (23 – 11 + 1 = 13), 1129 (29 – 11 + 1 = 19), 1153 (53 – 11 + 1 = 43), 1171 (71 – 11 + 1 = 61), 1723 (23 – 17 + 1 = 7), 1741 (41 – 17 + 1 = 25 = 5^2), 1747 (47 – 17 + 1 = 31), 1753 (53 – 17 + 1 = 37), 1759 (59 – 17 + 1 = 43), 1783 (83 – 17 + 1 = 67), 1789 (89 - 17 + 1 = 73), 2311 (23 – 11 + 1 = 13), 2341 (41 – 23 + 1 = 19), 2347 (47 – 23 + 1 = 25 = 5^2), 2371 (71 – 23 + 1 = 49 = 7^2), 2383 (83 – 23 + 1 = 61), 2389 (89 – 23 + 1 = 67), 2971 (71 – 29 + 1 = 43), 4111 (41 – 11 + 1 = 31), 4129 (41 – 29 + 1 = 13), 4153 (53 – 41 + 1 = 13), 4159 (59 – 41 + 1 = 19), 4723 (47 – 23 + 1 = 25 = 5^2), 4729 (47 – 29 + 1 = 19), 4759 (59 – 47 + 1 = 13), 4783 (83 – 47 + 1 = 37), 4789 (89 – 47 + 1 = 43), 5101 (101 – 5 + 1 = 97), 5107 (107 – 5 + 1 = 103), 5113 (113 – 5 + 1 = 109), 5167 (167 – 5 + 1 = 163), 5179 (179 – 5 + 1 = 173), 5197 (197 – 5 + 1 = 193), 5227 (227 – 5 + 1 = 223), 5233 (233 – 5 + 1 = 227), 5323 (53 – 23 + 1 = 31), 5347 (53 – 47 + 1 = 7), 5647 (647 – 5 + 1 = 643), 5743 (743 – 5 + 1 = 739), 5827 (827 – 5 + 1 = 823), 5857 (857 – 5 + 1 = 853), 5881 (881 – 5 + 1 = 877), 5923 (59 – 23 + 1 = 37), 7129 (71 – 29 + 1 = 43), 7159 (71 – 59 + 1 = 13) (...)

Example of larger p:
: p = 499711 where 4997 – 11 + 1 = 4987, prime.

Note:

Almost all from the first 65 primes obtained from m = 6*x + 1, prime, concatenated with n = 6*y + 1, prime (exceptions: 3779, 4373, 6173, 6719, 6779), and all the first 65 primes obtained from m = 6*x - 1, prime, concatenated with n = 6*y - 1, prime, belong to one of the 4 sequences considered by the conjectures above.

Note:

Up to the number 7349 there are 65 primes obtained concatenated two primes of the form 6*k + 1 and 65 primes obtained concatenated two primes of the form 6*k – 1!