Seventeen sequences of Poulet numbers characterized by a certain set of Smarandache-Coman divisors

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Abstract. In a previous article I defined the Smarandache-Coman divisors of order k of a composite integer n with m prime factors and I sketched some possible applications of this concept in the study of Fermat pseudoprimes. In this paper I make few conjectures about few possible infinite sequences of Poulet numbers, characterized by a certain set of Smarandache-Coman divisors.

Conjecture 1:

There is an infinity of 2-Poulet numbers which have the set of SC divisors of order 1 equal to {p, p}, where p is prime.

The sequence of this 2-Poulet numbers is:
341, 2047, 3277, 5461, 8321, 13747, 14491, 19951, 31417, ...
(see the lists below)

Conjecture 2:

There is an infinity of 2-Poulet numbers which have the set of SC divisors of order 2 equal to {p, p + 20*k}, where p is prime and k is non-null integer.

The sequence of this 2-Poulet numbers is:
4033, 4681, 10261, 15709, 23377, 31609, ...
(see the lists below)

Conjecture 3:

There is an infinity of 2-Poulet numbers which have the set of SC divisors of order 2 equal to {a, b}, where a + b + 1 is prime.

The sequence of this 2-Poulet numbers is:
1387, 2047, 2701, 3277, 4369, 4681, 8321, 13747, 14491, 18721, 31417, 31609, ...
(see the lists below)

Note: This is the case of twelve from the first twenty 2-Poulet numbers.

Conjecture 4:

There is an infinity of 2-Poulet numbers which have the set of SC divisors of order 2 equal to {a, b}, where a + b - 1 is prime.

The sequence of this 2-Poulet numbers is:
4033, 8321, 10261, 13747, 14491, 15709, 19951, 23377, 31417, ...
(see the lists below)

Conjecture 5:

There is an infinity of 2-Poulet numbers which have the set of SC divisors of order 2 equal to {a, b}, where a + b - 1 and a + b + 1 are twin primes.

The sequence of this 2-Poulet numbers is:
13747, 14491, 23377, 31417, ...
(see the lists below)

Conjecture 6:

There is an infinity of pairs of 2-Poulet numbers which have the set of SC divisors of order 2 equal to {a, b}, respectively to {c, d}, where a + b = c + d and a, b, c, d are primes.

Such pair of 2-Poulet numbers is:
(4681, 7957), because 29 + 149 = 71 + 107 = 178.

Conjecture 7:

There is an infinity of pairs of 2-Poulet numbers which have the set of SC divisors of order 2 equal to {a, b}, respectively to {c, d}, where a + b + 1 = c + d – 1.

Such pairs of 2-Poulet numbers are:
(3277, 8321), because 9 + 37 + 1 = 17 + 31 - 1 = 47;
(19951, 5461), because 23 + 31 + 1 = 41 + 15 – 1 = 55.

Conjecture 8:

There is an infinity of 2-Poulet numbers which have the set of SC divisors of order 6 equal to {p, q}, where abs{p – q} = 6*k, where p and q are primes and k is non-null positive integer.

The sequence of this 2-Poulet numbers is:
1387, 2047, 2701, 3277, 4033, 4369, 7957, 13747, 14491, 15709, 23377, 31417, 31609, ...
(see the lists below)

Note: This is the case of thirteen from the first twenty 2-Poulet numbers.

Conjecture 9:

There is an infinity of 2-Poulet numbers which have the set of SC divisors of order 6 equal to {a, b}, where abs{a – b} = p and p is prime.

The sequence of this 2-Poulet numbers is:
341, 4681, 10261, ...
(see the lists below)

Conjecture 10:

There is an infinity of 2-Poulet numbers which have the set of SC divisors of order 6 equal to {p, q}, where one from the numbers p and q is prime and the other one is twice a prime.

The sequence of this 2-Poulet numbers is:
341, 4681, 5461, 10261, ...
(see the lists below)

Conjecture 11:

There is an infinity of 3-Poulet numbers which have the set of SC divisors of order 1 equal to {a, b, c}, where a + b + c is prime and a, b, c are primes.

The sequence of this 2-Poulet numbers is:
561, 645, 1729, 1905, 2465, 6601, 8481, 8911, 10585, 12801, 13741, ...
(see the lists below)

Note: This is the case of eleven from the first twenty 2-Poulet numbers.

Conjecture 12:

There is an infinity of 3-Poulet numbers which have the set of SC divisors of order 1 equal to {a, b, c}, where a + b + c – 1 and a + b + c + 1 are twin primes.

The sequence of this 3-Poulet numbers is:
2821, 4371, 16705, 25761, 30121, ...
(see the lists below)

Conjecture 13:

There is an infinity of 3-Poulet numbers which have the set of SC divisors of order 1 equal to {n, n, n}.

Such 3-Poulet number is 13981.

Conjecture 14:

There is an infinity of 3-Poulet numbers which have the set of SC divisors of order 2 equal to {5, p, q}, where p and q are primes and q = p + 6*k, where k is non-null positive integer.

Such 3-Poulet numbers are:
1729, because $SCD_2(1729)$ = {5, 11, 17} and 17 = 11 + 6*1;
2821, because $SCD_2(2821)$ = {5, 11, 29} and 29 = 11 + 6*3;
6601, because $SCD_2(6601)$ = {5, 7, 13} and 13 = 7 + 6*1;
13741, because $SCD_2(13741)$ = {5, 11, 149} and 149 = 11 + 6*23;
15841, because $SCD_2(15841)$ = {5, 29, 71} and 71 = 29 + 6*7;
30121, because $SCD_2(30121)$ = {5, 11, 329} and 329 = 11 + 6*53.

Conjecture 15:

There is an infinity of Poulet numbers divisible by 15 which have the set of SC divisors of order 1 equal to {2, 4, 7, n_1, ..., n_i}, where n_1, ..., n_i are non-null positive integers and i > 0.

The sequence of this 3-Poulet numbers is:
18705, 55245, 72855, 215265, 831405, 1246785, ...
(see the lists below)

Conjecture 16:

There is an infinity of Poulet numbers divisible by 15 which have the set of SC divisors of order 1 equal to {2, 4, 23, n_1, ..., n_i}, where n_1, ..., n_i are non-null positive integers and i > 0.

The sequence of this 3-Poulet numbers is:
62745, 451905, ...
(see the lists below)

Conjecture 17:

There is an infinity of Poulet numbers which are multiples of any Poulet number divisible by 15 which has the set of SC divisors of order 1 equal to {2, 4, n_1, ..., n_i}, where $n_1 = n_2 =...= n_i = 7$ and i > 0.

Examples:
The Poulet number 645 = 3*5*43, having $SCD_1(645)$ = {2, 4, 7}, has the multiples the Poulet numbers 18705, 72885, which have SCD_1 = {2, 4, 7, 7}.
The Poulet number 1905 = 3*5*127, having $SCD_1(1905)$ = {2, 4, 7}, has the multiples 55245, 215265 which have SCD_1 = {2, 4, 7, 7}.

(see the sequence A215150 in OEIS for a list of Poulet numbers divisible by smaller Poulet numbers)

List of SC divisors of order 1 of the first twenty 2-Poulet numbers:
(see the sequence A214305 that I submitted to OEIS for a list of 2-Poulet numbers)

$SCD_1(341) = \{S(11 - 1), S(31 - 1)\} = \{S(10), S(30)\} = \{5, 5\}$;
$SCD_1(1387) = \{S(19 - 1), S(73 - 1)\} = \{S(18), S(72)\} = \{6, 6\}$;
$SCD_1(2047) = \{S(23 - 1), S(89 - 1)\} = \{S(22), S(88)\} = \{11, 11\}$;
$SCD_1(2701) = \{S(37 - 1), S(73 - 1)\} = \{S(36), S(72)\} = \{6, 6\}$;
$SCD_1(3277) = \{S(29 - 1), S(113 - 1)\} = \{S(28), S(112)\} = \{7, 7\}$;
$SCD_1(4033) = \{S(37 - 1), S(109 - 1)\} = \{S(36), S(108)\} = \{6, 9\}$;
$SCD_1(4369) = \{S(17 - 1), S(257 - 1)\} = \{S(16), S(256)\} = \{6, 10\}$;
$SCD_1(4681) = \{S(31 - 1), S(151 - 1)\} = \{S(30), S(150)\} = \{5, 10\}$;
$SCD_1(5461) = \{S(43 - 1), S(127 - 1)\} = \{S(42), S(126)\} = \{7, 7\}$;
$SCD_1(7957) = \{S(73 - 1), S(109 - 1)\} = \{S(72), S(108)\} = \{6, 9\}$;
$SCD_1(8321) = \{S(53 - 1), S(157 - 1)\} = \{S(52), S(156)\} = \{13, 13\}$;
$SCD_1(10261) = \{S(31 - 1), S(331 - 1)\} = \{S(30), S(330)\} = \{5, 11\}$;
$SCD_1(13747) = \{S(59 - 1), S(233 - 1)\} = \{S(58), S(232)\} = \{29, 29\}$;
$SCD_1(14491) = \{S(43 - 1), S(337 - 1)\} = \{S(42), S(336)\} = \{7, 7\}$;
$SCD_1(15709) = \{S(23 - 1), S(683 - 1)\} = \{S(22), S(682)\} = \{11, 31\}$;
$SCD_1(18721) = \{S(97 - 1), S(193 - 1)\} = \{S(96), S(192)\} = \{8, 8\}$;
$SCD_1(19951) = \{S(71 - 1), S(281 - 1)\} = \{S(70), S(280)\} = \{7, 7\}$;
$SCD_1(23377) = \{S(97 - 1), S(241 - 1)\} = \{S(96), S(240)\} = \{8, 6\}$;
$SCD_1(31417) = \{S(89 - 1), S(353 - 1)\} = \{S(88), S(352)\} = \{11, 11\}$;
$SCD_1(31609) = \{S(73 - 1), S(433 - 1)\} = \{S(72), S(432)\} = \{6, 9\}$.

List of SC divisors of order 2 of the first twenty 2-Poulet numbers:
(see the sequence A214305 that I submitted to OEIS for a list of 2-Poulet numbers)

$SCD_2(341) = \{S(11 - 2), S(31 - 2)\} = \{S(9), S(29)\} = \{6, 29\}$;
$SCD_2(1387) = \{S(19 - 2), S(73 - 2)\} = \{S(17), S(71)\} = \{17, 71\}$;
$SCD_2(2047) = \{S(23 - 2), S(89 - 2)\} = \{S(21), S(87)\} = \{7, 29\}$;
$SCD_2(2701) = \{S(37 - 2), S(73 - 2)\} = \{S(35), S(71)\} = \{7, 71\}$;
$SCD_2(3277) = \{S(29 - 2), S(113 - 2)\} = \{S(27), S(111)\} = \{9, 37\}$;
$SCD_2(4033) = \{S(37 - 2), S(109 - 2)\} = \{S(35), S(107)\} = \{7, 107\}$;
$SCD_2(4369) = \{S(17 - 2), S(257 - 2)\} = \{S(15), S(255)\} = \{5, 17\}$;
$SCD_2(4681) = \{S(31 - 2), S(151 - 2)\} = \{S(29), S(149)\} = \{29, 149\}$;
$SCD_2(5461) = \{S(43 - 2), S(127 - 2)\} = \{S(41), S(125)\} = \{41, 15\}$;
$SCD_2(7957) = \{S(73 - 2), S(109 - 2)\} = \{S(71), S(107)\} = \{71, 107\}$;
$SCD_2(8321) = \{S(53 - 2), S(157 - 2)\} = \{S(51), S(155)\} = \{17, 31\}$;
$SCD_2(10261) = \{S(31 - 2), S(331 - 2)\} = \{S(29), S(329)\} = \{29, 47\}$;
$SCD_2(13747) = \{S(59 - 2), S(233 - 2)\} = \{S(57), S(231)\} = \{19, 11\}$;
$SCD_2(14491) = \{S(43 - 2), S(337 - 2)\} = \{S(41), S(335)\} = \{41, 67\}$;
$SCD_2(15709) = \{S(23 - 2), S(683 - 2)\} = \{S(21), S(681)\} = \{7, 227\}$;
$SCD_2(18721) = \{S(97 - 2), S(193 - 2)\} = \{S(95), S(191)\} = \{19, 191\}$;
$SCD_2(19951) = \{S(71 - 2), S(281 - 2)\} = \{S(69), S(279)\} = \{23, 31\}$;
$SCD_2(23377) = \{S(97 - 2), S(241 - 2)\} = \{S(95), S(239)\} = \{19, 239\}$;
$SCD_2(31417) = \{S(89 - 2), S(353 - 2)\} = \{S(87), S(351)\} = \{29, 13\}$;
$SCD_2(31609) = \{S(73 - 2), S(433 - 2)\} = \{S(71), S(431)\} = \{71, 431\}$.

List of SC divisors of order 6 of the first twenty 2-Poulet numbers:
(see the sequence A214305 that I submitted to OEIS for a list of 2-Poulet numbers)

SCD_6(341) = {S(11 – 6), S(31 – 6)} = {S(5), S(25)} = {5, 10};
SCD_6(1387) = {S(19 – 6), S(73 – 6)} = {S(13), S(67)} = {13, 67};
SCD_6(2047) = {S(23 – 6), S(89 – 6)} = {S(17), S(83)} = {17, 83};
SCD_6(2701) = {S(37 – 6), S(73 – 6)} = {S(31), S(67)} = {31, 67};
SCD_6(3277) = {S(29 – 6), S(113 – 6)} = {S(23), S(107)} = {23, 107};
SCD_6(4033) = {S(37 – 6), S(109 – 6)} = {S(31), S(103)} = {31, 103};
SCD_6(4369) = {S(17 – 6), S(257 – 6)} = {S(11), S(251)} = {11, 251};
SCD_6(4681) = {S(31 – 6), S(151 – 6)} = {S(25), S(145)} = {10, 29};
SCD_6(5461) = {S(43 – 6), S(127 – 6)} = {S(37), S(121)} = {37, 22};
SCD_6(7957) = {S(73 – 6), S(109 – 6)} = {S(67), S(103)} = {67, 103};
SCD_6(8321) = {S(53 – 6), S(157 – 6)} = {S(47), S(151)} = {47, 151};
SCD_6(10261) = {S(31 – 6), S(331 – 6)} = {S(25), S(325)} = {10, 13};
SCD_6(13747) = {S(59 – 6), S(233 – 6)} = {S(53), S(227)} = {53, 227};
SCD_6(14491) = {S(43 – 6), S(337 – 6)} = {S(37), S(331)} = {37, 331};
SCD_6(15709) = {S(23 – 6), S(683 – 6)} = {S(17), S(677)} = {17, 677};
SCD_6(18721) = {S(97 – 6), S(193 – 6)} = {S(91), S(187)} = {13, 17};
SCD_6(19951) = {S(71 – 6), S(281 – 6)} = {S(65), S(275)} = {13, 11};
SCD_6(23377) = {S(97 – 6), S(241 – 6)} = {S(91), S(235)} = {13, 47};
SCD_6(31417) = {S(89 – 6), S(353 – 6)} = {S(83), S(347)} = {83, 347};
SCD_6(31609) = {S(73 – 6), S(433 – 6)} = {S(67), S(427)} = {67, 61}.

List of SC divisors of order 1 of the first twenty 3-Poulet numbers:
(see the sequence A215672 that I submitted to OEIS for a list of 3-Poulet numbers)

SCD_1(561) = SCD_1(3*11*17) = {S(2), S(10), S(16)} = {2, 5, 6};
SCD_1(645) = SCD_1(3*5*43) = {S(2), S(4), S(42)} = {2, 4, 7};
SCD_1(1105) = SCD_1(5*13*17) = {S(4), S(12), S(16)} = {4, 4, 6};
SCD_1(1729) = SCD_1(7*13*19) = {S(6), S(12), S(18)} = {3, 4, 6};
SCD_1(1905) = SCD_1(3*5*127) = {S(2), S(4), S(126)} = {2, 4, 7};
SCD_1(2465) = SCD_1(5*17*29) = {S(4), S(16), S(28)} = {4, 6, 7};
SCD_1(2821) = SCD_1(7*13*31) = {S(6), S(12), S(30)} = {3, 4, 5};
SCD_1(4371) = SCD_1(3*31*47) = {S(2), S(30), S(46)} = {2, 5, 23};
SCD_1(6601) = SCD_1(7*23*41) = {S(6), S(22), S(40)} = {3, 11, 5};
SCD_1(8481) = SCD_1(3*11*257) = {S(2), S(10), S(256)} = {2, 5, 10};
SCD_1(8911) = SCD_1(7*19*67) = {S(6), S(18), S(66)} = {3, 19, 67};
SCD_1(10585) = SCD_1(5*29*73) = {S(4), S(28), S(72)} = {4, 7, 6};
SCD_1(12801) = SCD_1(3*17*251) = {S(2), S(16), S(250)} = {2, 6, 15};
SCD_1(13741) = SCD_1(7*13*151) = {S(6), S(12), S(150)} = {3, 4, 10};
SCD_1(13981) = SCD_1(11*31*41) = {S(10), S(30), S(40)} = {5, 5, 5};
SCD_1(15841) = SCD_1(7*31*73) = {S(6), S(30), S(72)} = {3, 5, 6};
SCD_1(16705) = SCD_1(5*13*257) = {S(4), S(12), S(256)} = {4, 4, 10};
SCD_1(25761) = SCD_1(3*31*277) = {S(2), S(30), S(276)} = {2, 5, 23};
SCD_1(29341) = SCD_1(13*37*61) = {S(12), S(36), S(60)} = {4, 6, 5};
SCD_1(30121) = SCD_1(7*13*331) = {S(6), S(12), S(330)} = {3, 4, 11}.

List of SC divisors of order 2 of the first twenty 3-Poulet numbers:
(see the sequence A215672 that I submitted to OEIS for a list of 3-Poulet numbers)

$SCD_2(561) = SCD_1(3*11*17) = \{S(1), S(9), S(15)\} = \{1, 6, 5\}$;
$SCD_2(645) = SCD_1(3*5*43) = \{S(1), S(3), S(41)\} = \{1, 3, 41\}$;
$SCD_2(1105) = SCD_1(5*13*17) = \{S(3), S(11), S(15)\} = \{3, 11, 5\}$;
$SCD_2(1729) = SCD_1(7*13*19) = \{S(5), S(11), S(17)\} = \{5, 11, 17\}$;
$SCD_2(1905) = SCD_1(3*5*127) = \{S(1), S(3), S(125)\} = \{1, 3, 15\}$;
$SCD_2(2465) = SCD_1(5*17*29) = \{S(3), S(15), S(27)\} = \{3, 5, 9\}$;
$SCD_2(2821) = SCD_1(7*13*31) = \{S(5), S(11), S(29)\} = \{5, 11, 29\}$;
$SCD_2(4371) = SCD_1(3*31*47) = \{S(1), S(29), S(45)\} = \{1, 29, 6\}$;
$SCD_2(6601) = SCD_1(7*23*41) = \{S(5), S(21), S(29)\} = \{5, 7, 13\}$;
$SCD_2(8481) = SCD_1(3*11*257) = \{S(1), S(9), S(255)\} = \{1, 6, 17\}$;
$SCD_2(8911) = SCD_1(7*19*67) = \{S(5), S(17), S(65)\} = \{5, 17, 13\}$;
$SCD_2(10585) = SCD_1(5*29*73) = \{S(3), S(27), S(71)\} = \{3, 9, 71\}$;
$SCD_2(12801) = SCD_1(3*17*251) = \{S(1), S(15), S(249)\} = \{1, 5, 83\}$;
$SCD_2(13741) = SCD_1(7*13*151) = \{S(5), S(11), S(149)\} = \{5, 11, 149\}$;
$SCD_2(13981) = SCD_1(11*31*41) = \{S(9), S(29), S(39)\} = \{6, 29, 13\}$;
$SCD_2(15841) = SCD_1(7*31*73) = \{S(5), S(29), S(71)\} = \{5, 29, 71\}$;
$SCD_2(16705) = SCD_1(5*13*257) = \{S(3), S(111), S(255)\} = \{3, 11, 17\}$;
$SCD_2(25761) = SCD_1(3*31*277) = \{S(1), S(29), S(275)\} = \{1, 29, 11\}$;
$SCD_2(29341) = SCD_1(13*37*61) = \{S(11), S(35), S(59)\} = \{11, 7, 59\}$;
$SCD_2(30121) = SCD_1(7*13*331) = \{S(5), S(11), S(329)\} = \{5, 11, 329\}$.

List of SC divisors of order 1 of the first ten Poulet numbers divisible by 3 and 5:
(see the sequence A216364 that I submitted to OEIS for a list of Poulet numbers divisible by 15)

$SCD_1(645) = SCD_1(3*5*43) = \{2, 4, 7\}$;
$SCD_1(1905) = SCD_1(3*5*127) = \{2, 4, 7\}$;
$SCD_1(18705) = SCD_1(3*5*29*43) = \{2, 4, 7, 7\}$;
$SCD_1(55245) = SCD_1(3*5*29*127) = \{2, 4, 7, 7\}$;
$SCD_1(62745) = SCD_1(3*5*47*89) = \{2, 4, 23, 11\}$;
$SCD_1(72855) = SCD_1(3*5*43*113) = \{2, 4, 7, 7\}$;
$SCD_1(215265) = SCD_1(3*5*113*127) = \{2, 4, 7, 7\}$;
$SCD_1(451905) = SCD_1(3*5*47*641) = \{2, 4, 23, 8\}$;
$SCD_1(831405) = SCD_1(3*5*43*1289) = \{2, 4, 7, 23\}$;
$SCD_1(1246785) = SCD_1(3*5*43*1933) = \{2, 4, 7, 23\}$.