The Smarandache-Coman divisors of order k of a composite integer n with m prime factors

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Abstract. We will define in this paper the Smarandache-Coman divisors of order k of a composite integer n with m prime factors, a notion that seems to have promising applications, at a first glance at least in the study of absolute and relative Fermat pseudoprimes, Carmichael numbers and Poulet numbers.

Definition 1:

We call *the set of Smarandache-Coman divisors of order 1 of a composite positive integer n with m prime factors*, n = $d_1*d_2*...*d_m$, where the least prime factor of n, d_1, is greater than or equal to 2, the set of numbers defined in the following way:

$SCD_1(n) = \{S(d_1 - 1), S(d_2 - 1), ..., S(d_m - 1)\}$, where S is the Smarandache function.

Examples:

1. The set of SC divisors of order 1 of the number 6 is {S(2 - 1), S(3 - 1)} = {S(1), S(2)} = {1, 2}, because 6 = 2*3;
2. $SCD_1(429)$ = {S(3 - 1), S(11 - 1), S(13 - 1)} = {S(2), S(10), S(12)} = {2, 5, 4}, because 429 = 3*11*13.

Definition 2:

We call *the set of Smarandache-Coman divisors of order 2 of a composite positive integer n with m prime factors*, n = $d_1*d_2*...*d_m$, where the least prime factor of n, d_1, is greater than or equal to 3, the set of numbers defined in the following way:

$SCD_2(n) = \{S(d_1 - 2), S(d_2 - 2), ..., S(d_m - 2)\}$, where S is the Smarandache function.

Examples:

1. The set of SC divisors of order 2 of the number 21 is {S(3 - 2), S(7 - 2)} = {S(1), S(5)} = {1, 5}, because 21 = 3*7;
2. $SCD_2(2429)$ = {S(7 - 2), S(347 - 2)} = {S(5), S(345)} = {5, 23}, because 2429 = 7*347.

Definition 3:

We call *the set of Smarandache-Coman divisors of order k of a composite positive integer n with m prime factors*, n = $d_1*d_2*…*d_m$,

where the least prime factor of n, d_1, is greater than or equal to k + 1, the set of numbers defined in the following way:
$SCD_k(n) = \{S(d_1 - k), S(d_2 - k), \ldots, S(d_m - k)\}$, where S is the Smarandache function.

Examples:

1. The set of SC divisors of order 5 of the number 539 is {S(7 – 5), S(11 – 5)} = {S(2), S(6)} = {2, 3}, because 539 = 7^2*11;
2. $SCD_6(221)$ = {S(13 – 6), S(17 – 6)} = {S(7), S(11)} = {7, 11}, because 221 = 13*17.

Comment:

We obviously defined the sets of numbers above because we believe that they can have interesting applications, in fact we believe that they can even make us re-think and re-consider the Smarandache function as an instrument to operate in the world of number theory: while at the beginning its value was considered to consist essentially in that to be a criterion for primality, afterwards the Smarandache function crossed a normal process of substantiation, so it was constrained to evolve in a relatively closed (even large) circle of equalities, inequalities, conjectures and theorems concerning, most of them, more or less related concepts. We strongly believe that some of the most important applications of the Smarandache function are still undiscovered. We were inspired in defining the Smarandache-Coman divisors by the passion for Fermat pseudoprimes, especially for Carmichael numbers and Poulet numbers, by the Korselt's criterion, one of the very few (and the most important from them) instruments that allow us to comprehend Carmichael numbers, and by the encouraging results we easily obtained, even from the first attempts to relate these two types of numbers, Fermat pseudoprimes and Smarandache numbers.

Smarandache-Coman divisors of order 1 of the 2-Poulet numbers:

(See the sequence A214305 in OEIS, posted by us, for a list with Poulet numbers with two prime factors)

$SCD_1(341)$ = {S(11 – 1), S(31 – 1)} = {S(10), S(30)} = {5, 5};
$SCD_1(1387)$ = {S(19 – 1), S(73 – 1)} = {S(18), S(72)} = {6, 6};
$SCD_1(2047)$ = {S(23 – 1), S(89 – 1)} = {S(22), S(88)} ={11, 11};
$SCD_1(2701)$ = {S(37 – 1), S(73 – 1)} = {S(36), S(72)} = {6, 6};
$SCD_1(3277)$ = {S(29 – 1), S(113 – 1)} = {S(28), S(112)} = {7, 7};
$SCD_1(4033)$ = {S(37 – 1), S(109 – 1)} = {S(36), S(108)} = {6, 9};
$SCD_1(4369)$ = {S(17 – 1), S(257 – 1)} = {S(16), S(256)} = {6, 10};
$SCD_1(4681)$ = {S(31 – 1), S(151 – 1)} = {S(30), S(150)} = {5, 10};
$SCD_1(5461)$ = {S(43 – 1), S(127 – 1)} = {S(42), S(126)} = {7, 7};
$SCD_1(7957)$ = {S(73 – 1), S(109 – 1)} = {S(72), S(108)} = {6, 9};
$SCD_1(8321)$ = {S(53 – 1), S(157 – 1)} = {S(52), S(156)} = {13, 13}.

Comment:
It is notable how easily are obtained interesting results: from the first 11 terms of the 2-Poulet numbers sequence checked there are already foreseen few patterns.

Open problems:
1. Is for the majority of the 2-Poulet numbers the case that the two Smarandache-Coman divisors of order 1 are equal, as for the seven from the eleven numbers checked above?
2. Is there an infinity of 2-Poulet numbers for which the set of SCD of order 1 is equal to {6, 6}, the case of Poulet numbers 1387 and 2701, or with {6, 9}, the case of Poulet numbers 4033 and 7957?

Smarandache-Coman divisors of order 2 of the 2-Poulet numbers:

SCD_2(341) = {S(11 – 2), S(31 – 2)} = {S(9), S(29)} = {6, 29};
SCD_2(1387) = {S(19 – 2), S(73 – 2)} = {S(17), S(71)} ={17, 71};
SCD_2(2047) = {S(23 – 2), S(89 – 2)} = {S(21), S(87)} = {7, 29};
SCD_2(2701) = {S(37 – 2), S(73 – 2)} = {S(35), S(71)} = {7, 71};
SCD_2(3277) = {S(29 – 2), S(113 – 2)} = {S(27), S(111)} = {9, 37};
SCD_2(4033) = {S(37 – 2), S(109 – 2)} = {S(35), S(107)} = {7, 107};
SCD_2(4369) = {S(17 – 2), S(257 – 2)} = {S(15), S(255)} = {5, 17};
SCD_2(4681) = {S(31 – 2), S(151 – 2)} ={S(29), S(149)} = {29, 149};
SCD_2(5461) = {S(43 – 2), S(127 – 2)} = {S(41), S(125)} = {41, 15};
SCD_2(7957) = {S(73 – 2), S(109 – 2)} ={S(71), S(107)} = {71, 107};
SCD_2(8321) = {S(53 – 2), S(157 – 2)} = {S(52), S(156)} = {17, 31}.

Comment:
In the case of SCD of order 2 of the 2-Poulet numbers there are too foreseen few patterns.

Open problems:
1. Is for the majority of the 2-Poulet numbers the case that the two Smarandache-Coman divisors of order 2 are both primes, as for the eight from the eleven numbers checked above?
2. Is there an infinity of 2-Poulet numbers for which the set of SCD of order 2 is equal to {p, p + 20*k}, where p prime and k positive integer, the case of Poulet numbers 4033 and 4681?

Smarandache-Coman divisors of order 1 of the 3-Poulet numbers:

(See the sequence A215672 in OEIS, posted by us, for a list with Poulet numbers with two prime factors)

SCD_1(561) = SCD_1(3*11*17) = {S(2), S(10), S(16)} = {2, 5, 6};
SCD_1(645) = SCD_1(3*5*43) = {S(2), S(4), S(42)} = {2, 4, 7};
SCD_1(1105) = SCD_1(5*13*17) = {S(4), S(12), S(16)} = {4, 4, 6};
SCD_1(1729) = SCD_1(7*13*19) = {S(6), S(12), S(18)} = {3, 4, 6};
SCD_1(1905) = SCD_1(3*5*127) = {S(2), S(4), S(126)} = {2, 4, 7};

$SCD_1(2465) = SCD_1(5*17*29) = \{S(4), S(16), S(28)\} = \{4, 6, 7\}$;
$SCD_1(2821) = SCD_1(7*13*31) = \{S(6), S(12), S(30)\} = \{3, 4, 5\}$;
$SCD_1(4371) = SCD_1(3*31*47) = \{S(2), S(30), S(46)\} = \{2, 5, 23\}$;
$SCD_1(6601) = SCD_1(7*23*41) = \{S(6), S(22), S(40)\} = \{3, 11, 5\}$;
$SCD_1(8481) = SCD_1(3*11*257) = \{S(2), S(10), S(256)\} = \{2, 5, 10\}$;
$SCD_1(8911) = SCD_1(7*19*67) = \{S(6), S(18), S(66)\} = \{3, 19, 67\}$.

Open problems:

1. Is there an infinity of 3-Poulet numbers for which the set of SCD of order 1 is equal to {2, 4, 7}, the case of Poulet numbers 645 and 1905?
2. Is there an infinity of 3-Poulet numbers for which the sum of SCD of order 1 is equal to 13, the case of Poulet numbers 561 (2 + 5 + 6 = 13), 645 (2 + 4 + 7 = 13), 1729 (3 + 4 + 6 = 13), 1905 (2 + 4 + 7 = 13) or is equal to 17, the case of Poulet numbers 2465 (4 + 6 + 7 = 17) and 8481 (2 + 5 + 10 = 17)?
3. Is there an infinity of Poulet numbers for which the sum of SCD of order 1 is prime, which is the case of the eight from the eleven numbers checked above? What about the sum of SCD of order 1 plus 1, the case of Poulet numbers 2821 (3 + 4 + 5 + 1 = 13) and 4371 (2 + 5 + 23 + 1 = 31) or the sum of SCD of order 1 minus 1, the case of Poulet numbers 1105 (4 + 4 + 6 – 1 = 13), 2821 (3 + 4 + 5 – 1 = 11) and 4371 (2 + 5 + 23 – 1 = 29)?