On The Smarandache Semigroups
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Abstract*:*

We discusse in this paper *a Smarandache semigroups* , *a Smarandache normal subgroups* and *a Smarandache lagrange semigroup*.We prove some results about it and prove that the Smarandache semigroup Z_{p^n} with multiplication modulo p^n where p is a prime has the subgroup of order p^n-$p^{n-1}$and we prove that if p is an odd prime then Z_{p^n} is a *Smarandache weakly Lagrange semigruop* and if p is an even prime then Z_{p^n} is *a Smarandache Lagrange semigruop*

الخلاصة: ناقشنا في هذا البحث أشبا ه زمر سماراندش ,زمر سماراندش الناظميةو أشبا ه زمر لاكرانج سماراندش حيث برهنا مجموعة من النتائج المتعلقة بها .كما أثبتنا أن شبه الزمرة Z_{p^n} مع عملية الضرب معيار p^n حيث p عدد أولي تحوي مجموعة جزئية تكون زمرة ذات رتبة p^n-p^{n-1} كما أثبتنا انه إذا كانت p عدد أولي فردي فان شبه الزمرة أعلاه تكون شبه زمرة لاكرانج سماراندش بضعف أما إذا كانت p عدد أولي زوجي فان شبه الزمرة أعلاه تكون شبه زمرة لاكرانج سماراندش.

1.Introduction :

Padilla Raul introduced the notion of *Smarandache semigroups*[1], in the year 1998 in the paper entitled Smarandache Algebraic Structures .since groups are prefect structures under a single closed associative binary operation ,it has become infeasible to define Smarandache groups . Smarandache semigroups are the analog in the Smarandache ideologies of the groups where Smarandache semigroup is defined to be the semigroup A such that a proper subset of A is a group (with respect to the same binary operation).

In this paper we prove some results in *a Smarandache normal subgroups*[1],[2], *a Smarandache lagrange semigroups*[1],[2],*a Smarandache direct product semigroups*[2],*a Smarandache strong internal direct product semigeoups*[2] , *the S-semigroup homomorphism*[1],[2] and prove research problems in the references about the semigroup Z_{p^n} we solve then using Mathlab programming to check our results in this open problems .

2.Definitions and Notations:

Definition 2.1*:The Smarandache semigroup* (S-semigroup) is defined to be a semigroup A such that a proper subset of A is a group (with respect to the same binary operation),[2].

Example 2.1*:* Let z_{12}={0,1, … ,11} be the semigroup under multiplication module 12.Clearly the set A={1,11}$\subset z_{12}$ is a group under multiplication modulo 12,so z_{12} is the Smarandache semigroup,[2] .

Definition 2.2*:* Let S be a S-semigroup.Let A be a proper subset of S which is a group under the operation of S.We say S is *a Smarandache normal subgroup* of the S-semigroup if xA$\subseteq$A and Ax$\subseteq$A or xA={0} and Ax={0} $\forall x \in S$ and if 0 is an element in S then we have xA= {0} and Ax={0},[2].

Definition 2.3: Let S be a S-semigroup we say the proper subset $M \subset S$ is *the maximal subgroup* of S that is N if a subgroup such that $M \subset N$ then M=N is the only possibility,[2].

Definition 2.4: Let S be a S-semigroup.If S has only one maximal subgroup we call S *a Smarandache maximal semigroup*,[2] .

Example 2.2: Let $z_7=\{0,1, \dots ,6\}$ be the S-semigroup under multiplication module 7,the only maximal semigroup of z_7 is $G=\{1,2, \dots ,6\} \subset z_7$ so z_7 is a Smarandache maximal semigroup ,[2].

Definition 2.5: Let $S_1, \dots ,S_n$ be n S-semigroup , $S=S_1 \times S_2 \times \dots \times S_n=\{(s_1,s_2, \dots ,s_n):s_i \in S_i$ for $i=1, \dots ,n\}$ is called *the Smarandache direct product of the S-semigroups* $S_1, \dots ,S_n$ if S is a Smarandache maximal semigroup, and G is got from the $S_1, \dots ,S_n$ as $G=G_1 \times G_2 \times \dots \times G_n$ where each G_i is the maximal subgroup of the S-semigroup S_i for i=1, … ,n,[2].

Definition 2.6: Let S be a S-semigroup.If $S=B \bullet A_1 \bullet \dots \bullet A_n$ where B is a S-semigroup and $A_1,\dots,A_n$ are maximal subgroup of S then we say S is *a Smarandache strong internal direct product*,[2].

Definition 2.7: *A homomorphism* φ from a semigroup (S,.) to a semigroup $(S',*)$ is a mapping φ from the set S into the set S' such that $\varphi(x.y)= \varphi(x)* \varphi(y)$ for every $x,y \in S$.If φ is also a surjective mapping then φ is called a homomorphism from S onto S'.In case the mapping φ above is injective, it is called a one to one homomorphism .*An isomorphism* from S to S' is a homomorphism which is both surjective and injective,[3].

Definition 2.8: Let S and S' be any two S-semigroups .A map φ from S to S' is said to be *a S-semigroup homomorphism* if φ restricted to a subgroup $A \subset S \to A' \subset S'$ is a group homomorphism .The S-semigroup homomorphism is an isomorphism if $\varphi : A \to A'$ is one to one and onto . Similarly,one can define S-semigroup automorphism on S,[2].

Definition 2.9: Let S be a finite S-semigroup .We say S is *a Smarandache non-Lagrange semigroup* if the order of none of the subgroups of S divides the order of the S-semigroup,[2].

Definition 2.10: Let S be a finite S-semigroup.If the order of every subgroups of S divides the order of the S-semigroup S then we say S is *a Smarandache Lagrange semigroup* ,[2].

***Definition 2.11*:** Let S be a finite S-semigroup.If there exist at least one subgroup A that is a proper subset $(A \subset S)$ having the same operation of S whose order divides the order of S then we say that S is *a Smarandache weakly Lagrange semigroup* ,[2].

Definition 2.12: Two integers a and b, not both of which are zero, are said to be *relatively prime* whenever g.c.d.(a,b)=1,[4].

Definition 2.13: For $n \geq 1$,let $\varphi(n)$ denotes the number of positive integer not exceeding n that is relatively prime to n,[4].

Definition 2.14: Let $(G,*)$ be a finite group and p a prime .A subgroup $(P,*)$ of $(G,*)$ is said to be a Sylow p-subgroup if $(P,*)$ is a p-group and is not properly contained in any other p-subgroup of $(G,*)$ for the same prime number p,[5].

Theorem 2.1: If p is a prime and k> 0,then

$\varphi(p^k)=p^k-p^{k-1}$,[4].

Theorem (Euler) 2.2: If n is a positive integer and g.c.d.(a,n)=1 then

$a^{\varphi(n)} \equiv 1 \bmod n$,[4].

Theorem 2.3: If n is a positive integer then $ax \equiv 1 \bmod n$ has a unique solution iff g.c.d.(a,n)=1,[6].

Theorem 2.4: Let $a,b,n \in Z$,n >0.If g.c.d.(a,n)=1 then the congruence $az \equiv b \bmod n$ has a unique solution z; moreover , any integer z' is a solution iff $z \equiv z' \bmod n$,[7] .

Theorem (Sylow)2.5: Let $(G,*)$ be a finite group of order $p^k q$,where p is a prime not dividing q Then $(G,*)$ has a Sylow p-subgroup of order p^k ,[5].

3.A Smarandache normal subgroups:

Proposition 3.1: Let S be a S-semigroup and A is a proper subgroup of S which is a Smarandache normal subgroup ,if B is another subgroup of S have the same identity element as in A then B is not a Smarandache normal subgroup.

Proof: Let A ,B be a Smarandache normal subgroups of S with the identity element e. Let x $\in A \subset S$ so $0 \neq x$ since A is a group and $x=x.e \in B$ since B is a Smarandache normal subgroup so $A \subset B$, by similar way $B \subset A$ so A=B and this contradiction with our hypotheses, so B is not a Smarandache normal subgroup ▣

Remark: In particular case if $B \subset A$ clear that those subgroups have the same identity element so all subgroups of S which are subsets from a Smarandache normal subgroup of S will be not a Smarandache normal subgroup.

Proposition 3.2: Let S be S-semigroup with zero element (0) and A is a Smarandache normal subgroup of S such that the identity element in S such as in A then $A=S/\{0\}$ and A is a maximal subgroup of S.

Proof: For every $0 \neq x \in S$, $x=x.e \in A$ since A is a Smarandache normal subgroup of S and $x.e \neq 0$ where x.e=0 iff x=0 since e is the identity element of S and A, so $S/\{0\} \subset A$ and since A is a group of S so $A \subset S/\{0\}$ therefore
$A=S/\{0\}$ and by the definition of a maximal subgroup of S then A is a maximal subgroup of S ▣

Proposition 3.3: Let S be a S-semigroup without zero element then every proper subgroup of S have the same identity element as in S is not a Smarandache normal subgroup of S.

Proof: Suppose that A is a Smarandache normal subgroup of S and e is the identity element of S and A.
Let $x \in S$, $x=x.e \in A$ since A is a Smarandache normal subgroup of S,
where $x.e \neq 0$ since $0 \notin S$ and e is the identity element of S,
so $S \subset A$ therefore A=S and this is contradiction ,
so A is not a Smarandache normal subgroup of S ▣

proposition 3.4: Let $S=S_1 \times S_2 \times \dots \times S_n$ is a Smarandache direct product of the S-semigroup S_1, … ,S_n where no one of S_i has zero element and let $G=G_1 \times G_2 \times \dots \times G_n$ is the maximal subgroup of S where each G_i is a maximal subgroup of S_i and a Smarandache normal subgroup of S_i with identity element different from the identity element of S_i then G is a Smarandache normal subgroup of S, i=1, …, n .

proof: Let $S= S_1 \times S_2 \times \dots \times S_n$ is Smarandache direct product of the S- semigroup $S_1, \dots ,S_n$, $G= G_1 \times G_2 \times \dots \times G_n$ is a maximal subgroup of S where each G_i is a maximal subgroup of S_i and a Smarandache normal subgroup of S_i with identity elements different from the identity element of S_i ,i=1, … ,n.
Let $x \in S$, $y \in G$
$xy=(x_1, \dots ,x_n)(g_1, \dots ,g_n)$ where $g_i \in G_i$ and $x_i \in S_i$,i=1, … ,n.
So $xy=(x_1g_1, \dots ,x_ng_n) \in G_1 \times G_2 \times \dots \times G_n=G$

since G_i is a Smarandache normal subgroup of S_i and no one of $x_i g_i = 0$ because that S_i has no zero element , i=1, …, n , so $xG \subseteq G$,
by similar way $Gx \subseteq G$ so G is a Smarandache normal subgroup of S ▣

Remark: It is important to know that the condition in above proposition where S_i dosn't have the same identity elements of G_i where if S_i has the same identity element of G_i we have $G_i = S_i$ by proposition 3.3 and this contradiction ,also it is important to note that each S_i must not have zero element since ,for example if S_2 has zero element so $x=(x_1,0, \dots ,x_n) \in S$
let $y= (g_1, \dots ,g_n) \in G$ where $g_i \in G_i$ and $x_i \in S_i$,i=1, … ,n.
but $xy==(x_1g_1,0, \dots ,x_ng_n) \notin G_1 \times G_2 \times \dots \times G_n=G$, since $0 \notin G_2$ where G_2 is a group .

proposition 3.5: Let S and S' be the S-semigroups and let $f: S \longrightarrow S'$ be a homomorphism from S onto S', if f is a Smarandache onto homomorphism from A to A' where $A \subset S$ is a group ,$A' \subset S'$ is a group and A is a Smarandache normal subgroup of S then A' is a Smarandache normal subgroup of S'.

proof: Let $f: S \longrightarrow S'$ is a Smarandache onto homomorphism from A to A' so $f(A)=A'$.
Let $f: S \longrightarrow S'$ is the onto homomorphism on the semigroup S',
let $y \in S'$ and $a' \in f(A)$,
so $\exists\ x \in S$ such that f(x)=y,
since f is a Smarandache onto homomorphism from A to A'
$\exists\, a \in A$ such that $f(a)=a'$,
$y.\,a'=f(x).f(a)=f(x.a)$, now since A is a Smarandache normal subgroup of S then
either $x.a \in A$ so $f(x.a) \in f(A)$ so $y.\,a' \in f(A)$ and by similar way $a'.\,y \in f(A)$
or x.a=0 if S has zero element (0) so $y.\,a'= f(x.a)=f(0)=0'$ where $f(0)=0'$ is the zero element of S' if S has 0 as a zero element . Since for all $y \in S'$ $\exists\ x \in S$ such that f(x)=y so if S has (0) then
$f(x).f(0)=f(x.0)=f(0)=f(0).f(x)\ \ \forall y \in S'$, so $f(A)=A'$ is a Smarandache normal subgroup of S' ▣

proposition 3.6: Let S and S' be the S-semigroups.Let $f: S \longrightarrow S'$ be an isomorphism from S to S' ,if f is a Smarandache onto homomorphism from A to A' where $A \subset S$ is a group ,$A' \subset S'$ is a group and A' is a Smarandache normal subgroup of S' then $f^{-1}(A')$ is a Smarandache normal subgroup of S .

proof: Let $f: S \longrightarrow S'$ is an isomorphism so $f: A \longrightarrow A'$ is a one to one .
let f is a Smarandache onto homomorphism from A to A' ,
so $f(A)=A'$ and $f^{-1}(f(A)) = A$.
Let $a \in A$ and $x \in S$,
since A' is a Smarandache normal subgroup of S' then either $f(x).f(a) \in A'$
so $f(x.a) \in A'$ therefore $x.a \in A$, by similar way $a.x \in A$,
or $f(x).f(a)=f(0)=0'$ if S has zero element (0),
so f(x.a)=f(0) then x.a=0 where $f(0)=0'$ is the zero element of S' if S has 0 as a zero element , by similar way a.x=0 also x.0=0.x=0 for each $x \in S$ if S has zero element (0) therefore $f^{-1}(A')$ is a Smarandache normal subgroup of S ▣

proposition 3.7: Let S be a Smarandache strong internal direct product semigroup where $S=B \bullet A_1 \bullet \dots \bullet A_n$ such that $A_1, \dots ,A_n$ are the maximal subgroups of S and B is the S-semigroup, if

1- A_i ,i=1, .. ,n ; is a Smarandache normal subgroup of S commutative with each other .
2- If S has identity element it must be different from the identity elements of A_i which is also different from one to another ,i=1, .. ,n.

3- S has zero

Then G= $A_1 \bullet \ldots \bullet A_n = S/\{0\}$ and G is a Smarandache normal subgroup of S.

Proof: Since $A_1, \ldots ,A_n$ are the maximal subgroup of S which are commutative with each other so G= $A_1 \bullet \ldots \bullet A_n$ is a group.

Let $0 \neq x \in S$, $x=b \bullet x_1 \ldots \bullet x_n$ where $x_i \in A_i$ and $b \in B$ such that B is the S-semigroup, i=1,… ,n , so $x \in$ G since A_i is a Smarandache normal subgroup of S, i=1 ,… ,n,where if $b \bullet x_i=0$ for some i , then x=0 contradiction .

So $\forall\ 0 \neq x \in S$ we have $x \in G$ therefore $S/\{0\} \subset G$ so $S/\{0\}=G$.

To prove G is a Smarandache normal subgroup of S .

Let $x \in S$ and $y \in G$ such that $x= b \bullet x_1 \ldots \bullet x_n$ and $y=x_1' \bullet \ldots \bullet x_n'$, where $x_i ,x_i' \in A_i$ and $b \in B$.

since $A_1, \ldots, A_n$ are commutative with each other

so $x \bullet y= b \bullet x_1 \ldots \bullet x_n \bullet x_1' \ldots \bullet x_n' = b \bullet x_1'' \bullet \ldots \bullet x_n''$,where $x_i''= x_i \bullet x_i'$, i=1, …,n.

Now since A_i a Smarandache normal subgroup of S so either

$x \bullet y \in G$ if $b \bullet x_i'' \in A_i$ for some i or $x \bullet y=0$ if $b \bullet x_i'' =0$ for some I,

also $x \bullet 0=0 \bullet x=0\ \forall\ x \in S$.

So G is a Smarandache normal subgroup of S ▣

Remark: It is important to know that the condition in above proposition where A_i does not have the same identity because in such case we will have contradiction by proposition 3.1, also it is important the condition that S must have not the same identity as in A_i and S has zero element since if S has the same identity as in A_i and does not have zero element we have contradiction with proposition 3.3.

4-The S-semigroup $(Z_{p^n} ,.p^n)$

Proposition 4.1: Let Z_{p^n} be the semigruop (p is a prime ,n >1) under multiplication modulo p^n ,then Z_{p^n} has subset of order p^n-p^{n-1} which is a subgroup under multiplication modulo p^n .

Proof: Let (Z_{p^n} ,.p^n) be the semigroup under multiplication modulo p^n ,p is a prim, n >1.

The order of Z_{p^n} is p^n, now ,to compose a subset of Z_{p^n} without zero element in it we must cancel zero element because that $0.x=x.0=0\ \forall\ x \in Z_{p^n}$,[6],

and all elements give us zero element if multiplication modulo p^n with each other.Since among the first positive integer ,those which are divisible by p are

P,2p,3p, … ,kp, where k is the largest integer such that $kp \leq p^n$,if k= p^{n-1} then

$kp= p^{n-1}p= p^n=0$ since we multiplication modulo p^n so the number of those elements is p^{n-1} .We must cancel all those elements of Z_{p^n} since

$tp\ p^{n-1} =0$ where $t \leq k$, $1\leq t$ and $t \in Z_{p^n}$,

$tp^2\ p^{n-2} =0$ where $t \leq k$, $1\leq t$ and $t \in Z_{p^n}$,

and so on ,

$tp^n\ p =0$ where $t \leq k$,$1\leq t$. and $t \in Z_{p^n}$,

therefore p^n-p^{n-1} is the number of the reminder element of Z_{p^n} ,which are relatively prime with p^n where $p^n-p^{n-1}=\varphi(p^n)$,[3].

Now let H is the set of those elements ,i.e,

H={1, … ,p-1,p+1, … ,2p-1,2p+1, … ,kp-1} so H is a group of order p^n-p^{n-1} ,to prove this , if x ,y $\in H \subset Z_{p^n}$ then xy$\in Z_{p^n}$

and since xy didn't divisible by p because that x,y didn't divisible by p,since H contains each element of Z_{p^n} which didn't divisible by p so xy$\in$H,

it is clear that H is associative under multiplication modulo p^n and the identity element of H is 1,now to prove that all x $\in$H has inverse element ,
let x ,y$\in$H then yx=xy $\equiv$1 mod p^n has a unique solution modulo p^n
iff g.c.d(x, p^n)=1 ,[5], so x has y as inverse .In fact y may be obtained direct from

$$y \equiv x^{\varphi(p^n)-1} \bmod p^n .$$

Since an application of Euler's theorem leads immediately to

$$xy \equiv x^{\varphi(p^n)} \equiv 1 \bmod p^n .$$

so (H, .p^n) is a group ▣

One can check that by using Mathlap programming after determine the value of n and p so we can present all examples about this subject .

Proposition 4.2: Let Z_{p^n} be the semigruop (p is a prime ,n >1) under multiplication modulo p^n ,p is an odd prime then Z_{p^n} is a Smarandache weakly Lagrange semigroup.

Proof: Let (Z_{p^n} ,.p^n) be the semigroup under multiplication modulo p^n, p is an odd prime , n >1. By proposition 4.1 , Z_{p^n} has the subgroup H of order p^n-p^{n-1}, O(Z_{p^n})=p^n and o(H)= p^n-p^{n-1}= p^{n-1}(p-1), O(Z_{p^n})didn't divisible by o(H) since $\frac{p^n}{p^n - p^{n-1}} = \frac{p}{p-1}$,

also Z_{p^n} has a subgroup of order 2 which is {1, p^{n-1}} and O(Z_{p^n})didn't divisible by 2 since p is an odd prime ,but by Sylow theorem in group theory,[5], H has K
as a p- Sylow subgroup of order p^{n-1} where p-1 didn't divisible by p, since K is a subgroup of $H \subset Z_{p^n}$ so K is a subgroup of Z_{p^n} and

$\frac{o(Z_{p^n})}{o(K)} = \frac{p^n}{p^{n-1}} = p$,so by definition 2.11, Z_{p^n} is a Smarandache Weakly Lagrange semigroup ▣

Example 4.1 : Let Z_{3^3} = {0,1, … , 26}be a S- semigroup under multiplication modulo 27 , it is clear that the order of Z_{3^3} is 27 .

Now by proposition 4.1 it has H_1 as a subgroup of order $3^3-3^2=18$ where $H_1=\{1,2,4,5,7,8,10,11,13,14,16,17,19,20,22,23,25,26\}$and the table of H_1 is given by

.27	1	2	4	5	7	8	10	11	13	14	16	17	19	20	22	23	25	26
1	1	2	4	5	7	8	10	11	13	14	16	17	19	20	22	23	25	26
2	2	4	8	10	14	16	20	22	26	1	5	7	11	13	17	19	23	25
4	4	8	16	20	1	5	13	17	25	2	10	14	22	26	7	11	19	23
5	5	10	20	25	8	13	23	1	11	16	26	4	14	19	2	7	17	22
7	7	14	1	8	22	2	16	23	10	17	4	11	25	5	19	26	13	20
8	8	16	5	13	2	10	26	7	23	4	20	1	17	25	14	22	11	19
10	10	20	13	23	16	26	19	2	22	5	25	8	1	11	4	14	7	17
11	11	22	17	1	23	7	2	13	8	19	14	25	20	4	26	10	5	16
13	13	26	25	11	10	23	22	8	7	20	19	5	4	17	16	2	1	14
14	14	1	2	16	17	4	5	19	20	7	8	22	23	10	11	25	26	13
16	16	5	10	26	4	20	25	14	19	8	13	2	7	23	1	17	22	11
17	17	7	14	4	11	1	8	25	5	22	2	19	26	16	23	13	20	10
19	19	11	22	14	25	17	1	20	4	23	7	26	10	2	13	5	16	8
20	20	13	26	19	5	25	11	4	17	10	23	16	2	22	8	1	14	7
22	22	17	7	2	19	14	4	26	16	11	1	23	13	8	25	20	10	5
23	23	19	11	7	26	22	14	10	2	25	17	13	5	1	20	16	8	4
25	25	23	19	17	13	11	7	5	1	26	22	20	16	14	10	8	4	2
26	26	25	23	22	20	19	17	16	14	13	11	10	8	7	5	4	2	1

and 27 didn't divisible by 18 , Z_{3^3} has H_2 as a subgroup of order 2 where $H_2=\{1,26\}$ and the table of H_2 is given by

.27	1	26
1	1	26
26	26	1

and 27 didn't divisible by 2 but Z_{3^3} has H_3 as a subgroup of order 9 where H_3 is a subset of Z_{27} and satisfy the definition of the group and 27 divisible by 9 where $H_3=\{1,4,7,10,13,16,19,22,25\}$ and the table of H_3 is given by

.27	1	4	7	10	13	16	19	22	25
1	1	4	7	10	13	16	19	22	25
4	4	16	1	13	25	10	22	7	19
7	7	1	22	16	10	4	25	19	13
10	10	13	16	19	22	25	1	4	7
13	13	25	10	22	7	19	4	16	1
16	16	10	4	25	19	13	7	1	22
19	19	22	25	1	4	7	10	13	16
22	22	7	19	4	16	1	13	25	10
25	25	19	13	7	1	22	16	10	4

So Z_{3^3} is Smarandache weakly Lagrange semigroup ▣

***Proposition 4.3**:*Let Z_{p^n} be the semigruop (p is a prime ,n >1) under multiplication modulo p^n ,p is an even prime then Z_{p^n} is a Smarandache Lagrange semigroup.

Proof: Let (Z_{p^n} ,.p^n) be the semigroup under multiplication modulo p^n, p is an even prime , n >1. By proposition 4.1 , Z_{p^n} has the subgroup H of order p^n-p^{n-1},

O(Z_{p^n})=p^n ,since p is an even so p=2 and $\frac{p^n}{p^n - p^{n-1}} = \frac{p}{p-1}$ =p.

So Z_{p^n} divisible by o(H),also every other subgroups of Z_{p^n} must be a subgroup of H since as we see in proposition 4.1 H contains all elements which are not zero and the multiplication of one with each other not zero so any other subgroup of Z_{p^n} must be subset of H, by Lagrange theorem in the group theory,[8][9], the order of H divisible by the order of those subgroups.
So p^n divisible by the order of all subgroups of Z_{p^n} so and by definition of a Smarandache Lagrange semigroup Z_{p^n} is a Smarandache Lagrange semigroup, where p=2▣

Example 4.2: Let $Z_{2^5} = Z_{32}$={0,1, … , 31} be the S-semigroup of order 32 under multiplication modulo 32, Z_{32} have the following subgroups:
Three subgroup of order 2 which are {1,15},{1,17},{1,31} and the table of those subgroups as following:

.32	1	15
1	1	15
15	15	1

.32	1	17
1	1	17
17	17	1

.32	1	31
1	1	31
31	31	1

It is clear that each one of them is a subset of Z_{32} and satisfy the definition of the group so it is a subgroup of Z_{32} .
Two subgroups of order 4 which are {1,7,17,23},{1,9,17,25},{1,15,17,31} and the table of those subgroups as following:

.32	1	7	17	23
1	1	7	17	23
7	7	17	23	1
17	17	23	1	7
23	23	1	7	17

.32	1	9	17	25
1	1	9	17	25
9	9	17	25	1
17	17	25	1	9
25	25	1	9	17

.32	1	15	17	31
1	1	15	17	31
15	15	1	31	17
17	17	31	1	15
31	31	17	15	1

Two subgroups of order 8 which are {1,3,9,11,17,19,25,27},
{1,5, 9,13,17,21,25,29} and the table of those subgroups as following:

.32	1	5	9	13	17	21	25	29
1	1	5	9	13	17	21	25	29

5	5	25	13	1	21	9	29	17
9	9	13	17	21	25	29	1	5
13	13	1	21	9	29	17	5	25
17	17	21	25	29	1	5	9	13
21	21	9	29	17	5	25	13	1
25	25	29	1	5	9	13	17	21
29	29	17	5	25	13	1	21	9

.32	1	3	9	11	17	19	25	27
1	1	3	9	11	17	19	25	27
3	3	9	27	1	19	25	11	17
9	9	27	17	3	25	11	1	19
11	11	1	3	25	27	17	19	9
17	17	19	25	27	1	3	9	11
19	19	25	11	17	3	9	27	1
25	25	11	1	19	9	27	17	3
27	27	17	19	9	11	1	3	25

It is clear that each one of them is a subset of Z_{32} and satisfy the definition of the group so it is a subgroup of Z_{32} .

Also by proposition 4.1 has H as the subgroup of order 16 where H={1,3,5,7,9,11,13,15,17,19,21,23,25,27,29,31} and the table of H is given by the following table :

.32	1	3	5	7	9	11	13	15	17	19	21	23	25	27	29	31
1	1	3	5	7	9	11	13	15	17	19	21	23	25	27	29	31
3	3	9	15	21	27	1	7	13	19	25	31	5	11	17	23	29
5	5	15	25	3	13	23	1	11	21	31	9	19	29	7	17	27
7	7	21	3	17	31	13	27	9	23	5	19	1	15	29	11	25
9	9	27	13	31	17	3	21	7	25	11	29	15	1	19	5	23
11	11	1	23	13	3	25	15	5	27	17	7	29	19	9	31	21
13	13	7	1	27	21	15	9	3	29	23	17	11	5	31	25	19
15	15	13	11	9	7	5	3	1	31	29	27	25	23	21	19	17
17	17	19	21	23	25	27	29	31	1	3	5	7	9	11	13	15
19	19	25	31	5	11	17	23	29	3	9	15	21	27	1	7	13
21	21	31	9	19	29	7	17	27	5	15	25	3	13	23	1	11
23	23	5	19	1	15	29	11	25	7	21	3	17	31	13	27	9
25	25	11	29	15	1	19	5	23	9	27	13	31	17	3	21	7
27	27	17	7	29	19	9	31	21	11	1	23	13	3	25	15	5
29	29	23	17	11	5	31	25	19	13	7	1	27	21	15	9	3
31	31	29	27	25	23	21	19	17	15	13	11	9	7	5	3	1

We note that the order of all those subgroups of Z_{32} divides the order of Z_{32} so it is Lagrange semigroup .

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