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MADM Strategy Application of Bipolar Single Valued Heptapartitioned Neutrosophic Set

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Abstract

The fundamental goal of this study is to propose the concept of a bipolar single-valued heptapartitioned neutrosophic set (BSVHNS). We also outline the fundamental of BSVHNS traits and illustrate a few sample theorems. We define the fundamentals of the properties of the accuracy and scoring functions for the BSVHNS. The bipolar single-valued heptapartitioned mean in neutrosophic arithmetic (BSVHMNA) operator and the bipolar single-valued heptapartitioned mean in neutrosophic geometric (BSVHMNG) operator are defined and their fundamental properties are established in this article. We develop two Multi-Attribute Decision Making (MADM) strategies in the context of the BSVHNS environment: One is BSVHNS-MADM strategy which is on the BSVHMNA operator and another one is BSVHNS-MADM strategy which is on the BSVHMNG operator. Finally, we demonstrate the effectiveness of the developed procedures using a numerical example drawn from the actual world.

Keywords: Heptapartitioned set; Heptapartitioned Neutrosophic set; Bipolar single valued Heptapartitioned set; Bipolar single valued Heptapartitioned Neutrosophic set; MADM-Strategy.

1. Introduction

Fuzzy set theory was introduced by Zadeh [31]. It has widely used in uncertain situations for solving the problems. Atanassov [2, 3] introduced the concept of an intuitionistic fuzzy (IF) set characterized by a membership function and a non-membership function. Decision making is a process that is related as final outcome of decision problems and helps decision makers (DMs) for the selection of suitable alternative or a set of alternatives. In reality, researchers often focus on decision-making problems in uncertain and inexact situations. The multiple attribute decision making (MADM) has created an efficient frame for the comparison respecting to the assessment of multiple incompatible attributes. To address the uncertainty, indeterminacy, and inconsistent nature of this actual world of mathematical objects, Smarandache [25] defined the Neutrosophic set. Fuzzy set and Intuitionistic fuzzy set are the most generalized form of neutrosophic set by including levels of indeterminacy and rejection as independent components.

Smarandache proposed the concept of NS based on the FS and its extended notions (interval valued FS, intuitionistic FS, and so on) by adding an independent indeterminacy association function to the existing IFs model presented by Atanassov. Several NS extensions and special instances have been proposed in the literature. These situations include the single valued neutrosophic sets (SVNS), interval neutrosophic sets (INs), Neutrosophic Soft Set (NSS), INSS, Refined Neutrosophic Set (RNS), bipolar neutrosophic sets (BNS), and neutrosophic cube set. NSs have recently emerged as an intriguing study area that has garnered widespread interest. The introduction of SVNSs and INSs is one of the most significant advances in the research of NS.

Wang et al. [29] introduced the Single Valued Neutrosophic Set in 2010. (SVNS). In many fields, air surveillance included [8] , Dispute settlement [17], decision making [9-13], error diagnosis [30], segmenting an image [15] and others, the SVNSs, as well as its variants and extensions, have been used. In the works, specifics of NS applications

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and theoretical advancements are presented [4, 16, 18, 19, 26, 27].

The Bipolar Single Valued Neutrosophic Set is defined by Deli et al. [6] (SVBNS). Later, a great deal of researchers used the idea of SVBNS in the creation of models for Multi Attribute Decision Making (MADM) [1, 7, 20, 21, 22] issues. The concept of the Heptapartitioned Neutrosophic Set (HNS), which included seven separate components, was founded by Radha et al. [23] in 2021.

The Bipolar Single-Valued Heptapartitioned Neutrosophic Set (BSVHNS), created in this study by combining BSVNS and HNS, is introduced. Next, we define some of BSVHNS's fundamental characteristics. On the BSVHNS, a few illustrated instances are also given. Additionally, we suggest a few aggregation operators and demonstrate their fundamental characteristics. Also, in the BSVHNS context, we design two additional MADM techniques.
The rest of this article's description is as follows:

Several pertinent findings on HNS are displayed in Section 2. The BSVHNS is first mentioned in Section 3. The bipolar single-valued heptapartitioned mean neutrosophic arithmetic operator and the bipolar single-valued heptapartitioned mean neutrosophic geometric operator are two aggregating operators that are introduced in Section 4 of the paper. We obtain the concepts of the score function and accuracy function in the BSVHNS environment in Section 5. In the bipolar single-valued heptapartitioned neutrosophic arithmetic mean operator using BSVHNS environment, we develop a MADM method in Section 6. The bipolar single-valued heptapartitioned neutrosophic geometric mean operator is used in Section 7 to develop a MADM plan in an BSVHNS environment. We proposed MADM strategies in Section 8 by presenting a practical numerical illustration and contrasting the two MADM procedures. As a method to conclude the work, we state future study in the newly built set environment.

2. Some Preliminary Results

The main concepts of this study, it is important to review some fundamental definitions of the terms Neutrosophic Set, Bipolar Neutrosophic Set, and Heptapartitioned Neutrosophic Set.

Definition 2.1 [25] A The following is a definition of Neutrosophic Set A on X:

$$\boldsymbol{A = \{< x, T_A(x), I_A(x), F_A(x) >, x \in X\}}$$

where $\boldsymbol{T_A,\ I_A,\ F_A : U \rightarrow [0,1]}$ and $\boldsymbol{0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3}$

Here, $\boldsymbol{T_A(x)}$ is the degree of membership, $\boldsymbol{I_A(x)}$is the degree of inderminancy and $\boldsymbol{F_A(x)}$ is the degree of non-membership. Here, $\boldsymbol{T_A(x)}$ and $\boldsymbol{F_A(x)}$ are dependent neutrosophic elements and $\boldsymbol{I_A(x)}$ is an independent neutrosophic element.

Definition 2.2 [5] Let X represent a universe. An object of the form is a QNS, A on X with independent neutrosophic components

$$\boldsymbol{A = \{< x, T_A(x), C_A(x), U_A(x), F_A(x) >, x \in X\}}$$

and $\boldsymbol{0 \leq T_A(x) + C_A(x) + U_A(x) + F_A(x) \leq 4}$ Here, $\boldsymbol{T_A(x)}$ is the truth membership, $\boldsymbol{C_A(x)}$ is contradiction membership, $\boldsymbol{U_A(x)}$ is ignorance membership and $\boldsymbol{F_A(x)}$ is the false membership.

Definition 2.3 [24] A non-empty set $\boldsymbol{P}$ shall be used. Each element of $\boldsymbol{P}$ is defined by a PNS over $\boldsymbol{P}$ by a truth-membership function $\boldsymbol{T_A(x)}$, a contradiction membership function $\boldsymbol{C_A(x)}$ an ignorance membership function $\boldsymbol{G_A(x)}$ unknown membership function $\boldsymbol{U_A(x)}$ and a falsity membership function $\boldsymbol{F_A(x)}$ such that for each $\boldsymbol{p \in P}$, $\boldsymbol{0 \leq T_A(x) + C_A(x) + G_A(x) + U_A(x) + F_A(x) \leq 5}$.

Definition 2.4 [23] Let $\boldsymbol{R}$ be a non-empty Universe. A Heptapartitioned neutrosophic set (HNS) $\boldsymbol{A}$ over $\boldsymbol{R}$ characterizes each element $\boldsymbol{p}$ in $\boldsymbol{R}$ by an absolute truth-membership function $\boldsymbol{T_A}$, a relative truth membership function $\boldsymbol{M_A}$, a contradiction membership function $\boldsymbol{C_A}$, an ignorance membership function $\boldsymbol{I_A}$, an unknown membership function $\boldsymbol{U_A}$, an absolute falsity membership function $\boldsymbol{F_A}$ and a relative falsity membership function $\boldsymbol{K_A}$ such that for each $\boldsymbol{p \in R}$, $\boldsymbol{T_A, M_A, C_A, I_A, U_A, F_A, K_A \in [0,1]}$ and

$$\boldsymbol{A = [\, p, T_A(p), M_A(p), C_A(p), I_A(p), U_A(p), F_A(p), K_A(p) : p \in R\,]\ 0 \leq T_A(p) + M_A(p) + C_A(p) + I_A(p) + U_A(p) + F_A(p) + K_A(p) \leq 7.}$$

Definition 2.5 [23] A Heptapartitioned neutrosophic set $\boldsymbol{A}$ is said to absolute Heptapartitioned neutrosophic set Δ if and only if its absolute truth-membership, a relative truth membership, a contradiction membership, an ignorance membership, an unknown membership, an absolute falsity membership and a relative falsity membership are defined as follows, $\boldsymbol{T_A(p) = 1, M_A(p) = 1,\ C_A(p) = 1,\ U_A(p) = 1,\ I_A(p) = 0,\ K_A(p) = 0\ \textit{and}\ F_A(p) = 0.}$

Definition 2.6 [23] A Heptapartitioned neutrosophic set $\boldsymbol{A}$ is said to relative Heptapartitioned neutrosophic set ∅ if and only if its absolute truth-membership, a relative truth membership a contradiction membership, an ignorance membership, an unknown membership, an absolute falsity membership and a relative falsity membership are defined as follows, $\boldsymbol{T_A(p) = 0, M_A(p) = 0,\ C_A(p) = 0,\ U_A(p) = 0,\ I_A(p) = 1,\ K_A(p) = 1\ \textit{and}\ F_A(p) = 1.}$

Definition 2.7 [23] For any two Heptapartitioned neutrosophic sets $\boldsymbol{A}$ and $\boldsymbol{B}$ over $\boldsymbol{R}$, $\boldsymbol{A}$ is said to be contained in

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$\boldsymbol{B}$ iff $T_A(p) \le T_B(p)$, $M_A(p) \le M_B(p)$, $C_A(p) \le C_B(p)$, $U_A(p) \ge U_B(p)$, $I_A(p) \ge I_B(p)$, $K_A(p) \ge K_B(p)$ *and* $F_A(p) \ge F_B(p)$.

Definition 2.8 [25] The complement of Heptapartitioned neutrosophic sets $\boldsymbol{A}$ over the universe $\boldsymbol{R}$ is denoted by A^C and is defined as $A^C = [\,(p, F_A(p),\ K_A(p),\ I_A(p),\ 1-U_A(p),\ C_A(p),\ M_A(p),\ T_A(p) : \forall p \in R]$.

Definition 2.9 [23] The union of any two Heptapartitioned neutrosophic sets $\boldsymbol{A}$ and $\boldsymbol{B}$ over $\boldsymbol{R}$ is denoted by $A \cup B$ and is defined as

$$A \cup B = [\,p, (\max(T_A(p), T_B(p)), \max(M_A(p), M_B(p)), \max(C_A(p), C_B(p)), \min(U_A(p), U_B(p)), \min(I_A(p), I_B(p)), \min(K_A(p), K_B(p)) \text{ and } \min(F_A(p), F_B(p))) : p \in R].$$

Definition 2.10 [23] The union of any two Heptapartitioned neutrosophic sets $\boldsymbol{A}$ and $\boldsymbol{B}$ over $\boldsymbol{R}$ is denoted by $A \cap B$ and is defined as

$$A \cap B = [\,p, (\min(T_A(p), T_B(p)), \min(M_A(p), M_B(p)), \min(C_A(p), C_B(p)), \max(U_A(p), U_B(p)), \max(I_A(p), I_B(p)), \max(K_A(p), K_B(p)) \text{ and } \max(F_A(p), F_B(p))) : p \in R].$$

Example 2.1 Consider two HNSs over R, given as

$$A = [0.4, 0.3, 0.5, 0.6, 0.4, 0.2, 0.7]/p_1 + [0.6, 0.2, 0.9, 0.4, 0.7, 0.5, 0.2]/p_2 + [0.4, 0.2, 0.1, 0.9, 0.4, 0.5, 0.7]/p_3$$

$$B = [0.6, 0.5, 0.5, 0.4, 0.2, 0.4, 0.9]/p_1 + [0.2, 0.4, 0.1, 0.6, 0.4, 0.3, 0.5]/p_2 + [0.7, 0.4, 0.3, 0.4, 0.2, 0.1, 0.5]/p_3$$

$$A^C = [0.7, 0.2, 0.4, 0.4, 0.5, 0.3, 0.4]/p_1 + [0.2, 0.5, 0.7, 0.6, 0.9, 0.2, 0.6]/p_2 + [0.7, 0.5, 0.4, 0.1, 0.1, 0.2, 0.4]/p_3$$

$$A \cup B = [0.6, 0.5, 0.5, 0.4, 0.2, 0.2, 0.7]/p_1 + [0.6, 0.4, 0.9, 0.4, 0.4, 0.3, 0.2]/p_2 + [0.7, 0.4, 0.3, 0.4, 0.2, 0.1, 0.5]/p_3$$

$$A \cap B = [0.4, 0.3, 0.5, 0.6, 0.4, 0.4, 0.9]/p_1 + [0.2, 0.2, 0.1, 0.6, 0.7, 0.5, 0.5]/p_2 + [0.4, 0.2, 0.1, 0.9, 0.4, 0.5, 0.7]/p_3$$

Definition 2.11 [28] Suppose that $\lambda_1, \lambda_2, \dots \dots \lambda_n$ be n real numbers. The arithmetic mean (AM) of $\lambda_1, \lambda_2, \dots \dots \lambda_n$ is specified by $AM\ (\lambda_1, \lambda_2, \dots \dots \lambda_n) = \frac{1}{n}\sum_{i=1}^{n} \lambda_i$.

Definition 2.12 [28] Suppose that $\lambda_1, \lambda_2, \dots \dots \lambda_n$ be n real numbers. The geometric mean (GM) of $\lambda_1, \lambda_2, \dots \dots \lambda_n$ is specified by $GM\ (\lambda_1, \lambda_2, \dots \dots \lambda_n) = (\prod_{i=1}^{n} \lambda_i)^{\frac{1}{n}}$.

3. Bipolar Single-Valued Heptapartitioned Neutrosophic Set

We obtain the idea of BSVHNS in this section. We also look into many aspects of these kinds of sets properties. A few additional instances are provided as well.

Definition 3.1 A bipolar single-valued heptapartitioned neutrosophic set H over a non-empty set ϕ is specified as: $H = \{(\lambda, T_H^-(\lambda), M_H^-(\lambda), C_H^-(\lambda), U_H^-(\lambda), I_H^-(\lambda), K_H^-(\lambda), F_H^-(\lambda), T_H^+(\lambda), M_H^+(\lambda), C_H^+(\lambda), U_H^+(\lambda), I_H^+(\lambda), K_H^+(\lambda), F_H^+(\lambda) : \lambda \in \phi\}$, where $T_H^-(\lambda), M_H^-(\lambda), C_H^-(\lambda), U_H^-(\lambda), I_H^-(\lambda), K_H^-(\lambda), F_H^-(\lambda) \in [-1, 0]$ and $T_H^+(\lambda), M_H^+(\lambda), C_H^+(\lambda), U_H^+(\lambda), I_H^+(\lambda), K_H^+(\lambda), F_H^+(\lambda) \in [0, 1]$.

The negative membership degrees $T_H^-(\lambda), M_H^-(\lambda), C_H^-(\lambda), U_H^-(\lambda), I_H^-(\lambda), K_H^-(\lambda), F_H^-(\lambda)$ indicate the degree of absolute truth-membership function T_H, a relative truth membership function M_H, a contradiction membership function C_H, an ignorance membership function I_H, an unknown membership function U_A, an absolute falsity membership function F_A and a relative falsity membership function K_H respectively for $\lambda \in \phi$ corresponding to an BSVHNS. Again, the positive membership degrees, $T_H^+(\lambda), M_H^+(\lambda), C_H^+(\lambda), U_H^+(\lambda), I_H^+(\lambda), K_H^+(\lambda), F_H^+(\lambda)$ indicate as same in the above membership functions of corresponding to an BSVHNS.

Example 3.1

Let $\phi = \{p, q\}$ be a fixed set. Then, $P = \{(p, -0.4, -0.2, -0.3, -0.5, -0.3, -0.2, -0.4, 0.3, 0.2, 0.4, 0.7, 0.1, 0.3, 0.5), (q, -0.3, -0.3, -0.6, -0.5, -0.2, -0.2, -0.7, 0.3, 0.2, 0.7, 0.5, 0.1, 0.6, 0.4)\}$ is an BSVHNS over λ.

Definition 3.2

Let $H = \{(\lambda, T_H^-(\lambda), M_H^-(\lambda), C_H^-(\lambda), U_H^-(\lambda), I_H^-(\lambda), K_H^-(\lambda), F_H^-(\lambda), T_H^+(\lambda), M_H^+(\lambda), C_H^+(\lambda), U_H^+(\lambda), I_H^+(\lambda), K_H^+(\lambda), F_H^+(\lambda) : \lambda \in \phi\}$ be an BSVHNS.

Then, $[T_H^-(\lambda), M_H^-(\lambda), C_H^-(\lambda), U_H^-(\lambda), I_H^-(\lambda), K_H^-(\lambda), F_H^-(\lambda), T_H^+(\lambda), M_H^+(\lambda), C_H^+(\lambda), U_H^+(\lambda), I_H^+(\lambda), K_H^+(\lambda), F_H^+(\lambda)]$ is called a bipolar single-valued heptapartitioned neutrosophic number (BSVHNN), for each $\lambda \in \phi$.

Definition 3.3 Suppose that

$P = \{(\lambda, T_P^-(\lambda), M_P^-(\lambda), C_P^-(\lambda), U_P^-(\lambda), I_P^-(\lambda), K_P^-(\lambda), F_P^-(\lambda), T_P^+(\lambda), M_P^+(\lambda), C_P^+(\lambda), U_P^+(\lambda), I_P^+(\lambda), K_P^+(\lambda), F_P^+(\lambda) : \lambda \in \phi\}$ and

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$Q = \{(\lambda, T_Q^-(\lambda), M_Q^-(\lambda), C_Q^-(\lambda), U_Q^-(\lambda), I_Q^-(\lambda), K_Q^-(\lambda), F_Q^-(\lambda), T_Q^+(\lambda), M_Q^+(\lambda), C_Q^+(\lambda), U_Q^+(\lambda), I_Q^+(\lambda), K_Q^+(\lambda), F_Q^+(\lambda) : \lambda \epsilon \phi\}$ be any two BSVHNS over ϕ.

Example 3.2 Given two BSVHNS's

$P = \{(p, -0.4, -0.5, -0.2, -0.5, -0.2, -0.4, -0.2, 0.2, 0.4, 0.5, 0.4, 0.5, 0.3, 0.4), (q, -0.3, -0.4, -0.3, -0.6, -0.3, -0.5, -0.1)\}$ and

$Q = \{(p, -0.4, -0.4, -0.1, -0.6, -0.4, -0.6, -0.3, 0.4, 0.7, 0.6, 0.2, 0.3, 0.1, 0.3), (q, -0.2, -0.4, -0.1, -0.8, -0.4, -0.6, -0.2, 0.1, 0.5, 0.7, 0.1, 0.2, 0.2, 0.3)\}$ over $\phi = \{p, q\}$. Then $P \subseteq Q$.

Definition 3.4 Given two BSVHNS's

$P = \{(\lambda, T_P^-(\lambda), M_P^-(\lambda), C_P^-(\lambda), U_P^-(\lambda), I_P^-(\lambda), K_P^-(\lambda), F_P^-(\lambda), T_P^+(\lambda), M_P^+(\lambda), C_P^+(\lambda), U_P^+(\lambda), I_P^+(\lambda), K_P^+(\lambda), F_P^+(\lambda) : \lambda \epsilon \phi\}$ and

$Q = \{(\lambda, T_Q^-(\lambda), M_Q^-(\lambda), C_Q^-(\lambda), U_Q^-(\lambda), I_Q^-(\lambda), K_Q^-(\lambda), F_Q^-(\lambda), T_Q^+(\lambda), M_Q^+(\lambda), C_Q^+(\lambda), U_Q^+(\lambda), I_Q^+(\lambda), K_Q^+(\lambda), F_Q^+(\lambda) : \lambda \epsilon \phi\}$ be any two BSVHNS over ϕ.

Then, the intersection of P and Q is determined by:

$$P \cap Q = \{(\lambda, \min\left(T_P^-(\lambda), T_Q^-(\lambda)\right), \min\left(M_P^-(\lambda), M_Q^-(\lambda)\right), \min\left(C_P^-(\lambda), C_Q^-(\lambda)\right), \max\left(U_P^-(\lambda), U_Q^-(\lambda)\right), \max\left(I_P^-(\lambda), I_Q^-(\lambda)\right), \max\left(K_P^-(\lambda), K_Q^-(\lambda)\right), \max\left(F_P^-(\lambda), F_Q^-(\lambda)\right), \min\left(T_P^+(\lambda), T_Q^+(\lambda)\right), \min\left(M_P^+(\lambda), M_Q^+(\lambda)\right) \min\left(C_P^+(\lambda), C_Q^+(\lambda)\right), \max\left(U_P^+(\lambda), U_Q^+(\lambda)\right), \max\left(I_P^+(\lambda), I_Q^+(\lambda)\right), \max\left(K_P^+(\lambda), K_Q^+(\lambda)\right), \max\left(F_P^+(\lambda), F_Q^+(\lambda)\right) : \lambda \epsilon \phi\}.$$

Example 3.3

Given P and Q are two BSVHNS over $\phi = \{p, q\}$ such that

$P = \{(p, -0.7, -0.3, -0.4, -0.4, -0.3, -0.2, -0.6, 0.4, 0.5, 0.2, 0.4, 0.2, 0.3, 0.5), (q, -0.6, -0.4, -0.4, -0.3, -0.3, -0.2, -0.2, 0.3, 0.5, 0.4, 0.4, 0.7, 0.5, 0.4)\}$ and

$Q = \{(p, -0.6, -0.3, -0.5, -0.3, -0.6, -0.7, -0.5, 0.6, 0.1, 0.2, 0.2, 0.1, 0.4, 0.5), (q, -0.5, -0.4, -0.4, -0.5, -0.7, -0.5, -0.4, 0.5, 0.5, 0.7, 0.4, 0.4, 0.1, 0.2)\}$.

Then, their intersection is

$P \cap Q = \{(p, -0.7, -0.3, -0.5, -0.3, -0.3, -0.2, -0.5, 0.4, 0.1, 0.2, 0.4, 0.2, 0.4, 0.5), (q, -0.6, -0.4, -0.4, -0.3, -0.3, -0.2, -0.2,0.3, 0.5, 0.4, 0.4, 0.7, 0.5, 0.4)\}$.

Definition 3.5 Given P and Q are two BSVHNS,

$P = \{(\lambda, T_P^-(\lambda), M_P^-(\lambda), C_P^-(\lambda), U_P^-(\lambda), I_P^-(\lambda), K_P^-(\lambda), F_P^-(\lambda), T_P^+(\lambda), M_P^+(\lambda), C_P^+(\lambda), U_P^+(\lambda), I_P^+(\lambda), K_P^+(\lambda), F_P^+(\lambda) : \lambda \epsilon \phi\}$

and

$Q = \{(\lambda, T_Q^-(\lambda), M_Q^-(\lambda), C_Q^-(\lambda), U_Q^-(\lambda), I_Q^-(\lambda), K_Q^-(\lambda), F_Q^-(\lambda), T_Q^+(\lambda), M_Q^+(\lambda), C_Q^+(\lambda), U_Q^+(\lambda), I_Q^+(\lambda), K_Q^+(\lambda), F_Q^+(\lambda) : \lambda \epsilon \phi\}$ be any two BSVHNS over ϕ. Then, the union of P and Q is defined by:

$$P \cup Q = \{(\lambda, \max\left(T_P^-(\lambda), T_Q^-(\lambda)\right), \max\left(M_P^-(\lambda), M_Q^-(\lambda)\right), \max\left(C_P^-(\lambda), C_Q^-(\lambda)\right), \min\left(U_P^-(\lambda), U_Q^-(\lambda)\right), \min\left(I_P^-(\lambda), I_Q^-(\lambda)\right), \min\left(K_P^-(\lambda), K_Q^-(\lambda)\right), \min\left(F_P^-(\lambda), F_Q^-(\lambda)\right), \max\left(T_P^+(\lambda), T_Q^+(\lambda)\right), \max\left(M_P^+(\lambda), M_Q^+(\lambda)\right), \max\left(C_P^+(\lambda), C_Q^+(\lambda)\right), \min\left(U_P^+(\lambda), U_Q^+(\lambda)\right), \min\left(I_P^+(\lambda), I_Q^+(\lambda)\right), \min\left(K_P^+(\lambda), K_Q^+(\lambda)\right), \min\left(F_P^+(\lambda), F_Q^+(\lambda)\right) : \lambda \epsilon \phi\}.$$

Example 3.4

Given P and Q are two BSVHNS s over $\phi = \{p, q\}$ such that

$P = \{(p, -0.3, -0.5, -0.2, -0.6, -0.2, -0.4, -0.3, 0.2, 0.3, 0.5, 0.3, 0.2, 0.4, 0.6), (q, -0.2, -0.4, -0.4, -0.5, -0.2, -0.3, -0.4, 0.2, 0.4, 0.5, 0.2, 0.2, 0.3, 0.8)\}$ and

$Q = \{(p, -0.3, -0.2, -0.4, -0.6, -0.2, -0.1, -0.5, 0.3, 0.1, 0.3, 0.6, 0.3, 0.1, 0.6), (q, -0.4, -0.5, -0.5, -0.4, -0.3, -0.4, -0.1, 0.4, 0.2, 0.3, 0.2, 0.1, 0.3, 0.5)\}$.

Then, their union is

$P \cup Q = \{(p, -0.3, -0.2, -0.2, -0.6, -0.2, -0.4, -0.5, 0.3, 0.3, 0.5, 0.3, 0.2, 0.1, 0.6), (q, -0.2, -0.4, -0.4, -0.5, -0.3, -0.4, -0.4,0.4, 0.4, 0.5, 0.2, 0.1, 0.3, 0.5)\}$.

Definition 3.6

Let $P = \{(\lambda, T_P^-(\lambda), M_P^-(\lambda), C_P^-(\lambda), U_P^-(\lambda), I_P^-(\lambda), K_P^-(\lambda), F_P^-(\lambda), T_P^+(\lambda), M_P^+(\lambda), C_P^+(\lambda), U_P^+(\lambda), I_P^+(\lambda), K_P^+(\lambda), F_P^+(\lambda) : \lambda \epsilon \phi\}$ be an BSVHNSs over ϕ. Then, P^c is defined as:

$P^c = \{(\lambda, F_P^-(\lambda), K_P^-(\lambda), I_P^-(\lambda), -1 - U_P^-(\lambda), C_P^-(\lambda), M_P^-(\lambda), T_P^-(\lambda), F_P^+(\lambda), K_P^+(\lambda), I_P^+(\lambda), 1 - U_P^+(\lambda), C_P^+(\lambda), M_P^+(\lambda), T_P^+(\lambda)) : \lambda \epsilon \phi\}$.

Example 3.5

Given $P = \{(p, -0.4, -0.2, -0.3, -0.5, -0.3, -0.2, -0.4, 0.3, 0.2, 0.4, 0.7, 0.1, 0.3, 0.5), (q, -0.3, -0.3, -0.6, -0.5, -0.2, -0.2, -0.7, 0.3, 0.2, 0.7, 0.5, 0.1, 0.6, 0.4)\}$ be an BSVHNS over $\lambda = \{p, q\}$.

Then, P^c is

$P^c = \{(p, -0.4, -0.2, -0.3, -0.5, -0.3, -0.2, -0.4, 0.5, 0.3, 0.1, 0.3, 0.4, 0.2, 0.3), (q, -0.7, -0.2, -0.2, -0.5, -0.6, -0.3, -0.3, 0.4, 0.6, 0.1, 0.5, 0.7, 0.2, 0.3)\}$.

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Definition 3.7
The null BSVHNS (0^{BHN}) and the absolute BSVHNS (1^{BHN}) over ϕ are specified as given below:
(i) $0^{BHN} = \{(\lambda, 1, 1, 1, 0, 0, 0, 0, -1, -1, -1, 0, 0, 0, 0) : \lambda \in \phi\}$;
(ii) $1^{BHN} = \{(\lambda, 0, 0, 0, -1, -1, -1, -1, 0, 0, 0, 1, 1, 1, 1) : \lambda \in \phi\}$;
It is clearly know that,
$(i) 0^{BHN} \subseteq X \subseteq 1^{BHN}$, where X is an BSVHNS over ϕ;
$(ii) 0^{BHN^c} = 1^{BHN}$ & $1^{BHN^c} = 0^{BHN}$;
$(iii) 0^{BHN} \cup 1^{BHN} = 1^{BHN}$;
$(iv) 0^{BHN} \cap 1^{BHN} = 0^{BHN}$.

Definition 3.8 Given
$\lambda = [T_\phi^-(\lambda), M_\phi^-(\lambda), C_\phi^-(\lambda), U_\phi^-(\lambda), I_\phi^-(\lambda), K_\phi^-(\lambda), F_\phi^-(\lambda), T_\phi^+(\lambda), M_\phi^+(\lambda), C_\phi^+(\lambda), U_\phi^+(\lambda), I_\phi^+(\lambda), K_\phi^+(\lambda), F_\phi^+(\lambda)]$ and
$\gamma = [T_\phi^-(\gamma), M_\phi^-(\gamma), C_\phi^-(\gamma), U_\phi^-(\gamma), I_\phi^-(\gamma), K_\phi^-(\gamma), F_\phi^-(\gamma), T_\phi^+(\gamma), M_\phi^+(\gamma), C_\phi^+(\gamma), U_\phi^+(\gamma), I_\phi^+(\gamma), K_\phi^+(\gamma), F_\phi^+(\gamma)]$
be two BSVHNNSs. Then,

$(i) k.\lambda = [-\left(-T_\phi^-(\lambda)\right)^k, -\left(-M_\phi^-(\lambda)\right)^k, -\left(-C_\phi^-(\lambda)\right)^k, -\left(1-(1-(-U_\phi^-(\lambda))^k\right),$
$-\left(-I_\phi^-(\lambda)\right)^k, -\left(-K_\phi^-(\lambda)\right)^k, -\left(-F_\phi^-(\lambda)\right)^k, 1-\left(1-\left(T_\phi^+(\lambda)\right)^k\right), 1-\left(1-\left(M_\phi^+(\lambda)\right)^k\right), 1-\left(1-\right.$
$\left.\left(C_\phi^+(\lambda)\right)^k\right), \left(U_\phi^+(\lambda)\right)^k, 1-\left(1-\left(I_\phi^+(\lambda)\right)^k\right), 1-\left(1-\left(K_\phi^+(\lambda)\right)^k\right), 1-\left(1-\left(F_\phi^+(\lambda)\right)^k\right)],$
where $k > 0$.

$(ii) \lambda^k = \left[-\left(-T_\phi^-(\lambda)\right)^k, -\left(-M_\phi^-(\lambda)\right)^k, -\left(-C_\phi^-(\lambda)\right)^k, -\left(1-(1-(-U_\phi^-(\lambda)))^k\right),\right.$
$-\left(-I_\phi^-(\lambda)\right)^k, -\left(-K_\phi^-(\lambda)\right)^k, -\left(-F_\phi^-(\lambda)\right)^k, 1-\left(1-\left(T_\phi^+(\lambda)\right)^k\right), 1-\left(1-\left(M_\phi^+(\lambda)\right)^k\right),$
$1-\left(1-\left(C_\phi^+(\lambda)\right)^k\right), \left(U_\phi^+(\lambda)\right)^k, 1-\left(1-\left(I_\phi^+(\lambda)\right)^k\right), 1-\left(1-\left(K_\phi^+(\lambda)\right)^k\right), 1-\left(1-\left(F_\phi^+(\lambda)\right)^k\right)],$
where $k > 0$.

$(iii)\ \lambda + \gamma = [-(-T_\phi^-(\lambda) - T_\phi^-(\gamma) - T_\phi^-(\lambda).T_\phi^-(\gamma)), -(-M_\phi^-(\lambda) - M_\phi^-(\gamma) - M_\phi^-(\lambda).M_\phi^-(\gamma)),$
$-(-C_\phi^-(\lambda) - C_\phi^-(\gamma) - C_\phi^-(\lambda).C_\phi^-(\gamma)), -U_\phi^-(\lambda).U_\phi^-(\gamma), -(-I_\phi^-(\lambda) - I_\phi^-(\gamma)$
$- I_\phi^-(\lambda).I_\phi^-(\gamma)), -(-K_\phi^-(\lambda) - K_\phi^-(\gamma) - K_\phi^-(\lambda).K_\phi^-(\gamma)), -(-F_\phi^-(\lambda) - F_\phi^-(\gamma)$
$- F_\phi^-(\lambda).F_\phi^-(\gamma)), T_\phi^+(\lambda).T_\phi^+(\gamma), M_\phi^+(\lambda).M_\phi^+(\gamma), C_\phi^+(\lambda).C_\phi^+(\gamma), U_\phi^+(\lambda) + U_\phi^+(\gamma)$
$- U_\phi^+(\lambda).U_\phi^+(\gamma), I_\phi^+(\lambda).I_\phi^+(\gamma), K_\phi^+(\lambda).K_\phi^+(\gamma), F_\phi^+(\lambda).F_\phi^+(\gamma)]$

$(iv) \lambda.\gamma = \left[-T_\phi^-(\lambda).T_\phi^-(\gamma), -M_\phi^-(\lambda).M_\phi^-(\gamma), -C_\phi^-(\lambda).C_\phi^-(\gamma)\right), -\left(-U_\phi^-(\lambda) - U_\phi^-(\gamma)\right.$
$\left.- U_\phi^-(\lambda).U_\phi^-(\gamma)\right), -I_\phi^-(\lambda).I_\phi^-(\gamma), -K_\phi^-(\lambda).K_\phi^-(\gamma), -F_\phi^-(\lambda).F_\phi^-(\gamma), T_\phi^+(\lambda) + T_\phi^+(\gamma)$
$- T_\phi^+(\lambda).T_\phi^+(\gamma), M_\phi^+(\lambda) + M_\phi^+(\gamma) - M_\phi^+(\lambda).M_\phi^+(\gamma), C_\phi^+(\lambda) + C_\phi^+(\gamma)$
$- C_\phi^+(\lambda).C_\phi^+(\gamma), U_\phi^+(\lambda).U_\phi^+(\gamma), I_\phi^+(\lambda) + I_\phi^+(\gamma) - I_\phi^+(\lambda).I_\phi^+(\gamma), K_\phi^+(\lambda) + K_\phi^+(\gamma)$
$- K_\phi^+(\lambda).K_\phi^+(\gamma), F_\phi^+(\lambda) + F_\phi^+(\gamma) - F_\phi^+(\lambda).F_\phi^+(\gamma)]$

4. Bipolar Single-Valued Heptapartitioned Neutrosophic Operators of Aggregation

Definition 4.1 Assume that
$\lambda_i = [T_\phi^-(\lambda_i), M_\phi^-(\lambda_i), C_\phi^-(\lambda_i), U_\phi^-(\lambda_i), I_\phi^-(\lambda_i), K_\phi^-(\lambda_i), F_\phi^-(\lambda_i), T_\phi^+(\lambda_i), M_\phi^+(\lambda_i), C_\phi^+(\lambda_i), U_\phi^+(\lambda_i),$
$I_\phi^+(\lambda_i), K_\phi^+(\lambda_i), F_\phi^+(\lambda_i)], i = 1, 2, 3, \ldots \ldots n$ be a group of BSVHNNs, over ϕ. The bipolar single-valued heptapartitioned mean in neutrosophic arithmetic (BSVHMNA) operator is determined as follows:
BSVHMNA $(\lambda_1, \lambda_2, \lambda_3 \ldots \ldots \ldots . \lambda_n) = \frac{1}{n} \sum_{i=1}^{n} \lambda_i$ … … … … … … … … … … … … … … (1)

Theorem 4.1 Assume that
$\lambda_i = [T_\phi^-(\lambda_i), M_\phi^-(\lambda_i), C_\phi^-(\lambda_i), U_\phi^-(\lambda_i), I_\phi^-(\lambda_i), K_\phi^-(\lambda_i), F_\phi^-(\lambda_i), T_\phi^+(\lambda_i), M_\phi^+(\lambda_i), C_\phi^+(\lambda_i), U_\phi^+(\lambda_i),$
$I_\phi^+(\lambda_i), K_\phi^+(\lambda_i), F_\phi^+(\lambda_i)], i = 1, 2, 3, \ldots \ldots n$ be a group of BSVHNNs, over ϕ. The combined value BSVHMNA $(\lambda_1, \lambda_2, \lambda_3 \ldots \ldots \ldots . \lambda_n)$ is also an BSVHNN.

Proof: Assume that
$\lambda_i = [T_\phi^-(\lambda_i), M_\phi^-(\lambda_i), C_\phi^-(\lambda_i), U_\phi^-(\lambda_i), I_\phi^-(\lambda_i), K_\phi^-(\lambda_i), F_\phi^-(\lambda_i), T_\phi^+(\lambda_i), M_\phi^+(\lambda_i), C_\phi^+(\lambda_i), U_\phi^+(\lambda_i),$
$I_\phi^+(\lambda_i), K_\phi^+(\lambda_i), F_\phi^+(\lambda_i)], i = 1, 2, 3, \ldots \ldots n$ be a finite group of BSVHNNs, over ϕ. Therefore, λ_i is an BSVHNN.
Now,

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$\sum_{i=1}^{2}\lambda_i = (\lambda_1 + \lambda_2)$
$= [-(-T_\phi^-(\lambda_1) - T_\phi^-(\lambda_2) - T_\phi^-(\lambda_1).T_\phi^-(\lambda_2)), -(-M_\phi^-(\lambda_1) - M_\phi^-(\lambda_2) - M_\phi^-(\lambda_1).M_\phi^-(\lambda_2)),$
$-(-C_\phi^-(\lambda_1) - C_\phi^-(\lambda_2) - C_\phi^-(\lambda_1).C_\phi^-(\lambda_2)), -U_\phi^-(\lambda_1).U_\phi^-(\lambda_2)), -(-I_\phi^-(\lambda_1) - I_\phi^-(\lambda_2) - I_\phi^-(\lambda_1).I_\phi^-(\lambda_2)),$
$-(-K_\phi^-(\lambda_1) - K_\phi^-(\lambda_2) - K_\phi^-(\lambda_1).K_\phi^-(\lambda_2)), -(-F_\phi^-(\lambda_1) - F_\phi^-(\lambda_2) - F_\phi^-(\lambda_1).F_\phi^-(\lambda_2)),$
$(T_\phi^+(\lambda_1).T_\phi^+(\lambda_2), M_\phi^+(\lambda_1).M_\phi^+(\lambda_2), C_\phi^+(\lambda_1).C_\phi^+(\lambda_2), U_\phi^+(\lambda_1) + U_\phi^+(\lambda_2) - U_\phi^+(\lambda_1).U_\phi^+(\lambda_2),$
$I_\phi^+(\lambda_1).I_\phi^+(\lambda_2), K_\phi^+(\lambda_1).K_\phi^+(\lambda_2), F_\phi^+(\lambda_1).F_\phi^+(\lambda_2)]$
$= [T_\phi^-(\lambda_1,\lambda_2), M_\phi^-(\lambda_1,\lambda_2), C_\phi^-(\lambda_1,\lambda_2), U_\phi^-(\lambda_1,\lambda_2), I_\phi^-(\lambda_1,\lambda_2), K_\phi^-(\lambda_1,\lambda_2), F_\phi^-(\lambda_1,\lambda_2), T_\phi^+(\lambda_1,\lambda_2), M_\phi^+(\lambda_1,\lambda_2),$
$C_\phi^+(\lambda_1,\lambda_2), U_\phi^+(\lambda_1,\lambda_2), I_\phi^+(\lambda_1,\lambda_2), K_\phi^+(\lambda_1,\lambda_2), F_\phi^+(\lambda_1,\lambda_2)]$ (say), which is an BSVHNN.
Assume that $\sum_{i=1}^{n}\lambda_i$ is an BSVHNN over λ for $n = m$,
i.e., $\sum_{i=1}^{m}\lambda_i = [T_\phi^-(\lambda_1,\lambda_2,\ldots\ldots\lambda_m), M_\phi^-(\lambda_1,\lambda_2,\ldots\ldots\lambda_m), C_\phi^-(\lambda_1,\lambda_2,\ldots\ldots\lambda_m), U_\phi^-(\lambda_1,\lambda_2,\ldots\ldots\lambda_m),$
$I_\phi^-(\lambda_1,\lambda_2,\ldots\ldots\lambda_m), K_\phi^-(\lambda_1,\lambda_2,\ldots\ldots\lambda_m), F_\phi^-(\lambda_1,\lambda_2,\ldots\ldots\lambda_m), T_\phi^+(\lambda_1,\lambda_2,\ldots\ldots\lambda_m), M_\phi^+(\lambda_1,\lambda_2,\ldots\ldots\lambda_m),$
$C_\phi^+(\lambda_1,\lambda_2,\ldots\ldots\lambda_m), U_\phi^+(\lambda_1,\lambda_2,\ldots\ldots\lambda_m), I_\phi^+(\lambda_1,\lambda_2,\ldots\ldots\lambda_m), K_\phi^+(\lambda_1,\lambda_2,\ldots\ldots\lambda_m), F_\phi^+(\lambda_1,\lambda_2,\ldots\ldots\lambda_m)]$
is an BSVHNN.
$\sum_{i=1}^{m+1}\lambda_i = \sum_{i=1}^{m}\lambda_i + \lambda_{m+1}$
$= [T_\phi^-(\lambda_1,\lambda_2,\ldots\ldots\lambda_m), M_\phi^-(\lambda_1,\lambda_2,\ldots\ldots\lambda_m), C_\phi^-(\lambda_1,\lambda_2,\ldots\ldots\lambda_m), U_\phi^-(\lambda_1,\lambda_2,\ldots\ldots\lambda_m), I_\phi^-(\lambda_1,\lambda_2,\ldots\ldots\lambda_m),$
$I_\phi^-(\lambda_1,\lambda_2,\ldots\ldots\lambda_m), K_\phi^-(\lambda_1,\lambda_2,\ldots\ldots\lambda_m), F_\phi^-(\lambda_1,\lambda_2,\ldots\ldots\lambda_m), T_\phi^+(\lambda_1,\lambda_2,\ldots\ldots\lambda_m), M_\phi^+(\lambda_1,\lambda_2,\ldots\ldots\lambda_m),$
$C_\phi^+(\lambda_1,\lambda_2,\ldots\ldots\lambda_m), U_\phi^+(\lambda_1,\lambda_2,\ldots\ldots\lambda_m), I_\phi^+(\lambda_1,\lambda_2,\ldots\ldots\lambda_m), K_\phi^+(\lambda_1,\lambda_2,\ldots\ldots\lambda_m), F_\phi^+(\lambda_1,\lambda_2,\ldots\ldots\lambda_m)]$
$+ [T_\phi^-(\lambda_{m+1}), M_\phi^-(\lambda_{m+1}), C_\phi^-(\lambda_{m+1}), U_\phi^-(\lambda_{m+1}), I_\phi^-(\lambda_{m+1}), I_\phi^-(\lambda_{m+1}), K_\phi^-(\lambda_{m+1}), F_\phi^-(\lambda_{m+1}),$
$T_\phi^+(\lambda_{m+1}), M_\phi^+(\lambda_{m+1}), C_\phi^+(\lambda_{m+1}), U_\phi^+(\lambda_{m+1}), I_\phi^+(\lambda_{m+1}), K_\phi^+(\lambda_{m+1}), F_\phi^+(\lambda_{m+1})]$
$= \Big[-\Big(-T_\phi^-(\lambda_1,\lambda_2,\ldots\ldots\lambda_m) - T_\phi^-(\lambda_{m+1}) - T_\phi^-(\lambda_1,\lambda_2,\ldots\ldots\lambda_m).T_\phi^-(\lambda_{m+1})\Big), -(-M_\phi^-(\lambda_1,\lambda_2,\ldots\ldots\lambda_m)$
$- M_\phi^-(\lambda_{m+1}) - M_\phi^-(\lambda_1,\lambda_2,\ldots\ldots\lambda_m).M_\phi^-(\lambda_{m+1}),$
$-\Big(-C_\phi^-(\lambda_1,\lambda_2,\ldots\ldots\lambda_m) - C_\phi^-(\lambda_{m+1}) - C_\phi^-(\lambda_1,\lambda_2,\ldots\ldots\lambda_m).\ C_\phi^-(\lambda_{m+1})\Big),$
$- U_\phi^-(\lambda_1,\lambda_2,\ldots\ldots\lambda_m).U_\phi^-(\lambda_{m+1}), -\Big(-I_\phi^-(\lambda_1,\lambda_2,\ldots\ldots\lambda_m) - I_\phi^-(\lambda_{m+1})$
$- I_\phi^-(\lambda_1,\lambda_2,\ldots\ldots\lambda_m).I_\phi^-(\lambda_{m+1})\Big), -\Big(-K_\phi^-(\lambda_1,\lambda_2,\ldots\ldots\lambda_m) - K_\phi^-(\lambda_{m+1})$
$- K_\phi^-(\lambda_1,\lambda_2,\ldots\ldots\lambda_m).K_\phi^-(\lambda_{m+1})\Big), -\Big(-F_\phi^-(\lambda_1,\lambda_2,\ldots\ldots\lambda_m) - F_\phi^-(\lambda_{m+1})$
$- F_\phi^-(\lambda_1,\lambda_2,\ldots\ldots\lambda_m).F_\phi^-(\lambda_{m+1})\Big), T_\phi^+(\lambda_1,\lambda_2,\ldots\ldots\lambda_m).T_\phi^+(\lambda_{m+1}), M_\phi^+(\lambda_1,\lambda_2,\ldots\ldots\lambda_m).M_\phi^+(\lambda_{m+1}),$
$C_\phi^+(\lambda_1,\lambda_2,\ldots\ldots\lambda_m).C_\phi^+(\lambda_{m+1}), \Big(U_\phi^+(\lambda_1,\lambda_2,\ldots\ldots\lambda_m) + U_\phi^+(\lambda_{m+1})$
$- U_\phi^+(\lambda_1,\lambda_2,\ldots\ldots\lambda_m).\ U_\phi^+(\lambda_{m+1})\Big),$
$I_\phi^+(\lambda_1,\lambda_2,\ldots\ldots\lambda_m).I_\phi^+(\lambda_{m+1}), K_\phi^+(\lambda_1,\lambda_2,\ldots\ldots\lambda_m).K_\phi^+(\lambda_{m+1}), F_\phi^+(\lambda_1,\lambda_2,\ldots\ldots\lambda_m).F_\phi^+(\lambda_{m+1})\Big)]$
$= [T_\phi^-(\lambda_1,\lambda_2,\ldots\ldots\lambda_{m+1}), M_\phi^-(\lambda_1,\lambda_2,\ldots\ldots\lambda_{m+1}), C_\phi^-(\lambda_1,\lambda_2,\ldots\ldots\lambda_{m+1}), U_\phi^-(\lambda_1,\lambda_2,\ldots\ldots\lambda_{m+1}),$
$I_\phi^-(\lambda_1,\lambda_2,\ldots\ldots\lambda_{m+1}), K_\phi^-(\lambda_1,\lambda_2,\ldots\ldots\lambda_{m+1}), F_\phi^-(\lambda_1,\lambda_2,\ldots\ldots\lambda_{m+1}), T_\phi^+(\lambda_1,\lambda_2,\ldots\ldots\lambda_{m+1}),$
$M_\phi^+(\lambda_1,\lambda_2,\ldots\ldots\lambda_{m+1}), C_\phi^+(\lambda_1,\lambda_2,\ldots\ldots\lambda_{m+1}), U_\phi^+(\lambda_1,\lambda_2,\ldots\ldots\lambda_{m+1}), I_\phi^+(\lambda_1,\lambda_2,\ldots\ldots\lambda_{m+1}),$
$K_\phi^+(\lambda_1,\lambda_2,\ldots\ldots\lambda_{m+1}), F_\phi^+(\lambda_1,\lambda_2,\ldots\ldots\lambda_{m+1})]$ (say) which is an BSVHNN.
Therefore, $\sum_{i=1}^{m+1}\lambda_i$ is an BSVHNN. This becomes, $\sum_{i=1}^{n}\lambda_i$ is an BSVHNN for $n = m + 1$.
Hence, $\sum_{i=1}^{n}\lambda_i$ is an BSVHNN for $n = 1$ and $n = 2$.
Again, $\sum_{i=1}^{n}\lambda_i$ is an BSVHNN for $n = m + 1$, whenever it is an BSVHNN for $n = m$. Therefore, based on the idea of mathematical induction, we can say that $\sum_{i=1}^{n}\lambda_i$ is an BSVHNN for each n. Now, from definition 3.8, we can tell that $\frac{1}{n}\sum_{i=1}^{n}\lambda_i$ is an BSVHNN. Hence, BSVHMNA$(\lambda_1,\lambda_2,\ldots\ldots\lambda_m) = \frac{1}{n}\sum_{i=1}^{n}\lambda_i$ is an BSVHNN.

Example 4.1 Assume that
$p = (-0.8, -0.4, -0.6, -0.8, -0.4, -0.5, -0.3, 0.4, 0.2, 0.7, 0.8, 0.2, 0.3, 0.3)$
and $q = (-0.7, -0.4, -0.6, -0.3, -0.6, -0.4, -0.6, 0.4, 0.3, 0.6, 0.4, 0.4, 0.5, 0.8)$
be two BSVHNNs.
Then, BSVHMNA is given by,
$(p, q) = 0.5(p + q) =$
$0.5(-0.94, -0.64, -0.84, -0.24, -0.76, -0.7, -0.72, 0.16, 0.06, 0.42, 0.88, 0.08, 0.15, 0.24) =$
$(-0.9847, -0.8944, -0.9573, -0.0663, -0.9337, -0.9147, -0.9212, 0.0427, 0.0153, 0.1273,$
$0.9685, 0.0206, 0.0398, 0.0663)$. It is also an BSVHNN.

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Definition 4.2 Assume that
$\lambda_i = [\, T_\phi^-(\lambda_i), M_\phi^-(\lambda_i), C_\phi^-(\lambda_i), U_\phi^-(\lambda_i), I_\phi^-(\lambda_i), K_\phi^-(\lambda_i), F_\phi^-(\lambda_i), T_\phi^+(\lambda_i), M_\phi^+(\lambda_i), C_\phi^+(\lambda_i), U_\phi^+(\lambda_i), I_\phi^+(\lambda_i), K_\phi^+(\lambda_i), F_\phi^+(\lambda_i)\,], i = 1, 2, 3, \dots \dots n$ be the group of BSVHNNs, over ϕ. The Bipolar Single-valued Heptapartitioned Geometric Neutrosophic Mean (BSVHGNM) operator is determined as follows:

BSVHNGM $(\lambda_1, \lambda_2, \dots \dots \lambda_m) = (\prod_{i=1}^{n} \lambda_i)^{\frac{1}{n}}$ … … … … … … … … … … … … … … … (2)

Theorem 4.2 Assume that
$\lambda_i = [\, T_\phi^-(\lambda_i), M_\phi^-(\lambda_i), C_\phi^-(\lambda_i), U_\phi^-(\lambda_i), I_\phi^-(\lambda_i), K_\phi^-(\lambda_i), F_\phi^-(\lambda_i), T_\phi^+(\lambda_i), M_\phi^+(\lambda_i), C_\phi^+(\lambda_i), U_\phi^+(\lambda_i), I_\phi^+(\lambda_i), K_\phi^+(\lambda_i), F_\phi^+(\lambda_i)\,], i = 1, 2, 3, \dots \dots n$ be the group of BSVHNNs, over ϕ. The combined value BSVHGNM $(\lambda_1, \lambda_2, \dots \dots \lambda_n)$ is also an BSVHNN.

Proof: Assume that
$\lambda_i = [\, T_\phi^-(\lambda_i), M_\phi^-(\lambda_i), C_\phi^-(\lambda_i), U_\phi^-(\lambda_i), I_\phi^-(\lambda_i), K_\phi^-(\lambda_i), F_\phi^-(\lambda_i), T_\phi^+(\lambda_i), M_\phi^+(\lambda_i), C_\phi^+(\lambda_i), U_\phi^+(\lambda_i), I_\phi^+(\lambda_i), K_\phi^+(\lambda_i), F_\phi^+(\lambda_i)\,], i = 1, 2, 3, \dots \dots n$ be a finite group of BSVHNNs, over ϕ. Therefore, λ_1 is an BSVHNN.
Now,

$$\prod_{i=1}^{2} \lambda_i = \lambda_1 . \lambda_2$$

$$= [\, -T_\phi^-(\lambda_1).T_\phi^-(\lambda_2), -M_\phi^-(\lambda_1).M_\phi^-(\lambda_2), -C_\phi^-(\lambda_1).C_\phi^-(\lambda_2), -(-U_\phi^-(\lambda_1) - U_\phi^-(\lambda_2) - U_\phi^-(\lambda_1).U_\phi^-(\lambda_2)),$$
$$-I_\phi^-(\lambda_1).I_\phi^-(\lambda_2), -K_\phi^-(\lambda_1).K_\phi^-(\lambda_2), -F_\phi^-(\lambda_1).F_\phi^-(\lambda_2), T_\phi^+(\lambda_1) + T_\phi^+(\lambda_2) - T_\phi^+(\lambda_1).T_\phi^+(\lambda_2),$$
$$M_\phi^+(\lambda_1) + M_\phi^+(\lambda_2) - M_\phi^+(\lambda_1).M_\phi^+(\lambda_2), C_\phi^+(\lambda_1) + C_\phi^+(\lambda_2) - C_\phi^+(\lambda_1).C_\phi^+(\lambda_2), U_\phi^+(\lambda_1).U_\phi^+(\lambda_2),$$
$$I_\phi^+(\lambda_1) + I_\phi^+(\lambda_2) - I_\phi^+(\lambda_1).I_\phi^+(\lambda_2), K_\phi^+(\lambda_1) + K_\phi^+(\lambda_2) - K_\phi^+(\lambda_1).K_\phi^+(\lambda_2), F_\phi^+(\lambda_1) + F_\phi^+(\lambda_2) - F_\phi^+(\lambda_1).F_\phi^+(\lambda_2)\,]$$
$$= [\, T_\phi^-(\lambda_1, \lambda_2), M_\phi^-(\lambda_1, \lambda_2), C_\phi^-(\lambda_1, \lambda_2), U_\phi^-(\lambda_1, \lambda_2), I_\phi^-(\lambda_1, \lambda_2), K_\phi^-(\lambda_1, \lambda_2), F_\phi^-(\lambda_1, \lambda_2), T_\phi^+(\lambda_1, \lambda_2), M_\phi^+(\lambda_1, \lambda_2),$$
$C_\phi^+(\lambda_1, \lambda_2), U_\phi^+(\lambda_1, \lambda_2), I_\phi^+(\lambda_1, \lambda_2), K_\phi^+(\lambda_1, \lambda_2), F_\phi^+(\lambda_1, \lambda_2)\,]$ (say), which is an BSVHNN.

Suppose that, $\prod_{i=1}^{n} \lambda_i$ is an BSVHNN over ϕ for $n = m$,
i.e. $\prod_{i=1}^{m} \lambda_i = [\, T_\phi^-(\lambda_1, \lambda_2, \dots \dots \lambda_m), M_\phi^-(\lambda_1, \lambda_2, \dots \dots \lambda_m), C_\phi^-(\lambda_1, \lambda_2, \dots \dots \lambda_m), U_\phi^-(\lambda_1, \lambda_2, \dots \dots \lambda_m),$
$I_\phi^-(\lambda_1, \lambda_2, \dots \dots \lambda_m), K_\phi^-(\lambda_1, \lambda_2, \dots \dots \lambda_m), F_\phi^-(\lambda_1, \lambda_2, \dots \dots \lambda_m), T_\phi^+(\lambda_1, \lambda_2, \dots \dots \lambda_m), M_\phi^+(\lambda_1, \lambda_2, \dots \dots \lambda_m),$
$C_\phi^+(\lambda_1, \lambda_2, \dots \dots \lambda_m), U_\phi^+(\lambda_1, \lambda_2, \dots \dots \lambda_m), I_\phi^+(\lambda_1, \lambda_2, \dots \dots \lambda_m), K_\phi^+(\lambda_1, \lambda_2, \dots \dots \lambda_m), F_\phi^+(\lambda_1, \lambda_2, \dots \dots \lambda_m)\,]$ is an BSVHNN.
Now,

$$\prod_{i=1}^{m+1} \lambda_i = \lambda_{m+1} . \prod_{i=1}^{m} \lambda_i$$
$$= [\, T_\phi^-(\lambda_{m+1}), M_\phi^-(\lambda_{m+1}), C_\phi^-(\lambda_{m+1}), U_\phi^-(\lambda_{m+1}), I_\phi^-(\lambda_{m+1}), K_\phi^-(\lambda_{m+1}), F_\phi^-(\lambda_{m+1}),$$
$$T_\phi^+(\lambda_{m+1}), M_\phi^+(\lambda_{m+1}), C_\phi^+(\lambda_{m+1}), U_\phi^+(\lambda_{m+1}), I_\phi^+(\lambda_{m+1}), K_\phi^+(\lambda_{m+1}), F_\phi^+(\lambda_{m+1})] . [\, T_\phi^-(\lambda_1, \lambda_2, \dots \dots \lambda_m),$$
$$U_\phi^-(\lambda_1, \lambda_2, \dots \dots \lambda_m), I_\phi^-(\lambda_1, \lambda_2, \dots \dots \lambda_m), K_\phi^-(\lambda_1, \lambda_2, \dots \dots \lambda_m), F_\phi^-(\lambda_1, \lambda_2, \dots \dots \lambda_m), T_\phi^+(\lambda_1, \lambda_2, \dots \dots \lambda_m),$$
$$M_\phi^+(\lambda_1, \lambda_2, \dots \dots \lambda_m), C_\phi^+(\lambda_1, \lambda_2, \dots \dots \lambda_m), U_\phi^+(\lambda_1, \lambda_2, \dots \dots \lambda_m), I_\phi^+(\lambda_1, \lambda_2, \dots \dots \lambda_m), K_\phi^+(\lambda_1, \lambda_2, \dots \dots \lambda_m),$$
$$F_\phi^+(\lambda_1, \lambda_2, \dots \dots \lambda_m)\,]$$
$$= [-T_\phi^-(\lambda_{m+1}) . T_\phi^-(\lambda_1, \lambda_2, \dots \dots \lambda_m), -M_\phi^-(\lambda_{m+1}) . M_\phi^-(\lambda_1, \lambda_2, \dots \dots \lambda_m), -C_\phi^-(\lambda_{m+1}) . C_\phi^-(\lambda_1, \lambda_2, \dots \dots \lambda_m),$$
$$-\left(-U_\phi^-(\lambda_{m+1}) - U_\phi^-(\lambda_1, \lambda_2, \dots \dots \lambda_m) - U_\phi^-(\lambda_{m+1}) . U_\phi^-(\lambda_1, \lambda_2, \dots \dots \lambda_m)\right),$$
$$-K_\phi^-(\lambda_{m+1}) . K_\phi^-(\lambda_1, \lambda_2, \dots \dots \lambda_m), -F_\phi^-(\lambda_{m+1}) . F_\phi^-(\lambda_1, \lambda_2, \dots \dots \lambda_m),$$
$$T_\phi^+(\lambda_{m+1}) + T_\phi^+(\lambda_1, \lambda_2, \dots \dots \lambda_m) - T_\phi^+(\lambda_{m+1}).T_\phi^+(\lambda_1, \lambda_2, \dots \dots \lambda_m),$$
$$M_\phi^+(\lambda_{m+1}) + M_\phi^+(\lambda_1, \lambda_2, \dots \dots \lambda_m) - M_\phi^+(\lambda_{m+1}).M_\phi^+(\lambda_1, \lambda_2, \dots \dots \lambda_m),$$
$$C_\phi^+(\lambda_{m+1}) + C_\phi^+(\lambda_1, \lambda_2, \dots \dots \lambda_m) - C_\phi^+(\lambda_{m+1}).C_\phi^+(\lambda_1, \lambda_2, \dots \dots \lambda_m),$$
$$U_\phi^+(\lambda_{m+1}) . U_\phi^+(\lambda_1, \lambda_2, \dots \dots \lambda_m), I_\phi^+(\lambda_{m+1}) + I_\phi^+(\lambda_1, \lambda_2, \dots \dots \lambda_m) - I_\phi^+(\lambda_{m+1}).I_\phi^+(\lambda_1, \lambda_2, \dots \dots \lambda_m),$$
$$K_\phi^+(\lambda_{m+1}) + K_\phi^+(\lambda_1, \lambda_2, \dots \dots \lambda_m) - K_\phi^+(\lambda_{m+1}).K_\phi^+(\lambda_1, \lambda_2, \dots \dots \lambda_m),$$
$$F_\phi^+(\lambda_{m+1}) + F_\phi^+(\lambda_1, \lambda_2, \dots \dots \lambda_m) - F_\phi^+(\lambda_{m+1}).F_\phi^+(\lambda_1, \lambda_2, \dots \dots \lambda_m)\,]$$
$$= [\, T_\phi^-(\lambda_1, \lambda_2, \dots \dots \lambda_{m+1}), M_\phi^-(\lambda_1, \lambda_2, \dots \dots \lambda_{m+1}), C_\phi^-(\lambda_1, \lambda_2, \dots \dots \lambda_{m+1}), U_\phi^-(\lambda_1, \lambda_2, \dots \dots \lambda_{m+1}),$$
$$I_\phi^-(\lambda_1, \lambda_2, \dots \dots \lambda_{m+1}), K_\phi^-(\lambda_1, \lambda_2, \dots \dots \lambda_{m+1}), F_\phi^-(\lambda_1, \lambda_2, \dots \dots \lambda_{m+1}), T_\phi^+(\lambda_1, \lambda_2, \dots \dots \lambda_{m+1}),$$
$$M_\phi^+(\lambda_1, \lambda_2, \dots \dots \lambda_{m+1}), C_\phi^+(\lambda_1, \lambda_2, \dots \dots \lambda_{m+1}), U_\phi^+(\lambda_1, \lambda_2, \dots \dots \lambda_{m+1}), I_\phi^+(\lambda_1, \lambda_2, \dots \dots \lambda_{m+1}),$$
$K_\phi^+(\lambda_1, \lambda_2, \dots \dots \lambda_{m+1}), F_\phi^+(\lambda_1, \lambda_2, \dots \dots \lambda_{m+1})\,]$ (say) which is an BSVHNN.

Therefore, $\prod_{i=1}^{m+1} \lambda_i$ is an BSVHNN. This becomes, $\prod_{i=1}^{n} \lambda_i$ is an BSVHNN for $n = m + 1$.
Hence, $\prod_{i=1}^{n} \lambda_i$ is an BSVHNN for $n = 1$ and $n = 2$. Again, $\prod_{i=1}^{n} \lambda_i$ is an BSVHNN for $n = m + 1$, whenever it is an BSVHNN for $n = m$. Therefore, based on the idea of mathematical induction, we can tell that $\prod_{i=1}^{n} \lambda_i$ is an BSVHN for each n. Now, from definition 3.8. we can say that $(\prod_{i=1}^{n} \lambda_i)^{\frac{1}{n}}$ is an BSVHNN. Hence, BSVHGNM

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$(\lambda_1, \lambda_2, \dots \dots \lambda_n) = (\prod_{i=1}^{n} \lambda_i)^{\frac{1}{n}}$ is an BSVHNN.

Example 4.2 Assume that

$p = (-0.8, -0.4, -0.6, -0.8, -0.4, -0.5, -0.3, 0.4, 0.2, 0.7, 0.8, 0.2, 0.3, 0.3)$

and $q = (-0.7, -0.4, -0.6, -0.3, -0.6, -0.4, -0.6, 0.4, 0.3, 0.6, 0.4, 0.4, 0.5, 0.8)$

be two BSVHNNs. Then, BSVHGNM

$(p, q) = (p.q)^{0.5} =$

$(-0.56, -0.16, -0.36, -0.86, -0.24, -0.20, -0.18, 0.64, 0.44, 0.88, 0.32, 0.52, 0.65, 0.86)^{0.5} =$

$(-0.7483, -0.4, -0.6, -0.9274, -0.4899, -0.4472, -0.4243, 0.8, 0.6633, 0.9381, 0.5657, 0.7211,$
$0.8062, 0.9274)$. It is also an BSVHNN.

5. Score and Accuracy Functions under the BSVHNS Environment

Definition 5.1 Suppose that

$\lambda = [T_\phi^-(\lambda), M_\phi^-(\lambda), C_\phi^-(\lambda), U_\phi^-(\lambda), I_\phi^-(\lambda), K_\phi^-(\lambda), F_\phi^-(\lambda), T_\phi^+(\lambda), M_\phi^+(\lambda), C_\phi^+(\lambda), U_\phi^+(\lambda), I_\phi^+(\lambda), K_\phi^+(\lambda), F_\phi^+(\lambda)]$

be an BSVHNN over ϕ. Then, the score function and accuracy function are determined by:

$$SF(\lambda) = \frac{-T_\phi^-(\lambda) - M_\phi^-(\lambda) - C_\phi^-(\lambda) + 1 + U_\phi^-(\lambda) - I_\phi^-(\lambda) - K_\phi^-(\lambda) - F_\phi^-(\lambda) + 1 - T_\phi^+(\lambda) + 1 - M_\phi^+(\lambda) + 1 - C_\phi^+(\lambda) - U_\phi^+(\lambda) + 1 - I_\phi^+(\lambda) + 1 - K_\phi^+(\lambda) + 1 - F_\phi^+(\lambda)}{14}$$

$$AF(\lambda) = \frac{-T_\phi^-(\lambda) - C_\phi^-(\lambda) + U_\phi^-(\lambda) - K_\phi^-(\lambda) - F_\phi^-(\lambda) - T_\phi^+(\lambda) - C_\phi^+(\lambda) + U_\phi^+(\lambda) - K_\phi^+(\lambda) - F_\phi^+(\lambda)}{5}$$

Example 5.1 Given

$\lambda = (-0.8, -0.4, -0.6, -0.8, -0.4, -0.5, -0.3, 0.4, 0.2, 0.7, 0.8, 0.2, 0.3, 0.3)$ be an BSVHNN as specified in Example 4.1. Then, $SF(\lambda) = 0.55$ and $AF(\lambda) = 0.1$.

Definition 5.2. Given

$\lambda = [T_\phi^-(\lambda), M_\phi^-(\lambda), C_\phi^-(\lambda), U_\phi^-(\lambda), I_\phi^-(\lambda), K_\phi^-(\lambda), F_\phi^-(\lambda), T_\phi^+(\lambda), M_\phi^+(\lambda), C_\phi^+(\lambda), U_\phi^+(\lambda), I_\phi^+(\lambda), K_\phi^+(\lambda), F_\phi^+(\lambda)]$

and

$\gamma = [T_\phi^-(\gamma), M_\phi^-(\gamma), C_\phi^-(\gamma), U_\phi^-(\gamma), I_\phi^-(\gamma), K_\phi^-(\gamma), F_\phi^-(\gamma), T_\phi^+(\gamma), M_\phi^+(\gamma), C_\phi^+(\gamma), U_\phi^+(\gamma), I_\phi^+(\gamma), K_\phi^+(\gamma), F_\phi^+(\gamma)]$

be any two BSVHNNs over ϕ. Then,

$(i) SF(\lambda) > SF(\gamma) \Rightarrow \lambda > \gamma;$

$(ii) SF(\lambda) = SF(\gamma), \quad AF(\lambda) > AF(\gamma) \Rightarrow \lambda > \gamma;$

$(iii) SF(\lambda) = SF(\gamma), \quad AF(\lambda) = AF(\gamma), \quad T_\phi^+(\lambda) > T_\phi^+(\gamma), \; T_\phi^-(\lambda) < T_\phi^-(\gamma) \Rightarrow \lambda > \gamma.$

Theorem 5.1

An BSVHNN has bounds on both its score function and accuracy function.

Proof: Suppose that

$\gamma = [T_\phi^-(\gamma), M_\phi^-(\gamma), C_\phi^-(\gamma), U_\phi^-(\gamma), I_\phi^-(\gamma), K_\phi^-(\gamma), F_\phi^-(\gamma), T_\phi^+(\gamma), M_\phi^+(\gamma), C_\phi^+(\gamma), U_\phi^+(\gamma), I_\phi^+(\gamma), K_\phi^+(\gamma), F_\phi^+(\gamma)]$

be an BSVHNN.

Therefore, $-1 \le T_\phi^-(\gamma) \le 0, -1 \le M_\phi^-(\gamma) \le 0, \; -1 \le C_\phi^-(\gamma) \le 0, -1 \le U_\phi^-(\gamma) \le 0, -1 \le I_\phi^-(\gamma) \le 0, -1 \le K_\phi^-(\gamma) \le 0, \; -1 \le F_\phi^-(\gamma) \le 0, \; 0 \le T_\phi^+(\gamma) \le 1, 0 \le M_\phi^+(\gamma) \le 1, 0 \le C_\phi^+(\gamma) \le 1, 0 \le U_\phi^+(\gamma) \le 1, 0 \le I_\phi^+(\gamma) \le 1, 0 \le K_\phi^+(\gamma) \le 1, 0 \le F_\phi^+(\gamma) \le 1.$

This implies, $0 \le 1 + T_\phi^-(\gamma) + T_\phi^+(\gamma) \le 2, \; 0 \le 1 + M_\phi^-(\gamma) + M_\phi^+(\gamma) \le 2, \; 0 \le 1 + C_\phi^-(\gamma) + C_\phi^+(\gamma) \le 2, 0 \le 1 + U_\phi^-(\gamma) + U_\phi^+(\gamma) \le 2, 0 \le 1 + I_\phi^-(\gamma) + I_\phi^+(\gamma) \le 2, 0 \le 1 + K_\phi^-(\gamma) + K_\phi^+(\gamma) \le 2, 0 \le 1 + F_\phi^-(\gamma) + F_\phi^+(\gamma) \le 2.$

Therefore,

$$0 \le -T_\phi^-(\lambda) + 1 - T_\phi^+(\lambda) - M_\phi^-(\lambda) + 1 - M_\phi^+(\lambda) - C_\phi^-(\lambda) + 1 - C_\phi^+(\lambda) + 1 + U_\phi^-(\lambda) - U_\phi^+(\lambda) - I_\phi^-(\lambda) + 1 - I_\phi^+(\lambda) - K_\phi^-(\lambda) + 1 - K_\phi^+(\lambda) - F_\phi^-(\lambda) + 1 - F_\phi^+(\lambda) \le 14$$

$$\Rightarrow 0 \le -T_\phi^-(\lambda) - M_\phi^-(\lambda) - C_\phi^-(\lambda) + 1 + U_\phi^-(\lambda) - I_\phi^-(\lambda) - K_\phi^-(\lambda) - F_\phi^-(\lambda) + 1 - T_\phi^+(\lambda) + 1 - M_\phi^+(\lambda) + 1 - C_\phi^+(\lambda) - U_\phi^+(\lambda) + 1 - I_\phi^+(\lambda) + 1 - K_\phi^+(\lambda) + 1 - F_\phi^+(\lambda) \le 14$$

$$\Rightarrow 0 \le \frac{-T_\phi^-(\lambda) - M_\phi^-(\lambda) - C_\phi^-(\lambda) + 1 + U_\phi^-(\lambda) - I_\phi^-(\lambda) - K_\phi^-(\lambda) - F_\phi^-(\lambda) + 1 - T_\phi^+(\lambda) + 1 - M_\phi^+(\lambda) + 1 - C_\phi^+(\lambda) - U_\phi^+(\lambda) + 1 - I_\phi^+(\lambda) + 1 - K_\phi^+(\lambda) + 1 - F_\phi^+(\lambda)}{14} \le 1$$

$$SF(\lambda) = \frac{-T_\phi^-(\lambda) - M_\phi^-(\lambda) - C_\phi^-(\lambda) + 1 + U_\phi^-(\lambda) - I_\phi^-(\lambda) - K_\phi^-(\lambda) - F_\phi^-(\lambda) + 1 - T_\phi^+(\lambda) + 1 - M_\phi^+(\lambda) + 1 - C_\phi^+(\lambda) - U_\phi^+(\lambda) + 1 - I_\phi^+(\lambda) + 1 - K_\phi^+(\lambda) + 1 - F_\phi^+(\lambda)}{14}$$

$\Rightarrow 0 \le SF(\lambda) \le 1.$

The score function is hence bounded.

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Again,
$-1 \le T_\phi^-(\gamma) + T_\phi^+(\gamma) \le 1, -1 \le M_\phi^-(\gamma) + M_\phi^+(\gamma) \le 1, -1 \le C_\phi^-(\gamma) + C_\phi^+(\gamma) \le 1,$
$-1 \le U_\phi^-(\gamma) + U_\phi^+(\gamma) \le 1, -1 \le I_\phi^-(\gamma) + I_\phi^+(\gamma) \le 1, -1 \le K_\phi^-(\gamma) + K_\phi^+(\gamma) \le 1,$
$-1 \le F_\phi^-(\gamma) + F_\phi^+(\gamma) \le 1.$
This implies

$$-5 \le -T_\phi^-(\gamma) - T_\phi^+(\gamma) - C_\phi^-(\gamma) - C_\phi^+(\gamma) + U_\phi^-(\gamma) + U_\phi^+(\gamma) - K_\phi^-(\gamma) - K_\phi^+(\gamma) - F_\phi^-(\gamma) - F_\phi^+(\gamma) \le 5$$

$$\Rightarrow -1 \le \frac{-T_\phi^-(\gamma) - C_\phi^-(\gamma) + U_\phi^-(\gamma) - K_\phi^-(\gamma) - F_\phi^-(\gamma) - T_\phi^+(\gamma) - C_\phi^+(\gamma) + U_\phi^+(\gamma) - K_\phi^+(\gamma) - F_\phi^+(\gamma)}{5} \le 1$$

$\Rightarrow -1 \le AF(\gamma) \le 1$
Hence, the accuracy function is bounded.

Theorem 5.2

The score function and accuracy function of an BSVHNN are monotonic increasing.

Proof: Given that

$\lambda = [T_\phi^-(\lambda), M_\phi^-(\lambda), C_\phi^-(\lambda), U_\phi^-(\lambda), I_\phi^-(\lambda), K_\phi^-(\lambda), F_\phi^-(\lambda), T_\phi^+(\lambda), M_\phi^+(\lambda), C_\phi^+(\lambda), U_\phi^+(\lambda), I_\phi^+(\lambda), K_\phi^+(\lambda), F_\phi^+(\lambda)]$
and
$\gamma = [T_\phi^-(\gamma), M_\phi^-(\gamma), C_\phi^-(\gamma), U_\phi^-(\gamma), I_\phi^-(\gamma), K_\phi^-(\gamma), F_\phi^-(\gamma), T_\phi^+(\gamma), M_\phi^+(\gamma), C_\phi^+(\gamma), U_\phi^+(\gamma), I_\phi^+(\gamma), K_\phi^+(\gamma), F_\phi^+(\gamma)]$
be any two BSVHNNs over ϕ such that $\lambda \subseteq \gamma$.
Therefore,

$$T_\phi^-(\lambda) \ge T_\phi^-(\gamma), M_\phi^-(\lambda) \ge M_\phi^-(\gamma), C_\phi^-(\lambda) \ge C_\phi^-(\gamma), U_\phi^-(\lambda) \le U_\phi^-(\gamma), I_\phi^-(\lambda) \ge I_\phi^-(\gamma), K_\phi^-(\lambda) \ge K_\phi^-(\gamma), F_\phi^-(\lambda) \ge F_\phi^-(\gamma), T_\phi^+(\lambda) \ge T_\phi^+(\gamma), M_\phi^+(\lambda) \ge M_\phi^+(\gamma), C_\phi^+(\lambda) \ge C_\phi^+(\gamma), U_\phi^+(\lambda) \le U_\phi^+(\gamma), I_\phi^+(\lambda) \ge I_\phi^+(\gamma), K_\phi^+(\lambda) \ge K_\phi^+(\gamma), F_\phi^+(\lambda) \ge F_\phi^+(\gamma).$$

It is known that,

$$SF(\lambda) = \frac{-T_\phi^-(\lambda) - M_\phi^-(\lambda) - C_\phi^-(\lambda) + 1 + U_\phi^-(\lambda) - I_\phi^-(\lambda) - K_\phi^-(\lambda) - F_\phi^-(\lambda) + 1 - T_\phi^+(\lambda) + 1 - M_\phi^+(\lambda) + 1 - C_\phi^+(\lambda) - U_\phi^+(\lambda) + 1 - I_\phi^+(\lambda) + 1 - K_\phi^+(\lambda) + 1 - F_\phi^+(\lambda)}{14}$$

$$SF(\gamma) = \frac{-T_\phi^-(\gamma) - M_\phi^-(\gamma) - C_\phi^-(\gamma) + 1 + U_\phi^-(\gamma) - I_\phi^-(\gamma) - K_\phi^-(\gamma) - F_\phi^-(\gamma) + 1 - T_\phi^+(\gamma) + 1 - M_\phi^+(\gamma) + 1 - C_\phi^+(\gamma) - U_\phi^+(\gamma) + 1 - I_\phi^+(\gamma) + 1 - K_\phi^+(\gamma) + 1 - F_\phi^+(\gamma)}{14}$$

$$AF(\lambda) = \frac{-T_\phi^-(\lambda) - C_\phi^-(\lambda) + U_\phi^-(\lambda) - K_\phi^-(\lambda) - F_\phi^-(\lambda) - T_\phi^+(\lambda) - C_\phi^+(\lambda) + U_\phi^+(\lambda) - K_\phi^+(\lambda) - F_\phi^+(\lambda)}{5}$$

$$AF(\gamma) = \frac{-T_\phi^-(\gamma) - C_\phi^-(\gamma) + U_\phi^-(\gamma) - K_\phi^-(\gamma) - F_\phi^-(\gamma) - T_\phi^+(\gamma) - C_\phi^+(\gamma) + U_\phi^+(\gamma) - K_\phi^+(\gamma) - F_\phi^+(\gamma)}{5}$$

Now,

$$SF(\lambda) - SF(\gamma) = \frac{-T_\phi^-(\lambda) - M_\phi^-(\lambda) - C_\phi^-(\lambda) + 1 + U_\phi^-(\lambda) - I_\phi^-(\lambda) - K_\phi^-(\lambda) - F_\phi^-(\lambda) + 1 - T_\phi^+(\lambda) + 1 - M_\phi^+(\lambda) + 1 - C_\phi^+(\lambda) - U_\phi^+(\lambda) + 1 - I_\phi^+(\lambda) + 1 - K_\phi^+(\lambda) + 1 - F_\phi^+(\lambda)}{14} - \frac{-T_\phi^-(\gamma) - M_\phi^-(\gamma) - C_\phi^-(\gamma) + 1 + U_\phi^-(\gamma) - I_\phi^-(\gamma) - K_\phi^-(\gamma) - F_\phi^-(\gamma) + 1 - T_\phi^+(\gamma) + 1 - M_\phi^+(\gamma) + 1 - C_\phi^+(\gamma) - U_\phi^+(\gamma) + 1 - I_\phi^+(\gamma) + 1 - K_\phi^+(\gamma) + 1 - F_\phi^+(\gamma)}{14}$$

≥ 0 [since $\lambda \subseteq \gamma$]
This implies, $SF(\gamma) \ge SF(\lambda)$, i.e., The scoring function increases monotonically.
Now,

$$AF(\gamma) \ge AF(\lambda) = \frac{-T_\phi^-(\gamma) - C_\phi^-(\gamma) + U_\phi^-(\gamma) - K_\phi^-(\gamma) - F_\phi^-(\gamma) - T_\phi^+(\gamma) - C_\phi^+(\gamma) + U_\phi^+(\gamma) - K_\phi^+(\gamma) - F_\phi^+(\gamma)}{5} - \frac{-T_\phi^-(\lambda) - C_\phi^-(\lambda) + U_\phi^-(\lambda) - K_\phi^-(\lambda) - F_\phi^-(\lambda) - T_\phi^+(\lambda) - C_\phi^+(\lambda) + U_\phi^+(\lambda) - K_\phi^+(\lambda) - F_\phi^+(\lambda)}{5}$$

≥ 0 [since $\gamma \subseteq \lambda$]
This implies, $AF(\gamma) \ge AF(\lambda)$, i.e., the accuracy function increases monotonically.
In light of this, the accuracy and score functions are monotonically growing functions.

6. BSVHNS – MADM Strategy based on BSVHMNA Operator

Suppose that $E = \{E_1, E_2 \dots \dots \dots . E_n\}$ be a fixed set of alternatives, and $\mathcal{D} = \{\mathcal{D}_1, \mathcal{D}_2 \dots \dots \dots \mathcal{D}_m\}$ be a collection of attributes. Each alternative is evaluated by the decision maker who is involved in the decision-making process $E_{i,}$ $(i = 1,2 \dots \dots \dots . n)$ over the attribute P_j $(j = 1, 2, \dots \dots m)$ in terms of BSVHNNs. A decision matrix can express the entire evaluation information of all alternatives. The proposed BSVHNS-MADM plan (see Figure 1) is using the following steps:

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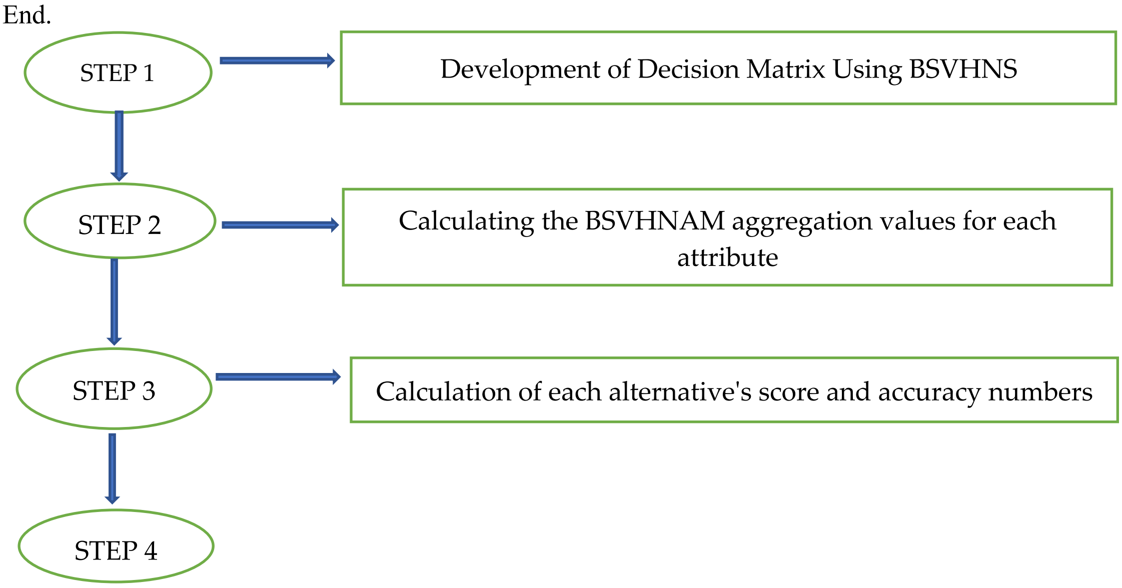
Step 1 : Create the decision matrix with BSVHNSs.
Each alternative's whole evaluation information E_i $(i = 1,2 \ldots\ldots n)$ based on the attributes $\mathcal{D}_j$ $(j = 1,2, \ldots\ldots m)$ is expressed in terms of BSVHNS

$$EI_{(E_i,\mathcal{D}_j)} = \{(\mathcal{D}_j, T_{ij}^{-}(E_i,\mathcal{D}_j), M_{ij}^{-}(E_i,\mathcal{D}_j), C_{ij}^{-}(E_i,\mathcal{D}_j), U_{ij}^{-}(E_i,\mathcal{D}_j), I_{ij}^{-}(E_i,\mathcal{D}_j), K_{ij}^{-}(E_i,\mathcal{D}_j), F_{ij}^{-}(E_i,\mathcal{D}_j), T_{ij}^{+}(E_i,\mathcal{D}_j), M_{ij}^{+}(E_i,\mathcal{D}_j), C_{ij}^{+}(E_i,\mathcal{D}_j), U_{ij}^{+}(E_i,\mathcal{D}_j), I_{ij}^{+}(E_i,\mathcal{D}_j), K_{ij}^{+}(E_i,\mathcal{D}_j), F_{ij}^{+}(E_i,\mathcal{D}_j))\}$$

indicate the evaluation data of E $(i = 1,2 \ldots\ldots n)$ base on $\mathcal{D}_j$ $(j = 1,2, \ldots\ldots m)$.
Then the Decision Matrix $(\mathcal{DM}[E|\mathcal{D}])$ can be stated as
$\mathcal{DM}[E|\mathcal{D}] =$
Step 2 : In this step, the decision maker determines the aggregation values $(E_i \mid \mathcal{D}_1, \mathcal{D}_2 \ldots \ldots \mathcal{D}_m) =$

	$\mathcal{D}_1$	$\mathcal{D}_2$	…	$\mathcal{D}_m$
E_1	$[T_{11}^{-}(E_1,\mathcal{D}_1), M_{11}^{-}(E_1,\mathcal{D}_1), C_{11}^{-}(E_1,\mathcal{D}_1), U_{11}^{-}(E_1,\mathcal{D}_1), I_{11}^{-}(E_1,\mathcal{D}_1), K_{11}^{-}(E_1,\mathcal{D}_1), F_{11}^{-}(E_1,\mathcal{D}_1), T_{11}^{+}(E_1,\mathcal{D}_1), M_{11}^{+}(E_1,\mathcal{D}_1), C_{11}^{+}(E_1,\mathcal{D}_1), U_{11}^{+}(E_1,\mathcal{D}_1), I_{11}^{+}(E_1,\mathcal{D}_1), K_{11}^{+}(E_1,\mathcal{D}_1), F_{11}^{+}(E_1,\mathcal{D}_1)]$	$[T_{12}^{-}(E_1,\mathcal{D}_2), M_{12}^{-}(E_1,\mathcal{D}_2), C_{12}^{-}(E_1,\mathcal{D}_2), U_{12}^{-}(E_1,\mathcal{D}_2), I_{12}^{-}(E_1,\mathcal{D}_2), K_{12}^{-}(E_1,\mathcal{D}_2), F_{12}^{-}(E_1,\mathcal{D}_2), T_{12}^{+}(E_1,\mathcal{D}_2), M_{12}^{+}(E_1,\mathcal{D}_2), C_{12}^{+}(E_1,\mathcal{D}_2), U_{12}^{+}(E_1,\mathcal{D}_2), I_{12}^{+}(E_1,\mathcal{D}_2), K_{12}^{+}(E_1,\mathcal{D}_2), F_{12}^{+}(E_1,\mathcal{D}_2)]$	…	$[T_{1m}^{-}(E_1,\mathcal{D}_m), M_{1m}^{-}(E_1,\mathcal{D}_m), C_{1m}^{-}(E_1,\mathcal{D}_m), U_{1m}^{-}(E_1,\mathcal{D}_m), I_{1m}^{-}(E_1,\mathcal{D}_m), K_{1m}^{-}(E_1,\mathcal{D}_m), F_{1m}^{-}(E_1,\mathcal{D}_m), T_{1m}^{+}(E_1,\mathcal{D}_m), M_{1m}^{+}(E_1,\mathcal{D}_m), C_{1m}^{+}(E_1,\mathcal{D}_m), U_{1m}^{+}(E_1,\mathcal{D}_m), I_{1m}^{+}(E_1,\mathcal{D}_m), K_{1m}^{+}(E_1,\mathcal{D}_m), F_{1m}^{+}(E_1,\mathcal{D}_m)]$
E_2	$[T_{21}^{-}(E_2,\mathcal{D}_1), M_{21}^{-}(E_2,\mathcal{D}_1), C_{21}^{-}(E_2,\mathcal{D}_1), U_{21}^{-}(E_2,\mathcal{D}_1), I_{21}^{-}(E_2,\mathcal{D}_1), K_{21}^{-}(E_2,\mathcal{D}_1), F_{21}^{-}(E_2,\mathcal{D}_1), T_{21}^{+}(E_2,\mathcal{D}_1), M_{21}^{+}(E_2,\mathcal{D}_1), C_{21}^{+}(E_2,\mathcal{D}_1), U_{21}^{+}(E_2,\mathcal{D}_1), I_{21}^{+}(E_2,\mathcal{D}_1), K_{21}^{+}(E_2,\mathcal{D}_1), F_{21}^{+}(E_2,\mathcal{D}_1)]$	$[T_{22}^{-}(E_2,\mathcal{D}_2), M_{22}^{-}(E_2,\mathcal{D}_2), C_{22}^{-}(E_2,\mathcal{D}_2), U_{22}^{-}(E_2,\mathcal{D}_2), I_{22}^{-}(E_2,\mathcal{D}_2), K_{22}^{-}(E_2,\mathcal{D}_2), F_{22}^{-}(E_2,\mathcal{D}_2), T_{22}^{+}(E_2,\mathcal{D}_2), M_{22}^{+}(E_2,\mathcal{D}_2), C_{22}^{+}(E_2,\mathcal{D}_2), U_{22}^{+}(E_2,\mathcal{D}_2), I_{22}^{+}(E_2,\mathcal{D}_2), K_{22}^{+}(E_2,\mathcal{D}_2), F_{22}^{+}(E_2,\mathcal{D}_2)]$	…	$[T_{2m}^{-}(E_2,\mathcal{D}_m), M_{2m}^{-}(E_2,\mathcal{D}_m), C_{2m}^{-}(E_2,\mathcal{D}_m), U_{2m}^{-}(E_2,\mathcal{D}_m), I_{2m}^{-}(E_2,\mathcal{D}_m), K_{2m}^{-}(E_2,\mathcal{D}_m), F_{2m}^{-}(E_2,\mathcal{D}_m), T_{2m}^{+}(E_2,\mathcal{D}_m), M_{2m}^{+}(E_2,\mathcal{D}_m), C_{2m}^{+}(E_2,\mathcal{D}_m), U_{2m}^{+}(E_2,\mathcal{D}_m), I_{2m}^{+}(E_2,\mathcal{D}_m), K_{2m}^{+}(E_2,\mathcal{D}_m), F_{2m}^{+}(E_2,\mathcal{D}_m)]$
…	…	…	…	…
E_n	$[T_{n1}^{-}(E_n,\mathcal{D}_1), M_{n1}^{-}(E_n,\mathcal{D}_1), C_{n1}^{-}(E_n,\mathcal{D}_1), U_{n1}^{-}(E_n,\mathcal{D}_1), I_{n1}^{-}(E_n,\mathcal{D}_1), K_{n1}^{-}(E_n,\mathcal{D}_1), F_{n1}^{-}(E_n,\mathcal{D}_1), T_{n1}^{+}(E_n,\mathcal{D}_1), M_{n1}^{+}(E_n,\mathcal{D}_1), C_{n1}^{+}(E_n,\mathcal{D}_1), U_{n1}^{+}(E_n,\mathcal{D}_1), I_{n1}^{+}(E_n,\mathcal{D}_1), K_{n1}^{+}(E_n,\mathcal{D}_1), F_{n1}^{+}(E_n,\mathcal{D}_1)]$	$[T_{n2}^{-}(E_n,\mathcal{D}_2), M_{n2}^{-}(E_n,\mathcal{D}_2), C_{n2}^{-}(E_n,\mathcal{D}_2), U_{n2}^{-}(E_n,\mathcal{D}_2), I_{n2}^{-}(E_n,\mathcal{D}_2), K_{n2}^{-}(E_n,\mathcal{D}_2), F_{n2}^{-}(E_n,\mathcal{D}_2), T_{n2}^{+}(E_n,\mathcal{D}_2), M_{n2}^{+}(E_n,\mathcal{D}_2), C_{n2}^{+}(E_n,\mathcal{D}_2), U_{n2}^{+}(E_n,\mathcal{D}_2), I_{n2}^{+}(E_n,\mathcal{D}_2), K_{n2}^{+}(E_n,\mathcal{D}_2), F_{n2}^{+}(E_n,\mathcal{D}_2)]$	…	$[T_{nm}^{-}(E_n,\mathcal{D}_m), M_{nm}^{-}(E_n,\mathcal{D}_m), C_{nm}^{-}(E_n,\mathcal{D}_m), U_{nm}^{-}(E_n,\mathcal{D}_m), I_{nm}^{-}(E_n,\mathcal{D}_m), K_{nm}^{-}(E_n,\mathcal{D}_m), F_{nm}^{-}(E_n,\mathcal{D}_m), T_{nm}^{+}(E_n,\mathcal{D}_m), M_{nm}^{+}(E_n,\mathcal{D}_m), C_{nm}^{+}(E_n,\mathcal{D}_m), U_{nm}^{+}(E_n,\mathcal{D}_m), I_{nm}^{+}(E_n,\mathcal{D}_m), K_{nm}^{+}(E_n,\mathcal{D}_m), F_{nm}^{+}(E_n,\mathcal{D}_m)]$

$BSVHNAM$ $(\mathcal{D}_1, \mathcal{D}_2 \ldots\ldots \mathcal{D}_m)$ of all the attributes for each alternative by using eqn (1). After the determination of aggregation values BSVHMNA $(\mathcal{D}_1, \mathcal{D}_2 \ldots\ldots \mathcal{D}_m)$, the decision maker makes an aggregate decision matrix aggregate-$\mathcal{DM}$.
Step 3 : In this step, the decision maker determine the score and accuracy values of each alternative by using the equation (3) and (4).
Step 4 : In this step, the decision maker ranks the alternatives by using Definition 5.1 and Definition 5.2.
Step 5 : End.



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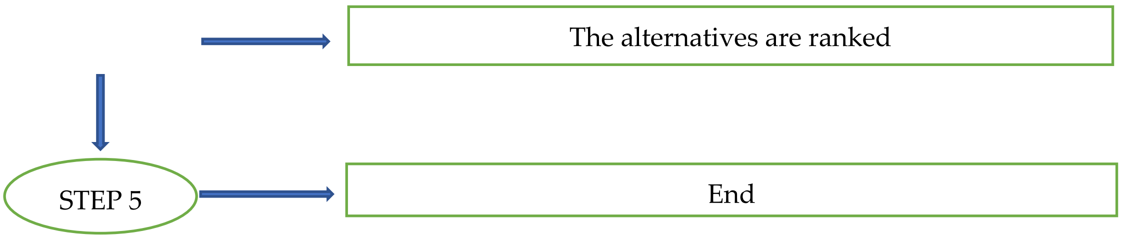

Figure 1 Flow chart of the BSVHNS-MADM Strategy based on BSVHMNA Operator

7. BSVHNS-MADM Strategy based on BSVHMNG Operator

Consider the same MADM problem that was discussed in Section 6. The proposed BSVHNS-MADM scheme (see Figure 2) can then be stated as follows:

Step 1 : Create the decision-making matrix with BSVHNSs.

It is comparable to step 1 in Section 6.

Step 2 : The decision makers determine the aggregation values in this step $(E_i \mid \mathcal{D}_1, \mathcal{D}_2, \dots \dots \dots \mathcal{D}_m) = BSVHMNG\ (\mathcal{D}_1, \mathcal{D}_2, \dots \dots \dots \mathcal{D}_m)$ of all the attributes for each alternative by using the equation (1). After the determination of aggregation values $BSVHMNG\ (\mathcal{D}_1, \mathcal{D}_2, \dots \dots \dots \mathcal{D}_m)$, The decision maker makes an aggregate DM for the decision makers.

Step 3 : In this stage, the decision maker uses the equations (3) and (4) to calculate the score and accuracy values for each alternative.

Step 4 : The decision maker ranks the choices using Definitions 5.1 and 5.2 in this step.

Step 5 : End.

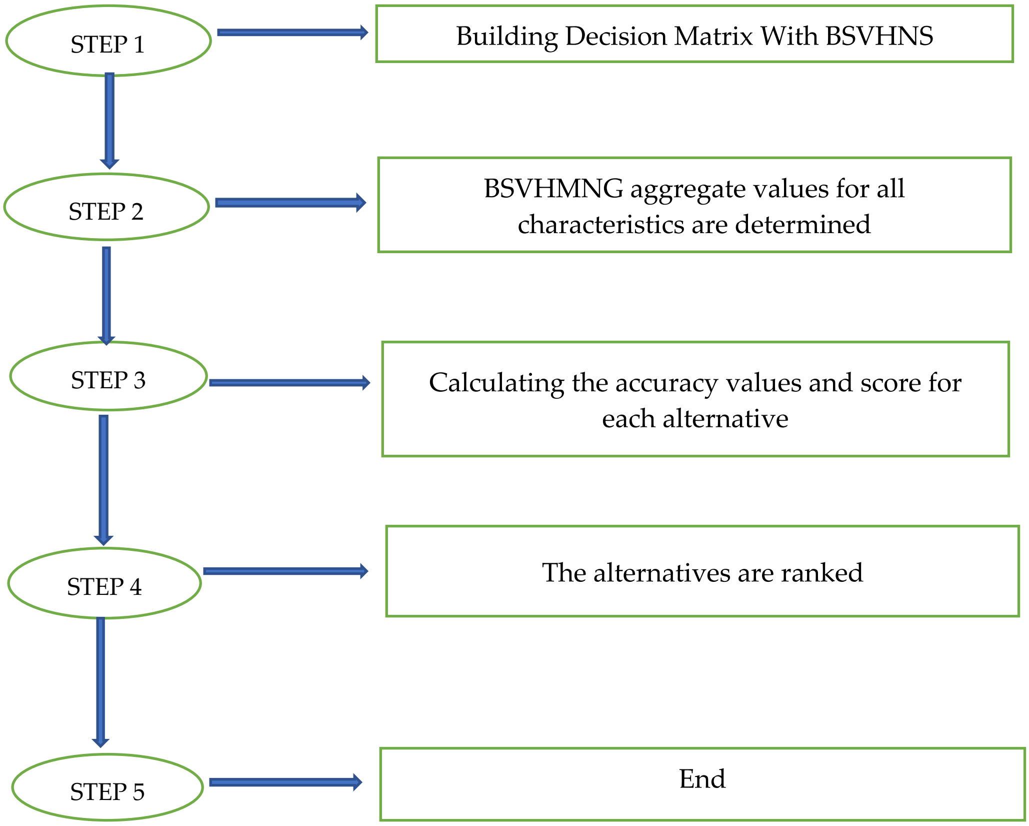

Figure 2: Flow chart of the BSVHNS-MADM Strategy based on BSVHMNG Operator

8. BSVHNS - MADM Strategy Validation

At this section, we offer a genuine scenario of "selection for good equipment in best hospital" to validate the suggested BSVHNS-MADM strategies based on both BSVHMNA and BSVHMNG operators. A good hospital should focus on

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making the patient's experience as seamless as possible, from appointment booking to discharge. Every government / private hospital requires Hospital Stretchers, Room rent per day, scanning process, X-Ray, ECG, and so on for the benefit of hospital users. To purchase a specific or all items, hospitals must select an appropriate private concern for providing some features. As a result, selecting the best private hospitals for acquiring the necessary items can be considered a MADM problem.

For the selection of suitable private hospital, the decision maker selects four major attributes namely

$\mathcal{D}_1$: The price of the products; $\mathcal{D}_2$: Product high quality; $\mathcal{D}_3$: Company support; $\mathcal{D}_4$: Safety

Table 1

In Table 2, We compute the aggregate values $(E_i \mid \mathcal{D}_1, \mathcal{D}_2, \mathcal{D}_3, \mathcal{D}_4)$ of all attributes for each alternative E_i, by using the BSVHMNA operator.

By using equation (2), we get $SF(E_1) = 0.63574$; $SF(E_2) = 0.635164$; $SF(E_3) = 0.7245$.
Therefore, $SF(E_2) < SF(E_1) < SF(E_3)$.

The ranking order is determined as follows: $E_2 < E_1 < E_3$.

As a result, E_3 is the best hospital in terms of quality goods and services among the alternatives (hospitals).

In table 3, we calculate the aggregation values $(E_i \mid \mathcal{D}_1, \mathcal{D}_2, \mathcal{D}_3, \mathcal{D}_4)$ of all attributes for each alternative E_i, by using the BSVHMNG operator.

Table 2: Aggregate – DM

$\mathcal{D}M$	$\mathcal{D}_1$	$\mathcal{D}_2$	$\mathcal{D}_3$	$\mathcal{D}_4$
E_1	$(-0.2, -0.4, -0.5, -0.7, -0.1, -0.3, -0.2, 0.3, 0.6, 0.5, 0.2, 0.4, 0.1, 0.3$	$(-0.7, -0.2, -0.3, -0.4, -0.5, -0.3, -0.6, 0.2, 0.4, 0.5, 0.4, 0.3, 0.6, 0.8)$	$(-0.4, -0.6, -0.2, -0.1, -0.5, -0.5, -0.3, 0.8, 0.7, 0.4, 0.2, 0.5, 0.3, 0.6)$	$(-0.2, -0.1, -0.8, -0.1, -0.6, -0.3, -0.1, 0.3, 0.2, 0.4, 0.3, 0.7, 0.5, 0.4)$
E_2	$(-0.2, -0.6, -0.7, -0.8, -0.4, -0.2, -0.2, 0.3, 0.4, 0.3, 0.1, 0.3, 0.1, 0.3)$	$(-0.6, -0.4, -0.4, -0.6, -0.4, -0.3, -0.5, 0.4, 0.2, 0.4, 0.4, 0.5, 0.7, 0.6)$	$(-0.7, -0.3, -0.1, -0.1, -0.6, -0.3, -0.1, 0.9, 0.6, 0.3, 0.1, 0.7, 0.4, 0.7)$	$(-0.5, -0.1, -0.8, -0.2, -0.7, -0.5, -0.4, 0.5, 0.4, 0.5, 0.3, 0.6, 0.6, 0.4)$
E_3	$(-0.4, -0.3, -0.5, -0.9, -0.2, -0.1, -0.5, 0.5, 0.2, 0.2, 0.1, 0.4, 0.2, 0.6)$	$(-0.4, -0.2, -0.2, -0.6, -0.7, -0.2, -0.5, 0.7, 0.5, 0.5, 0.4, 0.6, 0.4, 0.2)$	$(-0.7, -0.3, -0.1, -0.4, -0.6, -0.3, -0.7, 1.0, 0.6, 0.3, 0.1, 0.7, 0.4, 0.4)$	$(-0.4, -0.6, -0.9, -0.1, -0.5, -0.4, -0.8, 0.5, 0.4, 0.5, 0.2, 0.4, 0.3, 0.5)$

	$(E_i \mid \mathcal{D}_1, \mathcal{D}_2, \mathcal{D}_3, \mathcal{D}_4)$
E_1	$(-0.9699, -0.9537, -0.9857, -0.2300, -0.9767, -0.9541, -0.3299, 0.0036, 0.4281, 0.4472, 0.9102, 0.4527, 0.3080, 0.4899)$
E_2	$(-0.9878, -0.9598, -0.9918, -0.3130, -0.9890, -0.9469, -0.3183, 0.0138, 0.3722, 0.3663, 0.9013, 0.5009, 0.3600, 0.4738$
E_3	$(-0.9834, -0.9583, -0.9909, -0.2378, -0.9878, -0.9139, -0.6501, 0.047, 0.3936, 0.3499, 0.8842, 0.5091, 0.3130, 0.3936$

By using equation (2), we get $SF(E_1) = 0.38315$; $SF(E_2) = 0.3941$; $SF(E_3) = 0.4123$.

Therefore, $SF(E_1) < SF(E_2) < SF(E_3)$.

The ranking order is determined as follows: $E_1 < E_2 < E_3$.

As a result, E_3 is the best hospital for getting good services.

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Table 3: Aggregate – DM

Table 4 : Ranking order of alternatives

Strategies	Ranking order	Best alternative
BSVHNS – MADM strategy based on BHNAM operator	$E_2 < E_1 < E_3$	E_3
BSVHNS – MADM strategy based on BHNGM operator	$E_1 < E_2 < E_3$	E_3

Both BSVHNS - MADM techniques provide the same ranking order of the alternatives (See table 4), with E_3 being the best hospital for receiving decent treatment.

9. Sensitivity Analysis

Sensitivity analysis is a financial model that investigates how changes in other variables known as input variables

	$(E_i \mid \mathcal{D}_1, \quad \mathcal{D}_2, \quad \mathcal{D}_3, \quad \mathcal{D}_4)$
E_1	$(-0.3253, -0.2632, -0.3936, -0.3821, -0.3499, -0.3409, -0.2449, 0.4708, 0.5101, 0.4523, 0.2632, 0.4991, 0.4042, 0.5719)$
E_2	$(-0.4527, -0.2913, -0.3869, -0.5101, -0.5091, -0.3080, -0.2515, 0.6193, 0.4174, 0.3808, 0.1861, 0.5473, 0.4955, 0.5262)$
E_3	$(-0.4601, -0.3224, -0.3080, -0.2865, -0.4527, -0.2213, -0.6117, 1.000, 0.4434, 0.3883, 0.1682, 0.5441, 0.3299, 0.4434)$

affect target variables. It is a technique for predicting the outcome of a choice based on a set of variables. By building a given collection of variables, an analyst can determine how changes in one variable affect the outcome. In this model, we have reduced uncertainty to select the best hospital in the MADM scheme.

10. Comparative Analysis

Surapati Pramnik utilized five values and the MADM scheme in his Pentapartitioned neutrosophic set, whereas I used seven values and so many attributes in my Heptapartitioned neutrosophic set. I discovered score and accuracy functions for identifying the best service at the nearest hospital utilizing the MADM scheme by employing Bipolar Heptapartitioned Neutrosophic Set.

11. Conclusion

We define the concept of BSVHNS and demonstrate its fundamental attributes in this essay. We determine the BSVHNNs' score and accuracy functions and demonstrate their fundamental characteristics. In this section, we construct two aggregation operators the bipolar single-valued heptapartitioned neutrosophic arithmetic mean operator and the bipolar single-valued heptapartitioned neutrosophic geometric mean operator and demonstrate their fundamental characteristics. We create two new MADM scheme based on these two operators and provide a numerical illustration in an BSVHNS environment to demonstrate the usefulness of BSVHNS in MADM.

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