International Journal of Mathematical Archive-4(10), 2013, 225-238
Available online through www.ijma.info ISSN 2229 – 5046

# FUZZY NEUTROSOPHIC SOFT TOPOLOGICAL SPACES

**I. Arockiarani*, I. R. Sumathi* and J. Martina Jency****

**Department of Mathematics, Nirmala College for women, Coimbatore, (T.N.), India.*

***Department of Mathematics, Coimbatore Institute of Engineering and Technology, Coimbatore, (T.N.), India.*

*(Received on: 22-09-13; Revised & Accepted on: 07-10-13)*

*ABSTRACT*

*The focus of this paper is to propose a new notion of Fuzzy Neutrosophic soft set and to study some basic operations and results in Fuzzy Neutrosophic soft spaces. Further we develop a systematic study on Fuzzy Neutrosophic soft set and obtain various properties induced by them. Some equivalent characterization and inter-relations among them are discussed with counter example.*

***Keywords:*** *Soft sets, Neutrosophic set, Neutrosophic soft set, Fuzzy Neutrosophic soft set and Fuzzy Neutrosophic soft topological space.*

***MSC 2000****: 03B99, 03E99.*

## 1. INTRODUCTION

In dealing with uncertainties many theories have been recently developed, including the theory of probability, theory of fuzzy sets, theory of intuitionistic fuzzy sets and theory of rough sets and so on. Although many new techniques have been developed as a result of these theories, yet difficulties are still there. The major difficulties arise due to inadequacy of parameters.

In 1999, Molodtsov[4] , initiated the novel concept of soft set theory, which was a completely new approach for modeling uncertainty and had a rich potential for application in several directions. This so- called soft set theory is free from the difficulties affecting existing methods. The fuzzy set was introduced by Zadeh [13] in 1965 where each element had a degree of membership. The intuitionistic fuzzy set (IFS for short) on a universe X was introduced by K.Atanaasov [1] in 1983 as a generalization of fuzzy set, where besides the degree of membership and the degree of non – membership of each element. The concept of Neutrosophic set was introduced by F. Smarandache [10] which is a mathematical tool for handling problems involving imprecise, indeterminacy and inconsistent data. Pabitra Kumar Maji [7] had combined the Neutrosophic set with soft sets and introduced a new mathematical model 'Neutrosophic soft set'.

In the present study, we have defined Fuzzy Neutrosophic soft set and we establish some related properties with supporting proofs and examples. We consider the topological structure of Neutrosophic soft set as a tool to develop Fuzzy Neutrosophic soft topological space and derive some of their topological properties. Here we recall the definitions that are prerequisite for this paper.

## 2. PRELIMINARIES

**Definition 2.1: [10]** A Neutrosophic set A on the universe of discourse X is defined as

$A=\langle x,T_A(x),I_A(x),F_A(x)\rangle, x\in X$ where $T,I,F:X\to\left[{}^-0,1^+\right]$ [and ${}^-0\le T_A(x)+I_A(x)+F_A(x)\le 3^+$.

**Definition 2.2: [8]** A neutrosophic set A is contained in another neutrosophic set B. (i.e.,)

(i) $A\subseteq B \Leftrightarrow T_A(x)\le T_B(x)$ , $I_A(x)\le I_B(x)$ , $F_A(x)\ge F_B(x)$ , $\forall x\in X$

*Corresponding author: I. R. Sumathi* and J. Martina Jency***
***Department of Mathematics, Coimbatore Institute of Engineering and Technology, Coimbatore, (T.N.), India.*

**Definition 2.3: [4]** Let U be the initial universe set and E be a set of parameters. Let P(U) denote the power set of U. Consider a non-empty set A, A ⊂ E .A pair (F,A) is called a soft set over U, where F is a mapping given by F: A → P(U)

**Definition 2.4: [7]** Let U be the initial universe set and E be a set of parameters. Consider a non-empty set A, A ⊂ E. Let P(U) denote the set of all neutrosophic sets of U. The collection (F, A) is termed to be the soft neutrosophic set over U, where F is a mapping given by F: A → P(U).

**Definition 2.5: [7]** The complement of a neutrosophic soft set (F, A) denoted by $(F, A)^c$ and is defined as

$(F, A)^c = (F^c, \rceil A)$ where $F^c: \rceil A \to P(U)$ is a mapping given by $F^c(\alpha)$=neutrosophic soft complement with $T_{F^c}(x) = F_F(x)$, $I_{F^c}(x) = I_F(x)$, $F_{F^c}(x) = T_F(x)$.

**Definition 2.6:[7]** Let (F, A) and (G, B) be two neutrosophic soft sets over the common universe U.(F,A) is said to be neutrosophic soft subset of (G,B) if A⊂B and $T_{F(e)}(x) \le T_{G(e)}(x), I_{F(e)}(x) \le I_{G(e)}(x), F_{F(e)}(x) \ge F_{G(e)}(x)$ ∀ e∈ A , x ∈ U. We denote it by (F, A) ⊆ (G, B).

(F, A) is said to be neutrosophic soft super set of (G, B) if (G, B) is a neutrosophic soft subset of (F, A).We denote it by (F, A) ⊇ (G, B).

**Definition 2.7:[7]** Two neutrosophic soft sets (F, A) and (G, B) over the common universe U are said to be equal if (F, A) ⊆ (G,B) and (G, B) ⊆ (F, A).We denote it by (F, A) = (G, B).

**Definition: 2.8 [7]** Union of two Neutrosophic soft sets (F, A) and (G, B) over (U, E) is Neutrosophic soft set where C = A∪B ∀ e∈ C.

$$H(e) = \begin{cases} F(e) & ; \text{ if } e \in A - B \\ G(e) & ; \text{ if } e \in B - A \\ F(e) \cup G(e) & ; \text{ if } e \in A \cap B \end{cases}$$ and is written as $(F,A) \tilde{\cup} (G,B) = (H,C)$.

**Definition: 2.9 [7]** Intersection of two Neutrosophic soft sets (F,A) and (G,B) over (U, E) is Neutrosophic soft set where C = A∩B ∀ e∈ C. H(e) = F(e) ∩ G(e) and is written as $(F,A) \tilde{\cap} (G,B) = (H,C)$.

## 3. FUZZY NEUTROSOPHIC SOFT SET

From philosophical point of view, the neutrosophic set takes the value from real standard or non-standard subset of $[-0, 1^+]$. But in real life application in scientific and Engineering problems it is difficult to use neutrosophic set with value from real standard or non-standard subset of $[^-0, 1^+]$. Hence we consider the neutrosophic set which takes the value from the subset of [0, 1].

**Definition 3.1:** A Fuzzy Neutrosophic set A on the universe of discourse X is defined as

$$A = \langle x, T_A(x), I_A(x), F_A(x) \rangle, x \in X$$ where $T, I, F : X \to [0,1]$ and $0 \le T_A(x) + I_A(x) + F_A(x) \le 3$.

**Definition 3.2:** Let U be the initial universe set and E be a set of parameters. Consider a non-empty set A, A ⊂ E. Let P (U) denote the set of all fuzzy neutrosophic sets of U. The collection (F, A) is termed to be the fuzzy neutrosophic soft set over U, where F is a mapping given by F: A → P(U).

Throughout this paper Fuzzy Neutrosophic soft set is denoted by FNS set / FNSS.

**Definition 3.3:** A fuzzy neutrosophic soft set A is contained in another neutrosophic set B. (i.e.) A ⊆ B if ∀ x ∈X, $T_A(x) \le T_B(x)$ , $I_A(x) \le I_B(x)$ , $F_A(x) \ge F_B(x)$.

**Definition 3.4:** The complement of a fuzzy neutrosophic soft set (F, A) denoted by $(F, A)^c$ and is defined as

$(F, A)^c = (F^c, \rceil A)$ where $F^c : \rceil A \to P(U)$ is a mapping given by

$F^c(\alpha) = \langle x, T_{F^c}(x) = F_F(x), I_{F^c}(x) = 1 - I_F(x), F_{F^c}(x) = T_F(x) \rangle$

**Definition 3.5:** Let X be a non empty set, and $A=\langle x, T_A(x), I_A(x), F_A(x)\rangle, B=\langle x, T_B(x), I_B(x), F_B(x)\rangle$ are fuzzy neutrosophic soft sets. Then

$$A \tilde{\cup} B=\langle x, \max(T_A(x), T_B(x)), \max(I_A(x), I_B(x)), \min(F_A(x), F_B(x))\rangle$$

$$A \tilde{\cap} B=\langle x, \min(T_A(x), T_B(x)), \min(I_A(x), I_B(x)), \max(F_A(x), F_B(x))\rangle$$

**Definition 3.6:** A fuzzy neutrosophic soft set (F,A) over the universe U is said to be empty fuzzy neutrosophic soft set with respect to the parameter A if $T_{F(e)}=0, I_{F(e)}=0, F_{F(e)}=1, \forall x \in U, \forall e \in A.$ It is denoted by $\tilde{0}_N$.

**Definition 3.7:** A FNS set (F, A), over the universe U is said to be universe FNS set with respect to the parameter A if $T_{F(e)}=1, I_{F(e)}=1, F_{F(e)}=0, \forall x \in U, \forall e \in A$.It is denoted by $\tilde{1}_N$.

**Note:** $\tilde{0}_N^c=\tilde{1}_N$ and $\left(\tilde{1}_N\right)^c=\tilde{0}_N$

**Definition 3.8:** Let A and B be two FNS sets then A\B may be defined as

A\B= $\langle x, \min(T_A(x), F_B(x)), \min(I_A(x), 1-I_B(x)), \max(F_A(x), T_B(x))\rangle$.

**Definition3.9:**

(i) $F_E$ is called absolute Fuzzy Neutrosophic soft set over U if F(e) = $\tilde{1}_N$ for any e $\in$ E. We denote it by $U_E$

(ii) $F_E$ is called relative null Fuzzy Neutrosophic soft set over U if F(e) = $\tilde{0}_N$ for any e $\in$ E. We denote it by $\phi_E$.

Obviously $\phi_E = U_E^c$ and $U_E=\phi_E^c$.

**Definition 3.10:** The complement of a fuzzy neutrosophic soft set (F, A) can also be defined as

$(F, A)^c = U_E \setminus F(e)$ for all e$\in$A

**Note:** We denote $U_E$ by U in the proofs of the proposition.

**Definition 3.11:** If (F, A) and (G, B) be two fuzzy Neutrosophic soft set then "(F, A) AND (G, B)" is a FNSS denoted by (F, A) $\wedge$ (G, B) and is defined by (F, A) $\wedge$ (G, B) = (H, A x B)

where H(a, b) = F(a) $\cap$ G(b) $\forall$ a$\in$A and $\forall$ b$\in$B where $\cap$ is the operation intersection of FNSS.

**Definition 3.12:** If (F, A) and (G, B) be two fuzzy Neutrosophic soft set then "(F, A) OR (G, B)" is a FNSS denoted by (F, A) $\vee$ (G, B) and is defined by (F, A) $\vee$ (G, B) = (K, A x B)

where K(a, b) = F(a) $\cup$ G(b) $\forall$ a$\in$A and $\forall$ b$\in$B where $\cup$ is the operation union of FNSS.

**Proposition 3.13:** Let (F,A) and (G,A) be FNSS in $FNSS(U)_A$ .Then the following are true.

(i) (F,A) $\tilde{\subseteq}$ (G,A) iff (F, A) $\tilde{\cap}$ (G,A) = (F, A)

(ii) (F,A) $\tilde{\subseteq}$ (G,A) iff (F, A) $\tilde{\cup}$ (G,A) = (F, A)

**Proof:**

**(i)** Suppose that (F,A) $\tilde{\subseteq}$ (G,A) Then F(e) $\subseteq$ G(e) for all e $\in$A. Let (F, A) $\tilde{\cap}$ (G,A) = (H, A)

Since H(e) = F(e) $\cap$ G(e) = F(e) for all e$\in$A ,by definition (H, A) = (F, A). Suppose that (F, A) $\tilde{\cap}$ (G,A) = (F, A). Let (F, A) $\tilde{\cap}$ (G,A) = (H, A).Since H(e) = F(e) $\cap$ G(e) = F(e) for all e$\in$A, we know that F(e) $\subseteq$ G(e) for all e $\in$A. Hence (F, A) $\tilde{\subseteq}$ (G,A)

**(ii)** The proof is similar to (i)

**Proposition 3.14:** Let (F, A) , (G, A), (H, A), (S, A) $\in$ FNSS$(U)_A$. Then the following are true

(i) If (F, A) $\tilde{\cap}$ (G,A) = $\phi_A$ then (F,A) $\tilde{\subseteq}$ $(G,A)^c$

(ii) If (F,A) $\tilde{\subseteq}$ (G,A) and (G,A) $\tilde{\subseteq}$ (H,A) then (F,A) $\tilde{\subseteq}$ (H,A)

(iii) If (F,A) $\tilde{\subseteq}$ (G,A) and (H,A) $\tilde{\subseteq}$ (S,A) then (F, A) $\tilde{\cap}$ (H,A) $\tilde{\subseteq}$ (G, A) $\tilde{\cap}$ (S,A)

(iv) (F,A) $\tilde{\subseteq}$ (G,A) iff $(G,A)^c$ $\tilde{\subseteq}$ $(F,A)^c$

**Proof:**

**(i)** Suppose that (F, A) $\tilde{\cap}$ (G,A) = $\phi_A$ .Then F(e) $\cap$ G(e) = $\phi$ . So F(e) $\subseteq$ U\G(e) =$G^c(e)$ for all e$\in$A.

Therefore we have (F, A) $\tilde{\subseteq}$ $(G, A)^c$. Proof of (ii) and (iii) are obvious.

**(iv)** (F,A) $\tilde{\subseteq}$ (G,A)$\Leftrightarrow$ F(e) $\subseteq$ G(e) for all e$\in$A

$\Leftrightarrow (G(e))^c \subseteq (F(e))^c$ for all e$\in$A

$\Leftrightarrow G^c(e) \subseteq F^c(e)$ for all e$\in$A

$\Leftrightarrow (G, A)^c \tilde{\subseteq} (F,A)^c$

**Definition 3.15:** Let I be an arbitrary index set and $\{(F_i , A)\}_{i\in I}$ be a subfamily of FNSS$(U)_A$.

(i) The union of these FNSS is the FNSS (H, A) where $H(e) = \bigcup_{i\in I} F_i(e)$ for each e$\in$A.

We write $\tilde{\bigcup}_{i\in I} (F_i , A) = (H,A)$

(ii) The intersection of these FNSS is the FNSS (M, A) where $M(e) = \bigcap_{i\in I} F_i(e)$ for each e$\in$A.

We write $\tilde{\bigcap}_{i\in I} (F_i, A) = (M,A)$.

**Proposition 3.16:** Let I be an arbitrary index set and $\{(F_i , A)\}_{i\in I}$ be a subfamily of FNSS$(U)_A$ .

Then (i) $[\tilde{\bigcup}_{i\in I} (F_i , A)]^c = \tilde{\bigcap}_{i\in I} (F_i,A)^c$

(ii) $[\tilde{\bigcap}_{i\in I} (F_i,A)]^c = \tilde{\bigcup}_{i\in I} (F_i , A)^c$

**Proof:**

**(i)** $[\tilde{\bigcup}_{i\in I} (F_i , A)]^c = (H,A)^c$ ,By definition $H^c(e) = U_E \setminus H(e) = U_E \setminus \bigcup_{i\in I} F_i(e) = \bigcap_{i\in I} (U_E \setminus F_i(e))$ for all e$\in$A. On the other hand , $[\tilde{\bigcap}_{i\in I} (F_i, A)]^c$ =( K ,A) .By definition ,$K(e) = \bigcap_{i\in I} F_i^c(e) = \bigcap_{i\in I} (U - F_i(e))$ for all e$\in$A.

**(ii)** $[\tilde{\bigcap}_{i\in I} (F_i , A)]^c = (H,A)^c$ ,By definition $H^c(e) = U_E \setminus H(e) = U_E \setminus \bigcap_{i\in I} F_i(e) = \bigcup_{i\in I} (U_E \setminus F_i(e))$ for all e$\in$A. On the other hand , $[\tilde{\bigcup}_{i\in I} (F_i, A)]^c$ =( K ,A) .By definition ,$K(e) = \bigcup_{i\in I} F_i^c(e) = \bigcup_{i\in I} (U_E - F_i(e))$ for all e$\in$A.

**Note:** We denote $\phi_E$ by $\phi$ and $U_E$ by U

**Proposition 3.17:**

i) $(\phi ,A)^c = (U, A)$

ii) $(U, A)^c = (\phi ,A)$

**Proof:**

i) Let ($\phi$ ,A) = (F,A)

Then $\forall e \in A$,

$F(e) = \{(x, T_{F(e)}(x), I_{F(e)}(x), F_{F(e)}(x)): x \in U\}$

$= \{(x, 0, 0, 1): x \in U\}$

$(\phi , A)^c = (F,A)^c$

Then $\forall e \in A$,

$(F(e))^c = \{(x, T_{F(e)}(x), I_{F(e)}(x), F_{F(e)}(x)): x \in U\}^c$

$= \{(x, F_{(e)}(x), 1 - I_{F(e)}(x), T_{F(e)}(x)): x \in U\}$

$= \{(x, 1, 1, 0): x \in U\} = U$

Thus $(\phi , A)^c = (U, A)$

ii) Proof is similar to (i)

**Proposition 3.18:**

i) $(F,A) \tilde{\cup} (\phi ,A) = (F,A)$

ii) $(F,A) \tilde{\cup} (U ,A) = (U,A)$

Proof:

i) $(F,A) = \{e, (x, T_{F(e)}(x), I_{F(e)}(x), F_{F(e)}(x)): x \in U\} \forall e \in A$

$(\phi , A) = \{e(x, 0, 0, 1): x \in U\} \forall e \in A$

$(F,A) \tilde{\cup} (\phi , A) = \{e, (x, \max(T_{F(e)}(x), 0), \max(I_{F(e)}(x),0), \min( F_{F(e)}(x),1)): x \in U\} \forall e \in A$

$= \{e(x, T_{F(e)}(x), I_{F(e)}(x), F_{F(e)}(x)): x \in U\}\} \forall e \in A$

$= (F, A)$

ii) Proof is similar to (i)

**Proposition 3.19:**

i) $(F,A) \tilde{\cap} (\phi ,A) = (\phi,A)$

ii) $(F,A) \tilde{\cap} (U ,A) = (F,A)$

**Proof:**

i) $(F,A) = \{e, (x, T_{F(e)}(x), I_{F(e)}(x), F_{F(e)}(x)): x \in U\} \forall e \in A$

$(\phi , A) = \{e(x, 0, 0, 1): x \in U\} \forall e \in A$

$(F,A) \tilde{\cap} (\phi , A) = \{e, (x, \min(T_{F(e)}(x), 0), \min(I_{F(e)}(x),0), \max( F_{F(e)}(x),1)): x \in U\} \forall e \in A$

$= \{e(x, 0, 0, 1): x \in U\} \forall e \in A$

$= (\phi, A)$

Thus $(F, A) \tilde{\cap} (\phi , A) = (\phi, A)$

ii) Proof is similar to (i)

**Proposition 3.20:**

(i) $(F,A) \tilde{\cup} (\phi ,B) = (F,A)$ iff $B \subseteq A$

(ii) $(F,A) \tilde{\cup} (U ,B) = (U,A)$ iff $A \subseteq B$

 

**Proof:**
i) We have for (F,A)

$F(e) = \{(x, T_{F(e)}(x), I_{F(e)}(x), F_{F(e)}(x)): x \in U\} \forall e \in A$

Also let $(\phi, B) = (G, B)$ then

$G(e) = \{(x, 0, 0, 1): x \in U\} \forall e \in B$

Let $(F, A) \tilde{\cup} (\phi, B) = (F, A) \tilde{\cup} (G, B) = (H, C)$ where $C = A \cup B$ and $\forall e \in C$

$$H(e) = \begin{cases} \{(x, T_{F(e)}(x), I_{F(e)}(x), F_{F(e)}(x)) : x \in U\} \text{if } e \in A - B \\ \{(x, T_{G(e)}(x), I_{G(e)}(x), F_{G(e)}(x)) : x \in U\} \text{if } e \in B - A \\ \{(x, \max(T_{F(e)}(x), T_{G(e)}(x)), \max(I_{F(e)}(x), I_{G(e)}(x)), \min(F_{F(e)}(x), F_{G(e)}(x))) : x \in U\} \text{if } e \in A \cap B \end{cases}$$

$$= \begin{cases} \{(x, T_{F(e)}(x), I_{F(e)}(x), F_{F(e)}(x)) : x \in U\} \text{if } e \in A - B \\ \{(x, 0,0,1) : x \in U\} \text{if } e \in B - A \\ \{(x, \max(T_{F(e)}(x),0), \max(I_{F(e)}(x),0), \min(F_{F(e)}(x),1)) : x \in U\} \text{if } e \in A \cap B \end{cases}$$

$$= \begin{cases} \{(x, T_{F(e)}(x), I_{F(e)}(x), F_{F(e)}(x)) : x \in U\} \text{if } e \in A - B \\ \{(x, 0,0,1) : x \in U\} \text{if } e \in B - A \\ \{(x, T_{F(e)}(x), I_{F(e)}(x), F_{F(e)}(x)) : x \in U\} \text{if } e \in A \cap B \end{cases}$$

Let $B \subseteq A$

Then $$H(e) = \begin{cases} \{(x, T_{F(e)}(x), I_{F(e)}(x), F_{F(e)}(x)) : x \in U\} \text{if } e \in A - B \\ \{(x, T_{F(e)}(x), I_{F(e)}(x), F_{F(e)}(x)) : x \in U\} \text{if } e \in A \cap B \end{cases}$$

$= F(e) \ \forall e \in A$

Conversely Let $(F, A) \tilde{\cup} (\phi, B) = (F, A)$

Then $A = A \cup B \Rightarrow B \subseteq A$

(ii) Proof is similar to (i)

**Proposition 3.21:**
(i) $(F,A) \tilde{\cap} (\phi, B) = (\phi, A \cap B)$
(ii) $(F,A) \tilde{\cap} (U, B) = (F, A \cap B)$

**Proof:**
i) We have for (F,A)

$F(e) = \{(x, T_{F(e)}(x), I_{F(e)}(x), F_{F(e)}(x)): x \in U\} \forall e \in A$

let $(\phi, B) = (G,B)$ then

$G(e) = \{(x, 0, 0, 1): x \in U\} \forall e \in B$

Let $(F, A) \tilde{\cap} (\phi, B) = (F, A) \tilde{\cap} (G, B) = (H, C)$ where $C = A \cap B$ and $\forall e \in C$

$H(e) = \{(x, \min(T_{F(e)}(x), T_{G(e)}(x)), \min(I_{F(e)}(x), I_{G(e)}(x)), \max(F_{F(e)}(x), F_{G(e)}(x))) : x \in U\}$

 

$= \{(x, \min(T_{F(e)}(x),0), \min(I_{F(e)}(x),0), \max(F_{F(e)}(x),1)) : x \in U\}$

$= \{(x, 0,0,1) : x \in U\}$

$= (G, B) = (\phi, B)$

Thus $(F, A) \tilde{\cap} (\phi, B) = (\phi, B)$.

ii) Proof is similar to (i).

**Proposition 3.22:**

i) $((F,A) \tilde{\cup} (G,B))^c \tilde{\subseteq} (F,A)^c \tilde{\cup} (G,B)^c$

ii) $(F,A)^c \tilde{\cap} (G,B)^c \tilde{\subseteq} ((F,A) \tilde{\cap} (G,B))^c$

**Proof:** Let $(F,A) \tilde{\cup} (G,B) = (H,C)$ where $C = A \cup B$ and $\forall e \in C$

$$H(e) = \begin{cases} \{(x, T_{F(e)}(x), I_{F(e)}(x), F_{F(e)}(x)) : x \in U\} \text{if } e \in A\text{-}B \\ \{(x, T_{G(e)}(x), I_{G(e)}(x), F_{G(e)}(x)) : x \in U\} \text{if } e \in B\text{-}A \\ \{(x, \max(T_{F(e)}(x), T_{G(e)}(x)), \max(I_{F(e)}(x), I_{G(e)}(x)), \min(F_{F(e)}(x), F_{G(e)}(x))) : x \in U\} \text{if } e \in A \cap B \end{cases}$$

Thus $((F, A) \tilde{\cup} (G, B))^c = (H,C)^c$ where $C = A \cup B$ and $\forall e \in C$

$$(H(e))^c = \begin{cases} (F(e))^c \text{ if } e \in A - B \\ (G(e))^c \ e \in B - A \\ ((F(e) \cup G(e))^c \text{ if } e \in A \cap B \end{cases}$$

$$= \begin{cases} \{(x, F_{F(e)}(x), 1 - I_{F(e)}(x), T_{F(e)}(x)) : x \in U\} \text{if } e \in A\text{-}B \\ \{(x, F_{G(e)}(x), 1\text{-}I_{G(e)}(x), T_{G(e)}(x)) : x \in U\} \text{if } e \in B\text{-}A \\ \{(x, \min(F_{F(e)}(x), F_{G(e)}(x)), 1\text{-}\max(I_{F(e)}(x), I_{G(e)}(x)), \max(T_{F(e)}(x), T_{G(e)}(x))) : x \in U\} \text{if } e \in A \cap B \end{cases}$$

Again, $(F, A)^c \tilde{\cup} (G, B)^c = (I, J)$ say $J = A \cup B$ and $\forall e \in J$.

$$I(e) = \begin{cases} (F(e))^c \text{ if } e \in A - B \\ (G(e))^c \ e \in B - A \\ (F(e))^c \cup (G(e))^c \text{ if } e \in A \cap B \end{cases}$$

$$= \begin{cases} \{(x, F_{F(e)}(x), 1 - I_{F(e)}(x), T_{F(e)}(x)) : x \in U\} \text{if } e \in A - B \\ \{(x, F_{G(e)}(x), 1 - I_{G(e)}(x), T_{G(e)}(x)) : x \in U\} \text{if } e \in B - A \\ \{(x, \max(F_{F(e)}(x), F_{G(e)}(x)), \max(1 - I_{F(e)}(x), 1 - I_{G(e)}(x)), \min(T_{F(e)}(x), T_{G(e)}(x))) : x \in U\} \text{if } e \in A \cap B \end{cases}$$

$C \subseteq J \quad \forall e \in J, (H(e))^c \subseteq I(e)$

Thus $((F, A) \tilde{\cup} (G, B))^c \tilde{\subseteq} (F,A)^c \tilde{\cup} (G, B)^c$

ii) Let $(F, A) \tilde{\cap} (G, B) = (H, C)$ where $C = A \cap B$ and $\forall e \in C$

$H(e) = F(e) \cap G(e)$

$= \{(x, \min(T_{F(e)}(x), T_{G(e)}(x)), \min(I_{F(e)}(x), I_{G(e)}(x)), \max(F_{F(e)}(x), F_{G(e)}(x))\}$

 

Thus $((F,A)\ \tilde{\cap}\ (G,B))^c = (H,C)^c$ where $C = A\cap B$ and $\forall e\in C$

$(H(e))^c = \{(x, \min(T_{F(e)}(x), T_{G(e)}(x)), \min(I_{F(e)}(x), I_{G(e)}(x)), \max(F_{F(e)}(x), F_{G(e)}(x))\}^c$

$= \{(x, \max(F_{F(e)}(x), F_{G(e)}(x)), 1 - \min(I_{F(e)}(x), I_{G(e)}(x)), \min(T_{F(e)}(x), T_{G(e)}(x))\}$

Again $(F, A)^c\ \tilde{\cap}\ (G,B)^c = (I, J)$ say where $J = A \cap B$ and $\forall e\in J$

$I(e) = (F(e))^c \cap (G(e))^c$

$= \{(x, \min(F_{F(e)}(x), F_{G(e)}(x)), \min(1 - I_{F(e)}(x), 1 - I_{G(e)}(x)), \max(T_{F(e)}(x), T_{G(e)}(x))\}$

We see that $C = J$ and $\forall e\in J$, $I(e) \subseteq (H(e))^c$

Thus $(F, A)^c\ \tilde{\cap}\ (G, B)^c\ \tilde{\subseteq}\ ((F,A)\ \tilde{\cap}\ (G,B))^c$

**Proposition 3.23: (De Morgan's Laws)** For Fuzzy Neutrosophic soft sets (F, A) and (G, A) over the same universe U we have the following

i) $((F,A)\ \tilde{\cup}\ (G,A))^c = (F,A)^c\ \tilde{\cap}\ (G,A)^c$

ii) $((F,A)\ \tilde{\cap}\ (G,A))^c = (F,A)^c\ \tilde{\cup}\ (G,A)^c$

Proof: (i) Let $(F,A)\ \tilde{\cup}\ (G,A) = (H,A)$ where $\forall e\in A$

$H(e) = F(e) \cup G(e)$

$= \{(x, \max(T_{F(e)}(x), T_{G(e)}(x)), \max(I_{F(e)}(x), I_{G(e)}(x), \min(F_{F(e)}(x), F_{G(e)}(x))\}$

Thus $((F, A)\ \tilde{\cup}\ (G,A))^c = (H,A)^c$ where $\forall e\in A$

$(H(e))^c = (F(e) \cup G(e))^c$

$= \{(x, \max(T_{F(e)}(x), T_{G(e)}(x)), \max(I_{F(e)}(x), I_{G(e)}(x)), \min(F_{F(e)}(x), F_{G(e)}(x))\}^c$

$= \{(x, \min(F_{F(e)}(x), F_{G(e)}(x)), 1 - \max(I_{F(e)}(x), I_{G(e)}(x)), \max(T_{F(e)}(x), T_{G(e)}(x))\}$

Again $(F, A)^c\ \tilde{\cap}\ (G, A)^c = (I, A)$ say where $\forall e\in A$

$I(e) = (F(e))^c \cap (G(e))^c$

$= \{(x, \min(F_{F(e)}(x), F_{G(e)}(x)), \min(1 - I_{F(e)}(x), 1 - I_{G(e)}(x), \max(T_{F(e)}(x), T_{G(e)}(x))\}$

$= \{(x, \min(F_{F(e)}(x), F_{G(e)}(x)), 1 - \max(I_{F(e)}(x), I_{G(e)}(x)), \max(T_{F(e)}(x), T_{G(e)}(x))\}$

Thus $((F, A)\ \tilde{\cup}\ (G, A))^c = (F,A)^c\ \tilde{\cap}\ (G,A)^c$

(ii) Let $(F,A)\ \tilde{\cap}\ (G,A) = (H, A)$ where $\forall e\in A$

$H(e) = F(e) \cap G(e)$

$= \{x, \min(T_{F(e)}(x), T_{G(e)}(x)), \min(I_{F(e)}(x), I_{G(e)}(x)), \max(F_{F(e)}(x), F_{G(e)}(x))\}$ where $\forall e\in A$

$((F,A)\ \tilde{\cap}\ (G, A))^c = (H, A)^c$

$(H(e))^c = (F(e) \cap G(e))^c$

$=\{x, \max(F_{F(e)}(x), F_{G(e)}(x)), 1 - \min(I_{F(e)}(x), I_{G(e)}(x)), \min(T_{F(e)}(x), T_{G(e)}(x))\}$ where $\forall e\in A$

Again $(F, A)^c\ \tilde{\cup}\ (G,A)^c = (I, A)$ say where $\forall e\in A$

$I(e) = (F(e))^c \cup G(e))^c$

$= \{(x, \max(F_{F(e)}(x), F_{G(e)}(x)), \max(1 - I_{F(e)}(x), 1 - I_{G(e)}(x)), \min( T_{F(e)}(x), T_{G(e)}(x))\}$

$= \{( x, \max(F_{F(e)}(x), F_{G(e)}(x)), 1 - \min(I_{F(e)}(x), I_{G(e)}(x)), \min( T_{F(e)}(x), T_{G(e)}(x))\}$

Thus $((F,A) \tilde{\cap} (G ,A))^c = (F,A)^c \tilde{\cup} (G ,A)^c$ .

**Proposition 3.24:** For FNSS (F,A) and (G, B) over the same universe U, we have the following.
(i) $((F, A) \wedge (G, B))^c = (F,A)^c \vee (G,B)^c$
(ii) $((F, A) \vee (G, B))^c = (F,A)^c \wedge (G,B)^c$

**Proof:**

(i) Let $(F,A) \wedge (G,B) = (H, A \times B)$ where $H(a,b) = F(a) \cap G(b)$ $\forall$ $a \in A$ and $\forall$ $b \in B$ where $\cap$ is the operation intersection of FNSS.

Thus $H(a, b) = F(a) \cap G(b)$

$= \{x, \min(T_{F(a)}(x), T_{G(b)}(x)), \min(I_{F(a)}(x), I_{G(b)}(x)), \max( F_{F(a)}(x), F_{G(b)}(x))\}$

$((F,A) \wedge (G,B))^c = (H, A \times B)^c$ $\forall$ $(a, b) \in A \times B$

$(H(a,b))^c = \{x, \max(F_{F(a)}(x), F_{G(b)}(x)), 1 - \min(I_{F(a)}(x), I_{G(b)}(x)), \min( T_{F(a)}(x), T_{G(b)}(x))\}$

Let $(F, A)^c \vee (G,B)^c = (R, A\times B)$

where $R(a, b) = (F(a))^c \cup (G(b))^c$ $\forall$ $a \in A$ and $\forall$ $b \in B$ where $\cap$ is the operation intersection of FNSS.

$R(a, b) = \{x, \max(F_{F(a)}(x), F_{G(b)}(x)), \max(1 - I_{F(a)}(x), 1 - I_{G(b)}(x)), \min( T_{F(a)}(x), T_{G(b)}(x))\}$

$= \{x, \max(F_{F(a)}(x), F_{G(b)}(x)), 1 - \min(I_{F(a)}(x), I_{G(b)}(x)), \min( T_{F(a)}(x), T_{G(b)}(x))\}$

Thus $((F, A) \wedge (G, B))^c = (F,A)^c \vee (G,B)^c$

(ii) Similarly we can prove (ii).

## 4. FUZZY NEUTROSOPHIC SOFT TOPOLOGICAL SPACES

**Definition 4.1:** Let $(F_A, E)$ be FNS set on (U, E) and $\tau$ be a collection of Fuzzy Neutrosophic soft subsets of $(F_A , E)$. $(F_A, E)$ is called Fuzzy neutrosophic soft topology (FNST) if the following conditions hold.
(i) $\phi_E ,U_E \in \tau$
(ii) $F_E ,G_E \in \tau$ implies $F_E \tilde{\cap} G_E \in \tau$
(iii) $\{(F_\alpha)_E : \alpha \in \Gamma\} \subseteq \tau$ implies $\tilde{\cup}\{(F_\alpha)_E : \alpha \in \Gamma\} \in \tau$

The triplet $(U, \tau, E)$ is called an Fuzzy Neutrosophic soft topological space (FNSTS)over U.

Every member of $\tau$ is called an Fuzzy Neutrosophic soft open set in U.

$F_E$ is called an Fuzzy Neutrosophic soft closed set in U if $F_E \in \tau^c$ , where $\tau^c = \{F_E^C : F_E \in \tau\}$

**Example 4.2:** Let $U = \{b_1, b_2, b_3\}$ and $E= \{e_1, e_2\}$. Let $F_E, G_E, H_E, L_E$ be Neutrosophic soft set where

$H(e_1) = \{< b_1, 0.8,0.4,0.5>, < b_2, 0.7,0.7,0.3>, < b_3, 0.7,0.5,0.4>\}$

$H(e_2) = \{< b_1, 0.9,0.5,0.6>, < b_2, 0.8,0.8,0.4>, < b_3, 0.8,0.6,0.5>\}$

$F(e_1) = \{< b_1, 0.5,0.4,0.5>, < b_2, 0.6,0.7,0.3>, < b_3, 0.6,0.5,0.4>\}$

$F(e_2) = \{< b_1, 0.6,0.5,0.6>, < b_2, 0.7,0.8,0.4>, < b_3, 0.7,0.6,0.5>\}$

 

$G(e_1) = \{< b_1, 0.8,0.4,0.7>, < b_2, 0.7,0.3,0.4>, < b_3, 0.7,0.5,0.6>\}$

$G(e_2) = \{< b_1, 0.9,0.5,0.8>, < b_2, 0.8,0.4,0.5>, < b_3, 0.8,0.6,0.7>\}$

$L(e_1) = \{< b_1, 0.5,0.4,0.7>, < b_2, 0.6,0.3,0.4>, < b_3, 0.6,0.5,0.6>\}$

$L(e_2) = \{< b_1, 0.6,0.5,0.8>, < b_2, 0.7,0.4,0.5>, < b_3, 0.7,0.6,0.7>\}$

$\tau = \{F_E, G_E, H_E, L_E, \phi_E, U_E\}$ is an Fuzzy Neutrosophic soft topology on U.

**Proposition 4.3:** Let (U, $\tau_1$, E) and (U, $\tau_2$, E) be two Fuzzy Neutrosophic soft topological spaces.

Denote $\tau_1 \cap \tau_2 = \{F_E: F_E \in \tau_1 \text{ and } F_E \in \tau_2\}$ .Then $\tau_1 \cap \tau_2$ is an FNST on U.

**Proof:** Obviously $\phi_E, U_E \in \tau_1 \cap \tau_2$. Let $F_E, G_E \in \tau_1 \cap \tau_2$ .Then $F_E, G_E \in \tau_1$ and $F_E, G_E \in \tau_2$ . $\tau_1$ and $\tau_2$ are two FNST's on U. Then $F_E \cap G_E \in \tau_1$ and $F_E \cap G_E \in \tau_2$. Hence $F_E \cap G_E \in \tau_1 \cap \tau_2$.

Let $\{(F_\alpha)_E : \alpha \in \Gamma\} \subseteq \tau_1 \cap \tau_2$. Then $(F_\alpha)_E \in \tau_1$ and $(F_\alpha)_E \in \tau_2$ for any $\alpha \in \Gamma$.

Since $\tau_1$ and $\tau_2$ are two FNST's on U, $\tilde{\bigcup}\{(F_\alpha)_E : \alpha \in \Gamma\} \in \tau_1$ and $\tilde{\bigcup}\{(F_\alpha)_E : \alpha \in \Gamma\} \in \tau_2$.

Thus $\tilde{\bigcup}\{(F_\alpha)_E : \alpha \in \Gamma\} \in \tau_1 \cap \tau_2$.

Let $\tau_1$ and $\tau_2$ are two FNSTs on U. Denote

$\tau_1 \vee \tau_2 = \{F_E \,\tilde{\cup}\, G_E: F_E \in \tau_1 \text{ and } G \in \tau_2\}$.

$\tau_1 \wedge \tau_2 = \{F_E \,\tilde{\cap}\, G_E: F_E \in \tau_1 \text{ and } G \in \tau_2\}$.

**Example 4.2:** Let $F_E$ and $G_E$ be FNST as in example 4.3.

Define $\tau_1 = \{\phi_E, U_E, F_E\}$, $\tau_2 = \{\phi_E, U_E, G_E\}$

Then $\tau_1 \cap \tau_2 = \{\phi_E, U_E\}$ is FNSTs on U.

But $\tau_1 \cup \tau_2 = \{\phi_E, U_E, F_E, G_E\}$, $\tau_1 \vee \tau_2 = \{\phi_E, U_E, F_E, G_E, F_E \,\tilde{\cup}\, G_E\}$and

$\tau_1 \wedge \tau_2 = \{\phi_E, U_E, F_E, G_E, F_E \,\tilde{\cap}\, G_E\}$ are not FNSTs.

**Theorem 4.5:** Let (U,$\tau$, E) be a FNSTS & Let $e \in E$, $\tau(e) = \{F(e): F_E \in \tau\}$ is an FNST on U.

**Proof:** Let $e \in E$.

(i) $\phi_E, U_E \in \tau$, $\tilde{0}_N = \phi(e)$ and $\tilde{1}_N = U(e)$, we have $\tilde{0}_N$, $\tilde{1}_N \in \tau(e)$.

(ii) Let $V, W \in \tau(e)$. Then there exist $F_E, G_E \in \tau$, such that $V = F(e)$ and $W = G(e)$.
By $\tau$ be an FNST on U, $F_E \,\tilde{\cap}\, G_E \in \tau$.
Put $H_E = F_E \,\tilde{\cap}\, G_E$. Then $H_E \in \tau$.
Note that $V \cap W = F(e) \cap G(e) = H(e)$ and $\tau(e) = \{F(e): F_E \in \tau\}$ Then $V \cap W \in \tau(e)$.

(iii) Let $\{(V_\alpha)_E : \alpha \in \Gamma\} \subseteq \tau(e)$. Then for every $\alpha \in \Gamma$, there exist $\{(F_\alpha)_E \in \tau$
such that $V_\alpha = F_\alpha(e)$. By $\tau$ be an FNST on U, $\tilde{\bigcup}\{(F_\alpha)_E : \alpha \in \Gamma\} \in \tau$.

Put $F_E = \tilde{\bigcup}\{(F_\alpha)_E : \alpha \in \Gamma\}$, then $F_E \in \tau$. Note that $\bigcup_{\alpha \in \Gamma} V_\alpha = \{F_\alpha(e) : \alpha \in \Gamma\}$= F(e) and $\tau(e) = \{F(e) : F_E \in \tau\}$.Then $\bigcup_{\alpha \in \Gamma} V_\alpha \in \tau(e)$.

Therefore $\tau(e) = \{F(e): F_E \in \tau\}$ is an FNST on U.

**Definition 4.6:** Let $(U, \tau, E)$ be a FNSTS and $\mathcal{B} \subseteq \tau$. $\mathcal{B}$ is a basis on $\tau$ if for each $G_E \in \tau$, there exist $\mathcal{B}' \subseteq \mathcal{B}$ such that $G_E = \tilde{\cup}\ \mathcal{B}'$.

**Example 4.2:** Let $\tau$ be a FNST as in example 4.3. Then

$\mathcal{B} = \{F_E, G_E, L_E, \phi_E, U_E\}$ is a basis for $\tau$.

**Theorem 4.8:** Let $\mathcal{B}$ be a basis for FNST on$\tau$. Denote $\mathcal{B}_e = \{F(e): F_E \in \mathcal{B}\}$and $\tau(e) = \{F(e): F_E \in \tau\}$for and $e \in E$. Then $\mathcal{B}_e$ is a basis for fuzzy neutrosophic topology $\tau(e)$.

**Proof:** Let $e \in E$. For any $V \in \tau(e)$, $V = G(e)$ for $G_E \in \tau$. Here $\mathcal{B}$ is a basis for $\tau$. Then there exists $\mathcal{B}' \subseteq \mathcal{B}$ such that $G_E = \tilde{\cup}\ \mathcal{B}'$. So $V = \tilde{\cup}\ \mathcal{B}'$ where $\mathcal{B}_e' = \{F(e) : F_E \in \mathcal{B}'\} \subseteq \mathcal{B}_e$. Thus $\mathcal{B}_e$ is a basis for Fuzzy Neutrosophic topology $\tau(e)$ for and $e \in E$.

## 5. SOME PROPERTIES OF FUZZY NEUTROSOPHIC SOFT TOPOLOGICAL SPACES

In this section, we give some properties of Fuzzy Neutrosophic soft topological space.

**Definition 5.1:** Let $(U,\tau, E)$ be a FNSTS & Let $F_E \in FNSS(U)_E$. Then interior and closure of $F_E$ denoted respectively by $FNSInt(F_E)$ and $FNSCl(F_E)$ are defined as follows.

$FNSInt(F_E) = \tilde{\cup}\{G_E \in \tau : G_E\ \tilde{\subseteq}\ F_E\}$

$FNSCl(F_E) = \tilde{\cap}\{G_E \in \tau^c : F_E\ \tilde{\subseteq}\ G_E\}$.

**Example 4.2:** We consider the FNST given in example 4.3.

Let $M(e_1) = \{\langle b_1, 0.6,0.5,0.4\rangle, \langle b_2, 0.7,0.8,0.3\rangle, \langle b_3, 0.8,0.6,0.3\rangle\}$

$M(e_2) = \{\langle b_1, 0.7,0.6,0.4\rangle, \langle b_2, 0.8,0.9,0.4\rangle, \langle b_3, 0.8,0.7,0.3\rangle\}$

$FNSInt(M_E) = F_E$

$FNSCl(M_E) = F_E^c$.

**Theorem 5.3:** Let $(U,\tau, E)$ be FNST over U. Then the following properties hold.

(i) $U_E$ and $\phi_E$ are FNS closed sets over U.
(ii) The intersection of any number of FNS closed sets is a FNS closed set over U.
(iii) The union of any two FNS closed sets is an FNS closed set over U.

**Proof:** It is obvious from the proposition 3.12

**Theorem 5.4:** Let $(U,\tau, E)$ be a FNST & Let $F_E \in FNSS(U)_E$ .Then the following properties hold.

(i) $FNSInt(F_E)\ \tilde{\subseteq}\ F_E$
(ii) $F_E\ \tilde{\subseteq}\ G_E \Rightarrow FNSInt(F_E)\ \tilde{\subseteq}\ FNSInt(G_E)$.
(iii) $FNSInt(F_E) \in \tau$.
(iv) $F_E$ is a FNS open set $\Leftrightarrow FNSInt(F_E) = F_E$.
(v) $FNSInt(FNSInt(F_E)) = FNSInt(F_E)$
(vi) $FNSInt(\phi_E) = \phi_E$, $FNSInt(U_E) = U_E$.

**Proof:**

(i) and (ii) are obvious.

(iii) Obviously $\tilde{\cup}\{G_E \in \tau : G_E\ \tilde{\subseteq}\ F_E\} \in \tau$

Note that $\tilde{\cup}\{G_E \in \tau : G_E\ \tilde{\subseteq}\ F_E\} = FNSInt(F_E)$

$\Rightarrow FNSInt(F_E) \in \tau$.

 

(iv) Necessity: Let $F_E$ be a FNS open set. ie., $F_E \in \tau$.

By (i) and (ii) FNSInt ($F_E$) $\tilde{\subseteq} F_E$.

Since $F_E \in \tau$ and $F_E \tilde{\subseteq} F_E$

Then $F_E \tilde{\subseteq} \tilde{\bigcup} \{G_E \in \tau: G_E \tilde{\subseteq} F_E\}$= FNSInt ($F_E$).

ie ., $F_E \tilde{\subseteq}$ FNSInt($F_E$).

Thus FNSInt = $F_E$.

Sufficiency: Let FNSInt($F_E$) = $F_E$

By (iii) FNSInt($F_E$) $\in \tau$ , ie., $F_E$ is a FNS open set.

(v) To prove FNSInt (FNSInt ($F_E$)) = FNSInt ($F_E$)

By (iii) FNSInt ($F_E$) $\in \tau$.

By (iv) FNSInt (FNSInt ($F_E$)) = FNSInt ($F_E$).

(vi) We know that $U_E$ and $\phi_E \in \tau$.

By (iv) FNSInt ($\phi_E$) = $\phi_E$, FNSInt ($U_E$) = $U_E$.

**Theorem 5.5**: Let (U, $\tau$, E) be a FNST & Let $F_E \in$ FNSS $(U)_E$. Then the following properties hold.

(i) ($F_E$) $\tilde{\subseteq}$ FNSCl($F_E$)
(ii) $F_E \tilde{\subseteq} G_E \Rightarrow$ FNSCl ($F_E$) $\tilde{\subseteq}$ FNSCl ($G_E$).
(iii) $(\text{FNSCl}(F_E))^c \in \tau$.
(iv) $F_E$ is a FNS closed set $\Leftrightarrow$ FNSCl($F_E$) = $F_E$.
(v) FNSCl (FNSCl ($F_E$)) = FNSCl ($F_E$)
(vi) FNSCl ($\phi_E$) = $\phi_E$ , FNSCl($U_E$) = $U_E$.

**Proof:**
(i) and (ii) are obvious.

(iii) By theorem 5.4 (iii) FNSInt( $F_E^C$ )$\in\tau$.

Therefore $[(\text{FNSCl}(F_E)]^c = (\tilde{\bigcap}\{G_E \in \tau^c : F_E \tilde{\subseteq} G_E \})^c$

$$= \tilde{\bigcup}\{G_E \in \tau : G_E \tilde{\subseteq} F_E^C \}$$

$$= \text{FNSInt } F_E^C$$

Then $[(\text{FNSCl}(F_E)]^c \in \tau$.

(iv) Necessity: By theorem 5.5 (i) $F_E \tilde{\subseteq}$ FNSCl ($F_E$)

Let $F_E$ be a FNS closed set ie., $F_E \in \tau^c$ and $F_E \tilde{\subseteq} F_E$ .

FNSCl($F_E$) = $\tilde{\bigcap}\{G_E \in \tau^c : F_E \tilde{\subseteq} G_E \} \tilde{\subseteq} \{F_E \in \tau^c : F_E \tilde{\subseteq} F_E\}$.

Then FNSCl($F_E$) $\tilde{\subseteq} F_E$. Thus $F_E$ = FNSCl($F_E$).

Sufficiency: This holds by (iii).

(v) and (vi) hold by (iii) and (iv).

**Theorem 5.6**: Let (U, τ, E) be a FNST & Let $F_E$, $G_E \in$ FNSS $(U)_E$ .Then the following properties hold.

(i) FNSInt $(F_E)$ $\tilde{\cap}$ FNSInt $(F_E)$ = FNSInt $(F_E \tilde{\cap} G_E)$ .

(ii) FNSInt $(F_E)$ $\tilde{\cup}$ FNSInt $(F_E)$ $\tilde{\subseteq}$ FNSInt $(F_E \tilde{\cup} G_E)$

(iii) FNSCl $(F_E)$ $\tilde{\cup}$ FNSCl $(G_E)$ = FNSCl $(F_E \tilde{\cup} G_E)$

(iv) FNSCl $(F_E \tilde{\cup} G_E)$ $\tilde{\subseteq}$ FNSCl $(F_E)$ $\tilde{\cap}$ FNSCl $(G_E)$

(v) (FNSInt $(F_E))^c$ = FNSCl $(F_E{}^c)$

(vi) (FNSCl $(F_E))^c$ = FNSInt $(F_E{}^c)$

**Proof:**

(i) Since $F(e) \tilde{\cap} G(e) \tilde{\subseteq} F(e)$ for any $e \in E$,

We have $F_E \tilde{\cap} G_E \tilde{\subseteq} F_E$

By theorem 5.4 (ii) FNSInt $(F_E \tilde{\cap} G_E)$ $\tilde{\subseteq}$ FNSInt $(F_E)$.

Similarly FNSInt $(F_E \tilde{\cap} G_E)$ $\tilde{\subseteq}$ FNSInt $(G_E)$.

Thus FNSInt $(F_E \tilde{\cap} G_E)$ $\tilde{\subseteq}$ FNSInt $(F_E)$ $\tilde{\cap}$ FNSInt $(G_E)$

By theorem 5.4 (i), FNSInt $(F_E)$ $\tilde{\subseteq}$ $(F_E)$ and FNSInt $(G_E)$ $\tilde{\subseteq}$ $(G_E)$.

Then FNSInt $(F_E \tilde{\cap} G_E)$ $\tilde{\subseteq}$ $F_E \tilde{\cap} G_E$

So FNSInt $(F_E)$ $\tilde{\cap}$ FNSInt $(F_E)$ $\tilde{\subseteq}$ FNSInt $(F_E \tilde{\cap} G_E)$.

Similarly we can prove (ii), (iii) & (iv).

(v) (FNSInt $(F_E))^c$ = $\tilde{\cup}(\{G_E \in \tau : G_E \tilde{\subseteq} F_E\})^c$ = $\tilde{\cap}\{G_E \in \tau^c : F_E^C \tilde{\subseteq} G_E\}$

= FNSCl$(F_E^C)$

(vi) The proof is similar to (v).

**Example 5.7:** Let U = {$b_1$, $b_2$ } and E={$e_1$, $e_2$}. Let $F_E$ be Fuzzy Neutrosophic soft set where

$F(e_1)$ = {< $b_1$,0.2,0.6,0.8>, < $b_2$, 0.6,0.5,0.3>}

$F(e_2)$ = {< $b_1$, 0.2,0.4,0.5 >, < $b_2$, 0.9,0.7,0.1>}

Obviously τ = { $F_E$ ,$\phi_E$ ,$U_E$ } is an Fuzzy Neutrosophic soft topology on U.

$G_E$, $H_E$ are

$G(e_1)$= {< $b_1$,0.1,0.2,0.8> , < $b_2$, 0.6,0.5,0.3 >}

$G(e_2)$= {< $b_1$,0.2,0.4,0.5>, < $b_2$, 0.9,0.7,0.1>}

$H(e_1)$= {< $b_1$,0.2,0.6,0.8>, < $b_2$, 0.6,0.5,0.3>}

$H(e_2)$= {< $b_1$,0.2,0.4,0.5>, < $b_2$, 0.7,0.6,0.1>}

(i) FNSInt $(G_E)$ = $\phi_E$ = FNSInt $(H_E)$

$G_E \tilde{\cup} H_E = F_E$

FNSInt $(G_E)$ $\tilde{\cup}$ FNSInt $(H_E)$ = $\phi_E \tilde{\cup} \phi_E = \phi_E$ and FNSInt $(G_E \tilde{\cup} H_E)$ = FNSInt $(F_E)$ = $F_E$.

Therefore FNSInt $(G_E)$ $\tilde{\cup}$ FNSInt $(H_E)$ ≠ FNSInt $(G_E \tilde{\cup} H_E)$.

(ii) By theorem 5.6 (v)

$$FNSCl\ (G_E^c) = (FNSInt\ (G_E))^c = \phi_E^c = U_E$$

Similarly $FNSCl\ (H_E^c) = U_E$.

$$FNSCl\ (G_E^c)\ \tilde{\cap}\ FNSCl\ (H_E^c) = U_E\ \tilde{\cap}\ U_E = U_E.$$

Similarly $FNSCl(G_E^c\ \tilde{\cap}\ H_E^c) = FNSCl(G_E\ \tilde{\cup}\ H_E)^c$

$$= [FNSInt\ (G_E\ \tilde{\cup}\ H_E)]^c$$

$$= F_E^c$$

Thus $FNSCl(G_E^c\ \tilde{\cap}\ H_E^c) \neq FNSCl\ (G_E^c)\ \tilde{\cap}\ FNSCl\ (H_E^c)$.

## 6. CONCLUSION

We have introduced the concept of fuzzy Neutrosophic soft set and studied some of its properties. We have put forward some proposition based on this new notion. We have introduced topological structure on fuzzy Neutrosophic soft set and characterized some of its properties. We hope that this paper will promote the future study on FNSS and FNSTS to carry out a general framework for their application in practical life.

## REFERENCES


[1] K. Atanassov, Intuitionistic fuzzy sets, in V. Sgurev, ed., vii ITKRS Session, Sofia (June 1983 central Sci. and Techn. Library, Bulg. Academy of Sciences (1983)).

[2] M. Bora, T.J. Neog and D.K. Sut, "A study on some operations of fuzzy soft sets" International Journal of Mathematics Trends and Technology- Volume3 Issue- 2- 2012.

[3] M. Irfan Ali, F. Feng, X. Liu, W. K. Min and M. Shabir, "On some new operations in soft set theory", Comput. Math Appl.57 (2009) 1547-1553.

[4] D.Molodtsov, Soft set Theory - First Results, Comput.Math.Appl. 37 (1999)19-31.

[5] P.K.Maji , R. Biswas ans A. R. Roy, "Fuzzy soft sets", Journal of Fuzzy Mathematics, Vol. 9, no.3, pp – 589-602, 2001

[6] P. K. Maji, R. Biswas ans A. R. Roy, "Intuitionistic Fuzzy soft sets", The journal of fuzzy Mathematics, Vol. 9, (3)(2001), 677 – 692.

[7] Pabitra Kumar Maji, Neutrosophic soft set, Annals of Fuzzy Mathematics and Informatics, Volume 5, No.1, (2013).,157-168.

[8] A.A.Salama and S.A.Al – Blowi, Neutrosophic Set and Neutrosophic topological spaces, IOSR Journal of Math., Vol. (3) ISSUE4 (2012), 31 – 35

[9] M. Shabir and M. Naz, On soft topological spaces, Comput.Math.Appl. 61 (2011)1786 – 1799.

[10] F. Smarandache, Neutrosophy and Neutrosophic Logic, First International Conference on Neutrosophy, Neutrosophic Logic, Set, Probability and Statistics University of New Mexico, Gallup, NM 87301, USA (2002).

[11] F. Smarandache, Neutrosophic set, a generialization of the intuituionistics fuzzy sets, Inter. J. Pure Appl. Math., 24 (2005), 287 – 297.

[12] B. Tanay and M.B. Kandemir, Topological structure of fuzzy soft sets, Comput.Math.Appl. 61 (2001),2952 – 2957.

[13] L. A. Zadeh, Fuzzy Sets, Inform and Control 8(1965) 338 – 353.

[14] Zhaowen Li, Rongchen cui, On the topological structure of intuitionistic fuzzy soft sets, Annals of Fuzzy Mathematics and Informatics, Volume 5, No.1, (2013),229-239.

[15] Zorlutuna, M. Akdag, W. K. Min, S. Atmaca, "Remarks on soft topological spaces", Annals of Fuzzy Mathematics and Informatics, Volume3, No.2, (2012), pp.171-185.


**Source of support: Nil, Conflict of interest: None Declared**