Some considerations about Neutrosophic Logic

Angelo de Oliveira
Unir – Departamento de Matemática e Estatística
Rua Rio Amazonas, 351 – Jardim dos Migrantes
76.900-726, Ji-Paraná, RO
E-mail: mrxyztplk@gmail.com/angelo@unir.br

Abstract

This paper presents some very basics aspects of neutrosophic logic, in particular, exhibits the connectives of negation, conjunction and disjunction as found in [1], and checks some few properties and relationship between them. Also, discusses other forms of negation and how under some aspects one careful choose may be necessary to ensure consistency. Moreover, touch on (briefly) modal operators.

Keywords: *Neutrosophic Logic*, *Negation*, *De Morgan's Law*, *Modal Operators.*

1. Introduction

Neutrosophic logic is a relatively new non-classical logic dated from 1995, being an extension/combination of the fuzzy logic, intuitionistic logic, paraconsistent logic, and three-valued logics that use an indeterminate value [1]. To be more precise, one definition of neutrosophic logic is as follow.

Definition – Neutrosophic Logic: Let T, I, F be standard or non-standard real subsets of the non-standard unit interval $]^-0, 1^+[$, with sup T = t_sup, inf T = t_inf, sup I = i_sup, inf I = i_inf, sup F = f_sup, inf F = f_inf, and n_sup = t_sup + i_sup + f_sup, n_inf = t_inf + i_inf + f_inf.

A logic in which each proposition is estimated to have the percentage of truth in a subset T, the percentage of indeterminacy in a subset I, and the percentage of falsity in a subset F, where T, I, F are defined above, is called Neutrosophic Logic. The sets T, I and F are called neutrosophic components.

The subsets are not necessarily intervals, but any sets (discrete, continuous, open or closed or half-open/half-closed interval, intersections or unions of the previous sets, etc.) in accordance with the given proposition.

Among several differences between neutrosophic logic and others is the fact that there is a possibility of distinction between absolute truth (truth in all possible worlds) and relative truth (truth in at least one world) [2].

In neutrosophic logic, every logical variable x is described by an ordered triple x = (t, i, f), where t, i and f represents the degree of truth, indeterminacy and falsity, respectively.

In this paper, following what can be found in [1], the neutrosophic components T, I and F will be single elements (a proposal to simplify the neutrosophic sets into subsets belonging to R^3 can be seen in [3]). Thus, only the following three cases will be considered: I) the sum of the components t + i + f = 1; II) the sum of components t + i + f < 1; III) the sum of the components t + i + f > 1. In addition to this, the distinction between relative and absolute truth, falsehood and indeterminacy will not be considered.

2. Definitions and Considerations

The following definitions can be found in [1].

Definition – Logical Connectives ($\neg, \wedge, \vee$) in NL (Neutrosophic Logic)

Let (t_1, i_1, f_1) and (t_2, i_2, f_2) be elements of NL where the sum of the elements of the triplet is 1. The logical connectives of negation, conjunction and disjunction are defined as follow:

$\neg(t_1, i_1, f_1) = (f_1, i_1, t_1)$
$(t_1, i_1, f_1) \wedge (t_2, i_2, f_2) = (t = \min\{t_1, t_2\}, i = 1 – (t + f), f = \max\{f_1, f_1\})$
$(t_1, i_1, f_1) \vee (t_2, i_2, f_2) = (t = \max\{t_1, t_2\}, i = 1 – (t + f), f = \min\{f_1, f_2\})$.

As there are other ways to define the connectives, then, this logic will be denoted as NL1.

Definition – Logical Connectives ($\neg, \wedge, \vee$) in NL (Neutrosophic Logic) - II

Let (t_1, i_1, f_1) and (t_2, i_2, f_2) be elements of NL where the sum of the elements of the triplet is 1. The logical connectives of negation, conjunction and disjunction are defined as follow:

$\neg(t_1, i_1, f_1) = (f_1, i_1, t_1)$
$(t_1, i_1, f_1) \wedge (t_2, i_2, f_2) = (1 - \max\{i_1, i_2\} - \min\{1 - \max\{i_1, i_2\}, \max\{f_1, f_2\}\}, \max\{i_1, i_2\}, \min\{1 - \max\{i_1, i_2\}, \max\{f_1, f_2\}\})$
$(t_1, i_1, f_1) \vee (t_2, i_2, f_2) = (\min\{1 - \max\{i_1, i_2\}, \max \{t_1, t_2\}\}, \max \{i_1,i_2\}, 1 - \max \{i_1, i_2\} – \min\{1 - \max\{i_1, i_2\}, \max\{t_1, t_2\}\})$

The NL having these connectives will be mentioned as NL2.

Definition – Logical Connectives ($\neg, \wedge, \vee$) in INL (Intuitionistic Neutrosophic Logic)

An element of an Intuitionistic Neutrosophic Logic (INL) is a quadruple (t, i, f, u), where $t + i + f + u = 1$ and $u \geq 0$ are, respectively, the degree of truth, indeterminacy, falsehood and unawareness. Now, let the quadruples (t_1, i_1, f_1, u_1) and (t_2, i_2, f_2, u_2) be elements of INL. The logical connectives of negation, conjunction and disjunction are defined as follow:

$\neg(t_1, i_1, f_1, u_1) = (f_1, i_1, t_1, u_1)$
$(t_1, i_1, f_1, u_1) \wedge (t_2, i_2, f_2, u_2) = (t = \min\{t_1, t_2\}, i = \min\{i_1, i_2\}, f = \max\{f_1, f_2\}, u = 1 – t – i - f)$
$(t_1, i_1, f_1, u_1) \vee (t_2, i_2, f_2, u_2) = (t = \max\{t_1, t_2\}, i = \min\{i_1, i_2\}, f = \min\{f_1, f_2\}, u = 1 – t – i - f)$

The INL having these connectives will be referred as INL1.

Definition – Logical Connectives ($\neg, \wedge, \vee$) in INL (Intuitionistic Neutrosophic Logic) - II

Let the quadruples (t_1, i_1, f_1, u_1) and (t_2, i_2, f_2, u_2) be elements of INL. The logical connectives of negation, conjunction and disjunction are defined as follow:

$\neg(t_1, i_1, f_1, u_1) = (f_1, i_1, t_1, u_1)$
$(t_1, i_1, f_1, u_1) \wedge (t_2, i_2, f_2, u_2) = (t = \min\{t_1, t_2\}, i = 1 – t – f - u, f = \max\{f_1, f_2\}, u = \min\{u_1, u_2\})$
$(t_1, i_1, f_1, u_1) \vee (t_2, i_2, f_2, u_2) = (t = \max\{t_1, t_2\}, i = 1 – t – f - u, f = \min\{f_1, f_2\}, u = \min\{u_1, u_2\})$

The INL having these connectives will be referred as INL2.

Definition – Logical Connectives ($\neg$, $\wedge$, $\vee$) in PNL (Paraconsistent Neutrosophic Logic)

An element of a Paraconsistent Neutrosophic Logic (PNL) is a triple (t, i, f) where $t + i + f \geq 1$. Now, let the triples (t_1, i_1, f_1) and (t_2, i_2, f_2) be elements of PNL. The logical connectives of negation, conjunction and disjunction are defined as follow:

$\neg(t_1, i_1, f_1) = (f_1, i_1, t_1)$
$(t_1, i_1, f_1) \wedge (t_2, i_2, f_2) = (t = \min\{t_1, t_2\}, i = \max\{i_1, i_2\}, f = \max\{f_1, f_2\})$
$(t_1, i_1, f_1) \vee (t_2, i_2, f_2) = (t = \max\{t_1, t_2\}, i = \max\{i_1, i_2\}, f = \min\{f_1, f_2\})$

The PNL having these connectives will be referred as PNL1.

One thing to note at first sight is the fact that for NL1, NL2, INL1, INL2 and PNL1, the connective of negation ($\neg$) behaves like the definition of the operation of negation in classical logic, three-valued logic of Lukasiewicz, fuzzy logic as well as in ordinary intuitionistic fuzzy logic. As consequence, the double negation is a valid formula. In other words, $\neg\neg A = A$. Indeed:

$\neg\neg(t_1, i_1, f_1) = \neg(f_1, i_1, t_1) = (t_1, i_1, f_1)$ (NL1)
$\neg\neg(t_1, i_1, f_1) = \neg(f_1, i_1, t_1) = (t_1, i_1, f_1)$ (NL2)
$\neg\neg(t_1, i_1, f_1, u_1) = \neg(f_1, i_1, t_1, u_1) = (t_1, i_1, f_1, u_1)$ (INL1)
$\neg\neg(t_1, i_1, f_1, u_1) = \neg(f_1, i_1, t_1, u_1) = (t_1, i_1, f_1, u_1)$ (INL2)
$\neg\neg(t_1, i_1, f_1) = \neg(f_1, i_1, t_1) = (t_1, i_1, f_1)$ (PNL1)

Given that $\neg\neg A \neq A$ is a natural assumption when one think about intuitionism, it seems that or the neutrosophic logic is not a generalization of any intuitionist logic [3], or it is necessary a more precise definition of the word intuitionistic or that if the word can't be employed in general form, then must be clear in which context it can (from the definition of INL1 and INL2, the word intuitionistic is used when the sum of the degrees of truth (t), indeterminacy (i) and falsity (f) is less than 1 ($t + i + f < 1$) and the degree of unawareness (u) is not less than zero, that is, $u \geq 0$).

The negation above mentioned is not the only one way to define negation in neutrosophic logic. Other form is the following.

Let $A = (t_1, i_1, f_1)$ represent a logical variable in neutrosophic logic, then:

$\neg A = \neg(t_1, i_1, f_1) = (1 - t_1, 1 - i_1, 1 - f_1)$ [2]

But then again, $\neg\neg A = A$. Really:

$\neg\neg A = \neg\neg(t_1, i_1, f_1) = \neg(1 - t_1, 1 - i_1, 1 - f_1) = (1 – (1 - t_1), 1 - (1 - i_1), 1 - (1 - f_1)) =$
$= (1 – 1 + t_1), 1 - 1 + i_1), 1 - 1 + f_1)) = (t_1, i_1, f_1) = A$

Yet, there is another form of negation for neutrosophic logic [8].

Let $A = (t_1, i_1, f_1)$ represent a logical variable in neutrosophic logic, then:

$\neg A = \neg(t_1, i_1, f_1) = (f_1, 1 - i_1, t_1)$

Nonetheless, once more $\neg\neg A = A$. Effectively:

$\neg\neg A = \neg\neg(t_1, i_1, f_1) = \neg(f_1, 1 - i_1, t_1) = (t_1, 1 – (1 - i_1), f_1) = (t_1, 1 – 1 + i_1), f_1) = (t_1, i_1, f_1) = A$

To distinguish these forms of negations, they will be referred as $\neg_1$, $\neg_2$ and $\neg_3$, that is, being A = (t_1, i_1, f_1), then:

$\neg_1 A = (f_1, i_1, t_1)$
$\neg_2 A = (1 - t_1, 1 - i_1, 1 - f_1)$
$\neg_3 A = (f_1, 1 - i_1, t_1)$

As noted by [9] $\neg_2$ is a simple generalization of the most used negation in fuzzy logic. But a negative aspect is that truth and falsity is not interconnected by negation. That situation is circumvented using ¬1 and ¬3, but it seems that like ¬2, ¬3 yield some non intuitive results. The negation ¬1 seems to be the best choice.

The question of the negation ¬1, ¬2 and ¬3 have the same behavior found in classical logic was an issue (and maybe a trouble) found in intuitionistic fuzzy set [4], and as the neutrosophic logic intent is to be a generalization of IFL, one possible solution for this issue will be construct new negations following the same way found, for example, in [4][5][6].

Among many others aspects of NL1, NL2, INL1, INL2 and PNL are the following:

In NL1 the disjunction may be defined via $\neg_1$ and conjunction. Indeed:

Let $A = (t_1, i_1, f_1)$ and $B = (t_2, i_2, f_2)$ be elements of NL1, then

$\neg A = (t_1, i_1, f_1) = (f_1, i_1, t_1)$
$\neg B = (t_2, i_2, f_2) = (f_2, i_2, t_2)$
$\neg A \wedge \neg B = (f_1, i_1, t_1) \wedge (f_2, i_2, t_2) = (\min\{f_1, f_2\}, 1 - \min\{f_1, f_2\} - \max\{t_1, t_2\}, \max\{t_1, t_2\})$
$\neg(\neg A \wedge \neg B) = \neg(\min\{f_1, f_2\}, 1 - \min\{f_1, f_2\} - \max\{t_1, t_2\}, \max\{t_1, t_2\}) =$
$= (\max\{t_1, t_2\}, 1 - \min\{f_1, f_2\} - \max\{t_1, t_2\}, \min\{f_1, f_2\}) = A \vee B$

On the other hand, the conjunction may be defined using $\neg_1$ and disjunction:

$\neg A = (t_1, i_1, f_1) = (f_1, i_1, t_1)$
$\neg B = (t_2, i_2, f_2) = (f_2, i_2, t_2)$
$\neg A \vee \neg B = (f_1, i_1, t_1) \vee (f_2, i_2, t_2) = (\max\{f_1,f_2\}, 1 - (\max\{f_1,f_2\} + \min\{t_1, t_2\}), \min\{t_1, t_2\})$
$\neg(\neg A \vee \neg B) = \neg(\max\{f_1,f_2\}, 1 - (\max\{f_1,f_2\} + \min\{t_1, t_2\}), \min\{t_1, t_2\}) =$
$= (\min\{t_1, t_2\}, 1 - (\max\{f_1,f_2\} + \min\{t_1, t_2\}), \max\{f_1,f_2\}) = A \wedge B$

Given that the double negation it is true, then

$\neg(A \vee B) = \neg(\neg(\neg A \wedge \neg B)) = (\neg A \wedge \neg B) = \neg A \wedge \neg B$ and
$\neg(A \wedge B) = \neg(\neg(\neg A \vee \neg B)) = (\neg A \vee \neg B) = \neg A \vee \neg B$.

Thus, is verified that NL1 satisfies the De Morgan's law.

In NL2 the disjunction may be defined via $\neg_1$ and conjunction. Really:

Let $A = (t_1, i_1, f_1)$ and $B = (t_2, i_2, f_2)$ be elements of NL2, then

$\neg A = (t_1, i_1, f_1) = (f_1, i_1, t_1)$
$\neg B = (t_2, i_2, f_2) = (f_2, i_2, t_2)$
$\neg A \wedge \neg B = (f_1, i_1, t_1) \wedge (f_2, i_2, t_2) =$
$= (1 - \max\{i_1, i_2\} - \min\{1 - \max\{i_1, i_2\}, \max\{t_1, t_2\}\}, \max\{i_1, i_2\},$
$\min\{1 - \max\{i_1, i_2\}, \max\{t_1, t_2\}\})$

$\neg(\neg A \wedge \neg B) = (\min\{1 - \max\{i_1, i_2\}, \max\{t_1, t_2\}\}, \max\{i_1, i_2\},$
$1 - \max\{i_1, i_2\} - \min\{1 - \max\{i_1, i_2\}, \max\{t_1, t_2\}\}) = A \vee B$

As happen in NL1, the conjunction may be defined using $\neg_1$ and disjunction:

$\neg A = (t_1, i_1, f_1) = (f_1, i_1, t_1)$
$\neg B = (t_2, i_2, f_2) = (f_2, i_2, t_2)$
$\neg A \vee \neg B = (f_1, i_1, t_1) \vee (f_2, i_2, t_2) = (\min\{1 - \max\{i_1, i_2\}, \max\{f_1, f_2\}\}, \max\{i_1,i_2\},$
$1 - \max \{i_1, i_2\} - \min\{1 - \max\{i_1, i_2\}, \max\{f_1, f_2\}\})$
$\neg(\neg A \vee \neg B) = \neg(\min\{1 - \max\{i_1, i_2\}, \max\{f_1, f_2\}\}, \max\{i_1,i_2\},$
$1 - \max \{i_1, i_2\} - \min\{1 - \max\{i_1, i_2\}, \max\{f_1, f_2\}\}) =$
$= (1 - \max\{i_1, i_2\} - \min\{1 - \max\{i_1, i_2\}, \max\{f_1, f_2\}\}, \max\{i_1,i_2\},$
$\min\{1 - \max\{i_1, i_2\}, \max\{f_1, f_2\}\}) = A \wedge B$

Given that the double negation it is true, then

$\neg(A \vee B) = \neg(\neg(\neg A \wedge \neg B)) = (\neg A \wedge \neg B) = \neg A \wedge \neg B$ and
$\neg(A \wedge B) = \neg(\neg(\neg A \vee \neg B)) = (\neg A \vee \neg B) = \neg A \vee \neg B$.

In consequence, it's true that NL2 satisfies the De Morgan's law.

In INL1 the disjunction may be defined via $\neg_1$ and conjunction. In fact:

Let $A = (t_1, i_1, f_1, u_1)$ and $B = (t_2, i_2, f_2, u_2)$ be elements of INL1, then

$\neg A = (t_1, i_1, f_1, u_1) = (f_1, i_1, t_1, u_1)$
$\neg B = (t_2, i_2, f_2, u_2) = (f_2, i_2, t_2, u_2)$
$\neg A \wedge \neg B = ((f_1, i_1, t_1, u_1) \wedge (f_2, i_2, t_2, u_2) = (\min\{f_1, f_2\}, \min\{i_1, i_2\}, \max\{t_1, t_2\},$
$1 - \min\{f_1, f_2\} - \min\{i_1, i_2\} - \max\{t_1, t_2\})$
$\neg(\neg A \wedge \neg B) = (\max\{t_1, t_2\}, \min\{i_1, i_2\}, \min\{f_1, f_2\}, 1 - \min\{f_1, f_2\} - \min\{i_1, i_2\} - \max\{t_1, t_2\}) =$
$= A \vee B$

Also in INL1 the conjunction may be defined using $\neg_1$ and disjunction:

$\neg A = (t_1, i_1, f_1, u_1) = (f_1, i_1, t_1, u_1)$
$\neg B = (t_2, i_2, f_2, u_2) = (f_2, i_2, t_2, u_2)$
$\neg A \vee \neg B = (f_1, i_1, t_1, u_1) \vee (f_2, i_2, t_2, u_2) = (\max\{f_1, f_2\}, \min\{i_1, i_2\}, \min\{t_1, t_2\},$
$1 - \max\{f_1, f_2\} - \min\{i_1, i_2\} - \min\{t_1, t_2\})$
$\neg(\neg A \vee \neg B) = \neg(\max\{f_1, f_2\}, \min\{i_1, i_2\}, \min\{t_1, t_2\},$
$1 - \max\{f_1, f_2\} - \min\{i_1, i_2\} - \min\{t_1, t_2\}) =$
$= (\min\{t_1, t_2\}, \min\{i_1, i_2\}, \max\{f_1, f_2\}, 1 - \max\{f_1, f_2\} - \min\{i_1, i_2\} - \min\{t_1, t_2\}) = A \wedge B$

Given that the double negation it is true, then

$\neg(A \vee B) = \neg(\neg(\neg A \wedge \neg B)) = (\neg A \wedge \neg B) = \neg A \wedge \neg B$ and
$\neg(A \wedge B) = \neg(\neg(\neg A \vee \neg B)) = (\neg A \vee \neg B) = \neg A \vee \neg B$

Consequently, INL1 satisfies the De Morgan's law.

In INL2 the disjunction may be defined via $\neg_1$ and conjunction. It is sure that:

Let $A = (t_1, i_1, f_1, u_1)$ and $B = (t_2, i_2, f_2, u_2)$ be elements of INL2, then

$\neg A = (t_1, i_1, f_1, u_1) = (f_1, i_1, t_1, u_1)$
$\neg B = (t_2, i_2, f_2, u_2) = (f_2, i_2, t_2, u_2)$
$\neg A \wedge \neg B = (\min\{f_1, f_2\}, \min\{i_1, i_2\}, \max\{t_1, t_2\}, 1 - \min\{f_1, f_2\} - \min\{i_1, i_2\} - \max\{t_1, t_2\})$
$\neg(\neg A \wedge \neg B) = \neg(\max\{t_1, t_2\}, \min\{i_1, i_2\}, \min\{f_1, f_2\}, 1 - \max\{t_1, t_2\} - \min\{i_1, i_2\} -) = A \vee B$

As well as in INL2 the conjunction may be defined using $\neg_1$ and disjunction:

$\neg A = (t_1, i_1, f_1, u_1) = (f_1, i_1, t_1, u_1)$
$\neg B = (t_2, i_2, f_2, u_2) = (f_2, i_2, t_2, u_2)$
$\neg A \vee \neg B = (f_1, i_1, t_1, u_1) \vee (f_2, i_2, t_2, u_2) =$
$= (\max\{f_1, f_2\}, 1 - \max\{f_1, f_2\} - \min\{t_1, t_2\} - \min\{u_1, u_2\}, \min\{t_1, t_2\}, \min\{u_1, u_2\})$
$\neg(\neg A \vee \neg B) = \neg(\max\{f_1, f_2\}, 1 - \max\{f_1, f_2\} - \min\{t_1, t_2\} - \min\{u_1, u_2\}, \min\{t_1, t_2\},$
$\min\{u_1, u_2\}) =$
$= (\min\{t_1, t_2\}, 1 - \min\{t_1, t_2\} - \max\{f_1, f_2\} - \min\{u_1, u_2\}, \max\{f_1, f_2\},$
$\min\{u_1, u_2\}) = A \wedge B$

Given that the double negation it is true, then

$\neg(A \vee B) = \neg(\neg(\neg A \wedge \neg B)) = (\neg A \wedge \neg B) = \neg A \wedge \neg B$ and
$\neg(A \wedge B) = \neg(\neg(\neg A \vee \neg B)) = (\neg A \vee \neg B) = \neg A \vee \neg B$

Therefore, INL2 satisfies the De Morgan's law.

In PNL1 the disjunction may be defined via $\neg_1$ and conjunction. Indeed:

Let $A = (t_1, i_1, f_1)$ and $B = (t_2, i_2, f_2)$ be elements of PNL1, then

$\neg A = (t_1, i_1, f_1) = (f_1, i_1, t_1)$
$\neg B = (t_2, i_2, f_2) = (f_2, i_2, t_2)$
$\neg A \wedge \neg B = (f_1, i_1, t_1) \wedge (f_2, i_2, t_2) = (\min\{f_1, f_2\}, \max\{i_1, i_2\}, \max\{t_1, t_2\})$
$\neg(\neg A \wedge \neg B) = \neg(\max\{t_1, t_2\}, \max\{i_1, i_2\}, \min\{f_1, f_2\}) = A \vee B$

But the conjunction may be defined in PNL1 using $\neg_1$ and disjunction:

$\neg A = (t_1, i_1, f_1) = (f_1, i_1, t_1)$
$\neg B = (t_2, i_2, f_2) = (f_2, i_2, t_2)$
$\neg A \vee \neg B = (f_1, i_1, t_1) \wedge (f_2, i_2, t_2) = (\max\{f_1, f_2\}, \max\{i_1, i_2\}, \min\{t_1, t_2\})$
$\neg(\neg A \vee \neg B) = \neg(\max\{f_1, f_2\}, \max\{i_1, i_2\}, \min\{t_1, t_2\}) = (\min\{t_1, t_2\}, \max\{i_1, i_2\}, \max\{f_1, f_2\}) =$
$= A \wedge B$

Given that the double negation it is true, then

$\neg(A \vee B) = \neg(\neg(\neg A \wedge \neg B)) = (\neg A \wedge \neg B) = \neg A \wedge \neg B$ and
$\neg(A \wedge B) = \neg(\neg(\neg A \vee \neg B)) = (\neg A \vee \neg B) = \neg A \vee \neg B$

In this manner, PNL1 satisfies the De Morgan's law.

On the other side, if the $\neg_2$ or $\neg_3$ are used together with the definition of conjunction of NL1 (for example) to describe disjunction in the same way as above, then the result is:

$\neg_2 A = (t_1, i_1, f_1) = (1 - t_1, 1 - i_1, 1 - f_1)$
$\neg_2 B = (t_2, i_2, f_2) = (1 - t_2, 1 - i_2, 1 - f_2)$
$\neg_2 A \wedge \neg_2 B = (1 - t_1, 1 - i_1, 1 - f_1) \wedge (1 - t_2, 1 - i_2, 1 - f_2) =$

$= (\min\{1 - t_1, 1 - t_2\}, 1 – (\min\{1 - t_1, 1 - t_2\} + \max\{1 - f_1, 1 - f_1\}), \max\{1 - f_1, 1 - f_1\})$
$\neg_2(\neg_2 A \wedge \neg_2 B) =$
$= \neg_2(\min\{1 - t_1, 1 - t_2\}, 1 – (\min\{1 - t_1, 1 - t_2\} + \max\{1 - f_1, 1 - f_1\}), \max\{1 - f_1, 1 - f_1\}) =$
$= (1 - \min\{1 - t_1, 1 - t_2\}, \min\{1 - t_1, 1 - t_2\} + \max\{1 - f_1, 1 - f_1\}, 1 - \max\{1 - f_1, 1 - f_1\}) \neq$
$\neq A \vee_{NL1} B$

$\neg_3 A = (t_1, i_1, f_1) = (f_1, 1 - i_1, t_1)$
$\neg_3 B = (t_2, i_2, f_2) = (f_2, 1 - i_2, t_2)$
$\neg_3 A \wedge \neg_3 B = (\min\{f_1, f_2\}, \min\{i – i_1, 1 – i_2\}, \max\{t_1, t_2\})$
$\neg_3(\neg_3 A \wedge \neg_3 B) = \neg_3(\min\{f_1, f_2\}, \min\{i – i_1, 1 – i_2\}, \max\{t_1, t_2\}) =$
$= (\max\{t_1, t_2\}, 1 - \min\{i – i_1, 1 – i_2\}, \min\{f_1, f_2\}) \neq A \vee_{NL1} B$

As can readily be seen (the subscript NL1 appended to $\vee$ means that this connective it is the same found in NL1), when other types of negation are used, the conjunction give origin to another form of disjunction not dual with herself, that is, the De Morgan’s law not remain.

This outcome suggest that a careful choose between the connectives of negation, conjunction and disjunction must be done to ensure consistency, and given that these are used to produce others, this question is the capital importance.

Another question raised in [3] is the fact that seems that neutrosophic logic is not capable of maintaining modal operators, and this confronts with the affirmation given in the paragraph five of the first section of this paper. Given that in [2] is said that neutrosophic logic can distinguish between absolute truth (represented by 1^+ - truth in all worlds) and relative truth (represented by 1 - truth at least one world) and that this can be used in philosophy, the absence of adequate modal operators seems, at least, a limitation (in [2] the modal operators are only mentioned very briefly). Contrasting with this, in IFS (and IFL) operators with similarity of that found in modal logic, that is, necessity (□) and possibility (◊), are defined from the beginning [7]. On other side [9] believes that the neutrosophic formalism can be extended to deal with modal contexts.

3. Conclusion

Neutrosophic logic is a new type of non-classical logic that has very high aspirations, not to mention, the fact that his intent is to be a unifying field in logics. However, there are several controversial aspects in it, and at the moment, is not sure that this unification can be done. Firstly, the negations (as showed in this paper) behave in identical way in classical logic, and traditionally the invalidity of the double negation is an intuitionistic aspect. Still, negation, conjunction and disjunction must be carefully chosen to ensure consistency with other types of logic (if wished – a generalization must include all properties and add some others to be considered as being a generalization, right?). Other aspect is the fact that modal operators can be not properly defined in neutrosophic logic or may be impossible to define them in a proper manner.

In spite of this, some ideas in neutrosophic logic are interesting and deserve some additional research, and is not vain remember the wise words uttered by Valisiev [10]: “Será ofensivo para a nossa inteligência amadurecida se, ao nos chocarmos com outro tipo de operações lógicas diferentes das nossas, arbitrariamente deixarmos de chamá-las de lógicas”.

References

[1] ASHBACHER, C. *Introduction to Neutrosophic logic*. Rehoboth: American Research Press, 2002.

[2] SMARANDACHE, Florentin. *A Unifying Field in Logics: Neutrosophic Logic, Neutrosophy, Neutrosophic Set, Neutrosophic Probability and Statistics*. Rehoboth: American Research Press, 2005.

[3] GEORGIEV, K. *A simplification of the Neutrosophic Sets. Neutrosophic Logic and Intuitionistic Fuzzy Sets*. Notes on Intuitionistic Fuzzy Sets, v. 11, n. 2, p. 28-31, 2005.

[4] ATANASSOV, K. T. *On Intuitionistic Fuzzy Negations and Law for Excluded Middle*. 5th IEEE International Conference on Intelligent Systems (IS), p. 266-269, 2010.

[5] ATANASSOV, K. T. *On Some Intuitionistic Fuzzy Negations*. Notes on Intuitionistic Fuzzy Sets, v. 11, n. 6, p. 13-20, 2005.

[6] ATANASSOV, K. T. *On Some Types of Intuitionistic Fuzzy Negations*. Notes on Intuitionistic Fuzzy Sets, v. 11, n. 4, p. 170-172, 2005.

[7] ATANASSOV, K. T. *Intuitionistic Fuzzy Sets*. Fuzzy Sets and Systems, n. 20, p. 87-96, 1986.

[8] WANG, H., et all. *Interval Neutrosophic Sets and Logic: Theory and Applications in Computing*. Phoenix: Hexis, 2005.

[9] RIVIECCIO, U. *Neutrosophic Logics: Prospects and Problems*. Fuzzy Sets and Systems, n. 159, p. 1860-1868.

[10] ARRUDA, A. I. *N. A. Vasiliev e a Lógica Paraconsistente*. Coleção CLE, v. 7. Campinas: CLE, 1990.