ON THE PERFECT SQUARES IN SMARANDACHE CONCATENATED SQUARE SEQUENCE

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Abstract

Let n be positive integer, and let s(n) denote the n-th Smarandache concatenated squre number. In this paper we prove that if $n \equiv 2, 3, 4, 7, 8, 9, 11, 12, 14, 16, 17, 18, 20, 21, 22$, or $25 \pmod{27}$, then s(n) is not a square.

In [1], Marimutha defined the Smarandache concatenated square sequence $\{s(n)\}_{n=1}^{\infty}$ as follows:

(1) $s(1) = 1$, $s(2) = 14$, $s(3) = 149$, $s(4) = 14916$, $s(5) = 1491625, \ldots$.

Then we called s(n) the n-th Smarandache concatenated square number. Marimutha [1] conjectured that s(n) is never a perfect square. In this paper we prove the following result:

Theorem.

If $n \equiv 2, 3, 4, 7, 8, 9, 11, 12, 14, 16, 17, 18, 20, 21, 22$, or $25 \pmod{27}$, then s(n) is not a perfect square.

The above result implies that the density of perfect squares in Smarandache concatenated square sequence is at most 11/27.

Prof of Theorem. We now assume that s(n) is a perfect square. Then we have

$$s(n) = x^2, \tag{2}$$

were x is a positive integer. Notice that $10^k \equiv 1 \pmod 9$ for any positive integer k. We get from (1) and (2) that

$$s(n) \equiv 1^2 + 2^2 + \ldots + n^2 \equiv 1/6\, n(n+1)(2n+1) \equiv x^2 \pmod 9. \tag{3}$$

It implies that

(4) $$n(n+1)(2n+1) \equiv 6x^2 \pmod{27}.$$

If $n \equiv 2 \pmod{27}$, then from (4) we get $2*3*5 \equiv 6x^2 \pmod{27}$. It follows that

(5) $$x^2 \equiv 5 \pmod{9}.$$

Since 5 is not a square residue mod 3 , (5) is impossible. Therefore, if $n \equiv 2 \pmod{27}$, then s(n) is not a square.

By using some similarly elementary number theory methods, we can check that the congruence (4) does not hold for the remaining cases. The theorem is proved.

Reference:

1.H.Marimutha, "Smarandache concatenate type sequences", Bulletin of Pure and Applied Sciences, 16E (1997), No. 2, 225-226.