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Results in Mathematics

Smarandache n-Structure on CI-Algebras

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Abstract. In this paper, the notions of CI-algebras, Smarandache CI-algebra, Q-Smarandache filters and Q-Smarandache ideals are introduced. We show that a nonempty subset F of a CI-algebra X is a Q-Smarandache filter if and only if $A(x, y) \subseteq F$, which $A(x, y)$ is a Q-Smarandache upper set. Finally, we introduced the concepts of Smarandache BE-algebra, Smarandache dual BCK-algebra and Smarandache n-structure on CI-algebra.

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Keywords. CI-algebras, BE-algebra, dual BCK-algebra, implication algebra, Smarandache CI-algebra, Smarandache BE-algebra, (Q-Smarandache) Filter, (Q-Smarandache) ideal.

1. Introduction

The Smarandache algebraic structures theory was introduced in 1998 by Padilla [11]. In [6], Kandasamy studied of Smarandache groupoids, sub-groupoids, ideal of groupoids, seminormal sub groupoids, Smarandache Bol groupoids, and strong Bol groupoids and obtained many interesting results about them. Smarandache semigroups are very important for the study of congruences, and they were studied by Padilla [11]. In [5] Jun discussed the Smarandache structure in BCI-algebras. He introduced the notion of Smarandache (positive implicative, commutative, implicative) BCI-algebras, Smarandache subalgebras and Smarandache ideals and investigated some related properties. Smarandache BL-algebras have been invented by Borumand Saeid et al. [3], and they deal with Smarandache ideal structures in Smarandache BL-algebras.

Birkhäuser

Recently, Kim and Kim defined a BE-algebra [8]. Ahn and So [2] defined notion of ideals in BE-algebras and then stated and proved several characterizations of such ideals. In [10], Meng introduced the notion of an CI-algebra as a generalization of a BE-algebra.

In this paper, we discuss Smarandache structure on CI-algebras, and introduced of Smarandache filter and Smarandache ideals, then we obtain some related results which have been mentioned in the abstract.

2. Preliminaries

Definition 2.1. [8] An algebra $(X;*,1)$ of type $(2,0)$ is called a BE-algebra if
(BE1) $x*x=1$ for all $x\in X$;
(BE2) $x*1=1$ for all $x\in X$;
(BE3) $1*x=x$ for all $x\in X$;
(BE4) $x*(y*z)=y*(x*z)$ for all $x,y,z\in X$ (exchange).

A binary relation "$\leq$" on X is defined by $x\leq y$ if and only if $x*y=1$.

Proposition 2.2. [8] *If $(X;*,1)$ is a BE-algebra, then $x*(y*x)=1$, for any $x,y\in X$.*

Definition 2.3. [9] An algebra $(X;*,1)$ of type $(2,0)$ is called a CI-algebra if
(CI1) $x*x=1$ for all $x\in X$;
(CI2) $1*x=x$ for all $x\in X$;
(CI3) $x*(y*z)=y*(x*z)$ for all $x,y,z\in X$.

Definition 2.4. [7] An algebra $(X;*,1)$ of type $(2,0)$ is called a dual BCK-algebra if
(BE1) $x*x=1$ for all $x\in X$;
(BE2) $x*1=1$ for all $x\in X$;
(dBCK1) $x*y=y*x=1\Longrightarrow x=y$;
(dBCK2) $(x*y)*((y*z)*(x*z))=1$;
(dBCK3) $x*((x*y)*y)=1$.

Lemma 2.5. [7] *Let $(X;*,1)$ be a dual BCK-algebra and $x,y,z\in X$. Then*:
(a) $x*(y*z)=y*(x*z)$,
(b) $1*x=x$.

Proposition 2.6. [13] *Any dual BCK-algebra is a BE-algebra.*

Example 1. [13] Let $X=\{1,2,\ldots\}$ and the operation $*$ be defined as follows.

$$x*y=\begin{cases}y & \text{if } x=1\\ 1 & \text{otherwise}\end{cases}$$

Then $(X;*,1)$ is a BE-algebra, but it is not a dual BCK-algebra.

Definition 2.7. [4] An algebra $(X;*,1)$ of type $(2,0)$ is called an implication algebra if for all $x,y,z\in X$ the following identities hold:

(I1) $(x * y) * x = x$;
(I2) $(x * y) * y = (y * x) * x$;
(I3) $x * (y * z) = y * (x * z)$.

Proposition 2.8. [4] *If* $(X; *, 1)$ *is an implication algebra, then* $(X; *, 1)$ *is a dual* BCK*-algebra.*

Proposition 2.9. [13] *Any implication algebra is a* BE*-algebra.*

Definition 2.10. [8] Let $(X, *, 1)$ be a BE-algebra and F be a non-empty subset of X. Then F is said to be a filter of X, if

$(F1)$ $1 \in F$;
$(F2)$ $x * y \in F$ and $x \in F$ imply $y \in F$.

Definition 2.11. [1] A non-empty subset I of X is called an ideal of X if it satisfies:

(I1) $\forall x \in X$ and $\forall a \in I$ imply $x * a \in I$, i.e, $X * I \subseteq I$;
(I2) $\forall x \in X, \forall a, b \in I$ imply $(a * (b * x)) * x \in I$.

Lemma 2.12. [2] *A nonempty subset* I *of* X *is an ideal of* X *if and only if it satisfies*

1. $1 \in I$;
2. $(\forall x, y \in X)$ $(\forall y \in I)$ $(x * (y * z) \in I \Rightarrow x * z \in I)$.

Proposition 2.13. [1] *Let* I *be an ideal of* X*. If* $a \in I$ *and* $a \leq x$*, then* $x \in I$*.*

Definition 2.14. [9] Let X be a CI-algebra and $x, y \in X$. Define $A(x, y)$ by

$$A(x, y) := \{z \in X : x * (y * z) = 1\}$$

We call $A(x, y)$ an upper set of x and y.

Definition 2.15. [13] A CI-algebra X is said to be self distributive if $x*(y*z) = (x * y) * (x * z)$, for all $x, y, z \in X$

Definition 2.16. [8] A BE-algebra X is said to be self distributive if $x*(y*z) = (x * y) * (x * z)$, for all $x, y, z \in X$

Proposition 2.17. [12] *Let* X *be a self distributive* BE*-algebra. Then for all* $x, y, z \in X$ *the following statements hold*:

(1) *if* $x \leq y$*, then* $z * x \leq z * y$;
(2) $y * z \leq (x * y) * (x * z)$;

Definition 2.18. [13] A BE-algebra X is called commutative, if $(x * y) * y = (y * x) * x$ for any $x, y \in X$.

Theorem 2.19. [13] *If* $(X, *, 1)$ *is a commutative* BE*-algebra, then* $(X, *, 1)$ *is a dual* BCK*-algebra.*

3. Smarandache *CI*-Algebras

Note that every BE-algebra is a CI-algebra, but the converse is not true. A CI-algebra which is not a BE-algebra is called a proper CI-algebra.

Definition 3.1. A Smarandache CI-algebra X is defined to be a CI-algebra X in which there exists a proper subset Q of X such that

(S1) $1 \in Q$ and $|Q| \geq 2$;
(S2) Q is a BE-algebra under the operation of X.

Example 2. Let $X := \{1, a, b, c, d\}$ be a set with the following table.

$*$	1	a	b	c	d
1	1	a	b	c	d
a	1	1	b	b	d
b	1	a	1	a	d
c	1	1	1	1	d
d	d	d	d	d	1

Then $(X; *, 1)$ is a CI-algebra but is not a BE-algebra. If $Q := \{1, a, b, c\}$, then Q is a BE-algebra. So X is a Q-Smarandache CI-algebra.

Example 3. Let $X := \{1, a, b\}$ be a set with the following table.

$*$	1	a	b
1	1	a	b
a	a	1	b
b	b	b	1

then $(X; *, 1)$ is a CI-algebra but is not a Smarandache CI-algebra.

Definition 3.2. A nonempty subset F of CI-algebra X is called a Smarandache filter of X related to Q (or briefly, Q-Smarandache filter of X) if it satisfies:

(SF1) $1 \in F$;
(SF2) $(\forall y \in Q)(\forall x \in F)(x * y \in F \Rightarrow y \in F)$.

Example 4. In Example 2, $\{1, a\}$ and $\{1, b\}$ are Smarandache filter of X.

Note. If F is a Smarandache filter of CI-algebra X related to every BE-algebra contained in X, we simply say that F is a Smarandache filter of X.

Proposition 3.3. *If $\{F_\lambda : \lambda \in \Delta\}$ is an indexed set of Q-Smarandache filters of X, where $\Delta \neq \emptyset$, then $F = \cap\{F_\lambda : \lambda \in \Lambda\}$ is a Q-Smarandache filter of X.*

Proposition 3.4. *Any Filter F of CI-algebra X is a Q-Smarandache filter of X.*

Note. By the following example we show that the converse of above proposition is not correct in general.

Example 5. Let $X := \{1, a, b, c\}$ be a set with the following table.

$*$	1	a	b	c
1	1	a	b	c
a	1	1	b	c
b	1	a	1	c
c	c	c	c	1

Then X is a CI-algebra and $Q = \{1, a\}$ is BE-algebra which is properly contained in CI-algebra X and $F = \{1, b, c\}$ is a Q-Smarandache filter of X, but it is not a filter of X because $c * a = c \in F$ and $c \in F$, but $a \notin F$.

Proposition 3.5. *If F is a Q-Smarandache filter F of self distributive CI-algebra X, then* $(\forall x, y, z \in Q)(z * (y * x) \in F, z * y \in F \Rightarrow z * x \in F)$.

Proof. Since $z * (y * x) = (z * y) * (z * x) \in F$ and $z * y \in F$, then by $(SF2)$ we have $z * x \in F$. □

Proposition 3.6. *If F is a Q-Smarandache filter F of self distributive CI-algebra X, then* $(\forall x, y \in Q)(y * (y * x) \in F \Rightarrow y * x \in F)$.

Proof. Assume that $y * (y * x) \in F$, for all $x, y \in Q$ since $y * y = 1 \in F$, by $(SF1)$ and Proposition 3.5 we have $y * x \in F$. □

Proposition 3.7. *Let F be a Q-Smarandache filter of X. Then*:

(1) $F \neq \emptyset$.
(2) *If $x \in F, x \leq y, y \in Q$, then $y \in F$.*
(3) *If X is self distributive BE-algebra and $x, y \in F$, then $x * y \in F$.*

Proof. (1) Since F is a Q-Smarandache filter of X, therefore by $(SF1)$ we have $1 \in F$, then $F \neq \emptyset$.
(2) Let $x \in F, x \leq y$ and $y \in Q$. Then $x * y = 1 \in F$, therefore by $(SF2)$ we get that $y \in F$.
(3) We have $y * (x * (x * y)) = x * (y * (x * y)) = x * (x * 1) = x * 1 = 1$, thus $y * (x * (x * y)) \in F$, also $y \in F$, then by $(SF2) x * (x * y) \in F$, therefore by Proposition 3.6, $x * y \in F$. □

Proposition 3.8. *If F is a Q-Smarandache filter of CI-algebra X and Q satisfies $X * Q \subseteq Q$, then* $(\forall x, y \in F)(\forall z \in Q)(x * (y * z) = 1 \Rightarrow z \in F)$

Proof. Assume that $X * Q \subseteq Q$ and F be a Q-Smarandache filter of X. Suppose that $x*(y*z) = 1$, for all $x, y \in F$ and $z \in Q$, then $y*z \in Q$ by hypothesis and $x * (y * z) \in F$ we have $y * z \in F$. By $(SF2)$ since $y \in F$, it follows that $z \in F$. This completes the proof. □

Theorem 3.9. *Let Q_1 and Q_2 are BE-algebras which are properly contained in CI-algebra X and $Q_1 \subseteq Q_2$. Then every Q_2-Smarandache filter is a Q_1-Smarandache filter.*

Note. By the following example we show that the converse of above theorem is not correct in general.

Example 6. In Example 5, $Q_1 = \{1, a\}, Q_2 = \{1, a.b\}$ are BE-algebra which are properly contained in X. It is easily checked that $F = \{1, b, c\}$ is a Q_1-Smarandache filter of X and is not a Q_2-Smarandache filter of X, since $c * a = c \in F$, but $a \notin F$.

Proposition 3.10. *Let X be a self distributive Smarandache CI-algebra and F be a Q-Smarandache filter of X. Then $F_a = \{x : a * x \in F\}$ is a Q-Smarandache filter, for any $a \in X$.*

Proof. Since $a * a = 1 \in F$, then $a \in F_a$ and so $\emptyset \neq F_a$. Assume $x * y \in F_a$ and $x \in F_a$, then $a * (x * y) \in F$ and $a * x \in F$. By self distributivity we have $(a * x) * (a * y) \in F$ and $a * x \in F$. Thus $a * y \in F$ and so $y \in F_a$. Therefore F_a is a Q-Smarandache filter of X. □

Definition 3.11. Let X be a CI-algebra, $x, y \in X$ and $Q \subset X$ be a BE-algebra. Define

$$A(x, y) := \{z \in Q : x * (y * z) = 1\}$$

We call $A(x, y)$ a Q-Smarandache upper set of x and y.

Note. It is easy to see that $1, x, y \in A(x, y)$. The set $A(x, y)$, where $x, y \in Q$, need not be a Q-Smarandache filter of X in general. In Example 2, it is easy to check that $A(1, d) = \{d\}$, which means that $A(1, a)$ is not a Q-Smarandache filter of X.

Proposition 3.12. *Let X be a self distributive Smarandache CI-algebra. Then Q-Smarandache upper set $A(x, y)$ is a Q-Smarandache filter of X, where $x, y \in Q$.*

Proof. Since $x * 1 = 1$ for all $x \in Q$, then $1 \in A(x, y)$. Let $a * b \in A(x, y)$ and $a \in A(x, y)$, where $b \in Q$. Thus $1 = x * (y * (a * b))$ and $1 = x * (y * a)$. From self distributivity we have

$$\begin{aligned} 1 &= x * (y * (a * b)) \\ &= x * ((y * a) * (y * b)) \\ &= (x * (y * a)) * (x * (y * b)) \\ &= 1 * (x * (y * b)) \\ &= x * (y * b) \end{aligned}$$

Therefore $b \in A(x, y)$. This proves that $A(x, y)$ is a Q-Smarandache filter of X. □

Theorem 3.13. *Let F be a non-empty subset of a CI-algebra X. F is a Q-Smarandache filter of X if and only if $A(x, y) \subseteq F$, which $A(x, y)$ is a Q-Smarandache upper set.*

Proof. Assume that F is a Q-Smarandache filter of X and $x, y \in F$. If $z \in A(x, y)$, then $x*(y*z) = 1 \in F$. By Proposition 3.12 $z \in F$. Hence $A(x, y) \subseteq F$.

Conversely, suppose that $A(x, y) \subseteq F$ for all $x, y \in F$. Since $x * (y * 1) = x * 1 = 1, 1 \in A(x, y) \subseteq F$. Assume $a * b, a \in F$. Since $(a * b) * (a * b) = 1$, we have $b \in A(a * b, a) \subseteq F$. Hence F is a Q-Smarandache filter of X. □

Theorem 3.14. *If F is a Q-Smarandache filter of a CI-algebra X, then*

$$F = \cup_{x,y\in F} A(x, y).$$

Proof. Let F be a Q-Smarandache filter of X and $z \in F$. Since $z * (1 * z) = z * z = 1$, we have $z \in A(z, 1)$. Hence

$$F \subseteq \cup_{z\in F} A(z, 1) \subseteq \cup_{x,y\in F} A(x, y)$$

If $z \in \cup_{x,y\in F} A(x, y)$, then there exist $a, b \in F$ such that $z \in A(a, b)$. By Theorem 3.13 we get that $z \in F$. This means that $\cup_{x,y\in F} A(x, y) \subseteq F$. □

Theorem 3.15. *If F is a Q-Smarandache filter of CI-algebra X, then*

$$F = \cup_{x\in F} A(x, 1).$$

Proof. Let F be a Q-Smarandache filter of X and $z \in F$. Since $z * (1 * z) = z * z = 1$, we have $z \in A(z, 1)$. Hence

$$F \subseteq \cup_{z\in F} A(z, 1)$$

If $z \in \cup_{x\in F} A(x, 1)$, then there exists $a \in F$ such that $z \in A(a, 1)$, which means that $a * z = a * (1 * z) = 1 \in F$. Since F is Q-Smarandache filter of X and $a \in F$, we have $z \in F$. This means that $\cup_{x\in F} A(x, 1) \subseteq F$. □

Definition 3.16. A nonempty subset I of Smarandache CI-algebra X is called a Smarandache ideal of X related to Q (or briefly, Q-Smarandache ideal of X) if it satisfies:

(SI1) $\forall x \in Q$ and $\forall a \in I$ imply $x * a \in I$, i.e., $Q * I \subseteq I$

(SI2) $(\forall x \in Q)(\forall a, b \in I)$ imply $(a * (b * x)) * x \in I$.

Example 7. In Example 2, $Q = \{1, a, b\}$ is a BE-algebra of X and $I = \{1, a\}$ is a Smarandache ideal of Q, but $J = \{1, c\}$ is not a Q-Smarandache ideal of X because $c, 1 \in J$ and $a \in Q$, but $(c*(1*a))*a = (c*a)*a = 1*a = a \notin J$.

Lemma 3.17. *Let X be a CI-algebra. Then*

(1) *Every Q-Smarandache ideal I of X contains* 1;

(2) *If I is a Q-Smarandache ideal of X, then $(a * x) * x \in I$ for all $a \in I$ and $x \in Q$.*

Proof. (1) Let $\emptyset \neq I$ be a Q-Smarandache ideal of X. For $x \in I, 1 = x * x \in I * I \subseteq Q * I \subseteq I$. Thus $1 \in I$.

(2) Let $b := 1$ in $(SI2)$. Then $(a * (1 * x)) * x \in I$. Hence $(a * x) * x \in I$.

□

Lemma 3.18. *A nonempty subset I of Q-Smarandache of X is a Q-Smarandache ideal of X if and only if it satisfies*

1. $1 \in I$;
2. $(\forall x, y \in Q)(\forall y \in I)(x * (y * z) \in I \Rightarrow x * z \in I)$.

Theorem 3.19. *Let Q_1 and Q_2 are BE-algebras which are properly contained in X and $Q_1 \subseteq Q_2$. Then every Q_2-Smarandache ideal of X is a Q_1-Smarandache ideal.*

Definition 3.20. If X is a Q-Smarandache CI-algebra, X is said to be a Q-Smarandache commutative if Q is a commutative BE-algebra, i.e, for all $x, y \in Q, (x * y) * y = (y * x) * x$.

Example 8. Let $X := \{1, a, b, c, d\}$ be a set with the following table.

$*$	1	a	b	c	d
1	1	a	b	c	d
a	1	1	a	a	d
b	1	1	1	a	d
c	1	1	a	1	d
d	d	d	d	d	1

Then $(X; *, 1)$ is CI-algebra, but $Q := \{1, a, b, c\} \subseteq X$ is a commutative BE-algebra, so X is a Q-Smarandache commutative BE-algebra.

Proposition 3.21. *If X is a Q-Smarandache commutative CI-algebra, then for all $x, y \in Q, x * y = 1$ and $y * x = 1$ imply $x = y$.*

Proof. Let $x, y \in Q$ and $x * y = y * x = 1$. Then

$$x = 1 * x = (y * x) * x = (x * y) * y = 1 * y = y.$$

□

Theorem 3.22. *An algebra X is a Q-Smarandache commutative CI-algebra if and only if the following identities hold: for any $x, y, z \in Q$*

(1) $(y * 1) * x = x$;
(2) $(y * x) * (z * x) = (x * y) * (z * y)$;
(3) $x * (y * z) = y * (x * z)$.

Proof. Necessity. It suffices to prove (2). By $(BE4)$ and commutativity we have $(z * x) * (y * x) = y * ((z * x) * x) = y * ((x * z) * z) = (x * z) * (y * z)$.

Sufficiency. By (1) we have $1 * x = ((1 * 1) * 1) * x = x$. $(BE3)$

From (1) and $(BE3)$ we conclude $1 = 1 * 1 = ((1 * x) * 1) * (1 * 1) = (1 * (1 * x) * (1 * (1 * x)) = (1 * x) * (1 * x) = x * x$. $(BE1)$

By $(BE1)$ we have $1 = (x * 1) * (x * 1) = x * 1$, hence $(BE2)$ hold. It suffices to prove commutativity. From $(1), (2), (3)$, we have

$(y * x) * x = (y * x) * ((y * 1) * x) = (x * y) * ((y * 1) * y) = (x * y) * y$. Then Q is a commutative BE-algebra. □

Definition 3.23. A Smarandache BE-algebra X is defined to be a BE-algebra X in which there exists a proper subset Q of X such that
(S1) $1 \in Q$ and $|Q| \geq 2$;
(S2) Q is a dual BCK-algebra under the operation of X.

Example 9. Let $X := \{1, a, b, c\}$ be a set with the following table.

$*$	1	a	b	c
1	1	a	b	c
a	1	1	a	a
b	1	1	1	1
c	1	1	1	1

Then $(X, *, 1)$ is a BE-algebra, but it is not a dual dual BCK-algebra because $b * c = 1$ and $c * b = 1$, but $c \neq b$. On the other hand $Q := \{1, a, b, c\} \subset X$ is a dual BCK-algebra. Then X is a Q-Smarandache BE-algebra.

Definition 3.24. A Smarandache dual BCK-algebra X is defined to be a dual BCK-algebra X in which there exists a proper subset Q of X such that
(S1) $1 \in Q$ and $|Q| \geq 2$;
(S2) Q is an implication algebra under the operation of X.

Example 10. Let $X := \{1, a, b\}$ be a set with the following table.

$*$	1	a	b
1	1	a	b
a	1	1	a
b	1	1	1

Then $(X, *, 1)$ is a dual BCK-algebra, but it is not an implication algebra because $(a * b) * a = a * a = 1 \neq a$. On the other hand $Q := \{1, a\} \subset X$ is an implication algebra. Then X is a Q-Smarandache dual BCK-algebra.

Note. A Smarandache strong n-structure on a set S means a structure $\{W_0\}$ on a set S such that there exists a chain of proper subsets $P_{n-1} < P_{n-2} < \cdots < P_2 < P_1 < S$ where $<$ means "included in" whose corresponding structures verify the inverse chain $W_{n-1} > W_{n-2} > \cdots > W_2 > W_1 > W_0$ where $>$ signifies strictly stronger (i.e structure satisfying more axioms)

Definition 3.25. A Smarandache strong 3-structure of CI-algebra X is a chain $X_1 > X_2 > X_3 > X_4$ where X_1 is a CI-algebra, X_2 is a BE-algebra, X_3 is a dual BCK-algebra, X_4 is an implication algebra.

Example 11. Let $X := \{1, a, b, c, d\}$ be a set with the following table.

$*$	1	a	b	c	d
1	1	a	b	c	d
a	1	1	a	a	d
b	1	1	1	a	d
c	1	1	1	1	d
d	d	d	d	d	1

hence $X_1 = \{1, a, b, c, d\}$ is a CI-algebra, $X_2 = \{1, a, b, c\}$ is a BE-algebra, $X_3 = \{1, a, b\}$ is a dual BCK-algebra, $X_4 = \{1, a\}$ is an implication algebra. So, X is a Smarandache strong 3-structure.

4. Conclusion

In this paper, we have introduced the concept of Smarandache CI-algebras and investigated some of their useful properties. It is well known that the ideals and filters with special properties play an important role in the logic system. the aim of this article is to investigate Smarandache ideals and Smarandache filters in CI-algebra, we obtain the related properties and introduce Smarandache strong 3-structures CI-algebras.

We believe that these results are very useful in developing algebraic structures also these definitions and main results can be similarly extended to some other algebraic systems such as lattices and Lie algebras etc. In our future study of Smarandache structure of CI-algebras, may be the following topics should be considered:

(1) To get more results in Smarandache CI-algebras and application;
(2) To get more connection between CI-algebra and Smarandache CI-algebra;
(3) To define another Smarandache structure;
(4) To define fuzzy structure of Smarandache CI-algebras.

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