A CONJECTURE CONCERNING THE SMARANDACHE DUAL FUNCTION

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Abstract: In this paper we verify a conjecture concerning the Smarandache dual function.

Key words: Smarandache dual function; factorial; gap of primes

For any positive integers n, let $S^*(n)$ denote the greatest positive integer m such that $n \equiv 0 \pmod{m!}$. Then $S^*(n)$ is called the Smarandache dual function. In [2], Sandos conjectured that

$$S^*((2k-1)!(2k+1)!)=q-1, \quad (1)$$

Where k is a positive integer, q is the first prime following $2k+1$. In this paper we prove the following result.

Theorem. (1) holds for any positive integer k.

Proof. Since q is a prime with $q > 2k+1$, we have

$$(2k-1)!(2k+1)! \not\equiv 0 \pmod{q}. \quad (2)$$

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It implies that $S^*((2k-1)!(2k+1)!) \le q-1$. Further, since q is the least prime with $q > 2k+1$, by Bertrand Postulate (see [1, Theorem 418]), we have

$$q > 2(2k+1). \tag{3}$$

Hence, by (3), any prime divisor p of q-1 satisfies

$$p \le 2k-1. \tag{4}$$

For any positive integer a and any prime p, let $\text{ord}_p a$ denote the order of p in a. It is a well known fact that

$$\text{ord}_p n! = \sum_{r=1}^{\infty} \left[\frac{n}{p^r}\right], \tag{5}$$

where $[x]$ is the Gauss function of x. We now suppose that $S^*((2k-1)!(2k+1)!) < q-1$. Then there exists a prime p sucn that

$$\text{ord}_p(2k\text{-}1)! + \text{ord}_p(2k+1)! < \text{ord}_p(q\text{-}1)!. \tag{6}$$

Hence, by (5) and (6), we get

$$\left[\frac{2k-1}{p^r}\right] + \left[\frac{2k+1}{p^r}\right] < \left[\frac{q-1}{p^r}\right] \tag{7}$$

for a suitable positive integer r. From (7), we get

$$\left[\frac{2k-1}{p^r}\right] + \left[\frac{2k+1}{p^r}\right] + 1 \le \left[\frac{q-1}{p^r}\right], \tag{8}$$

whence we obtain

$$4k < q\text{-}1. \tag{8}$$

It follows that $q \ge 4k+2$, a contradiction with (3). Thus, we get $S^*((2k-1)!(2k+1)!) = q-1$. The theorem is proved.

References

[1] G.H.Hardy and E.M.Wright, An introduction to the theory of numbers, Oxford University Press, Oxford, 1938.

[2] J. Sandor, On certain generalizations of the Smarandache function, Smarandache Notions J. 11(2000), 2002-212.