A GENERAL RESULT ON THE SMARANDACHE STAR FUNCTION

(Amarnath Murthy ,S.E. (E &T), Well Logging Services,Oil And Natural Gas Corporation Ltd. ,Sabarmati, Ahmedbad, India- 380005.)

ABSTRCT: In this paper ,the result (theorem-2.6) Derived in REF. [2], the paper "Generalization Of Partition Function, Introducing 'Smarandache Factor Partition' which has been observed to follow a beautiful pattern has been generalized.

DEFINITIONS In [2] we define SMARANDACHE FACTOR PARTITION FUNCTION , as follows:

Let $\alpha_1, \alpha_2, \alpha_3, \ldots \alpha_r$ be a set of r natural numbers and $p_1, p_2, p_3, \ldots p_r$ be arbitrarily chosen distinct primes then $F(\alpha_1, \alpha_2, \alpha_3, \ldots \alpha_r)$ called the Smarandache Factor Partition of $(\alpha_1, \alpha_2, \alpha_3, \ldots \alpha_r)$ is defined as the number of ways in which the number

$$N = p_1^{\alpha 1} p_2^{\alpha 2} p_3^{\alpha 3} \ldots p_r^{\alpha r}$$

could be expressed as the product of its' divisors. For simplicity , we denote $F(\alpha_1, \alpha_2, \alpha_3, \ldots \alpha_r) = F'(N)$,where

$$N = p_1^{\alpha_1} p_2^{\alpha_2} p_3^{\alpha_3} \ldots p_r^{\alpha_r} \ldots p_n^{\alpha_n}$$

and p_r is the r^{th} prime. $p_1 = 2$, $p_2 = 3$ etc.

Also for the case

$$\alpha_1 = \alpha_2 = \alpha_3 = \ldots = \alpha_r = \ldots = \alpha_n = 1$$

we denote

$$F(1, 1, 1, 1, 1\ldots) = F(1\#n)$$
$\leftarrow$ n - ones $\rightarrow$

Smarandache Star Function

(1) $$F^{''}(N) = \sum_{d/N} F^{'}(d_r) \quad \text{where } d_r | N$$

(2) $$F'^{**}(N) = \sum_{d_r/N} F'^{*}(d_r)$$

d_r ranges over all the divisors of N.

If N is a square free number with n prime factors, let us denote

$$F'^{**}(N) = F^{**}(1\#n)$$

Here we generalise the above idea by the following definition

Smarandache Generalised Star Function

(3) $$F'^{n*}(N) = \sum_{d_r/N} F'^{(n-1)*}(d_r) \qquad n > 1$$

and d_r ranges over all the divisors of N.

For simplicity we denote

$$F'(Np_1p_2\ldots p_n) = F'(N@1\#n) \text{ , where}$$

$(N, p_i) = 1$ for $i = 1$ to n and each p_i is a prime.

F'(N@1#n) is nothing but the Smarandache factor partition of (a number N multiplied by n primes which are coprime to N).

In [3] a proof of the following result is given:

$$F'(Np_1p_2p_3) = F'^{*}(N) + 3F'^{2*}(N) + F'^{3*}(N)$$

The present paper aims at generalising the abve result.

DISCUSSION:

THEOREM(3.1)

$$F'(N@1\#n) = F'(Np_1p_2...p_n) = \sum_{m=0}^{n} [\ a_{(n,m)}\ F'^{m*}(N)]$$

where

$$a_{(n,m)} = (1/m!) \sum_{k=1}^{m} (-1)^{m-k}\ .{}^{m}C_k\ .k^n$$

PROOF:

Let the divisors of N be

$d_1, \quad d_2, \quad ..., \quad d_k$

Consider the divisors of $(Np_1p_2...p_n)$ arranged as follows

$d_1, \quad d_2, \quad ..., \quad d_k$ -------say type (0)

$d_1p_i, \quad d_2p_i, \quad ..., \quad d_kp_i$ -------say type (1)

$d_1p_ip_j, \quad d_2p_ip_j, \quad ..., \quad d_kp_ip_j$ -------say type (2)

$d_1p_ip_j..., d_2p_ip_j..., ..., d_kp_ip_j...$ -------say type (t)

(there are t primes in the term $d_1p_ip_j...$ apart from d_1)

$d_1p_1p_2...p_n, \quad d_2p_1p_2...p_n, \quad d_np_1p_2...p_n,$ -------say type (n)

There are ${}^{n}C_0$ divisors sets of the type (0)

There are ${}^{n}C_1$ divisors sets of the type (1)

There are ${}^{n}C_2$ divisors sets of the type (2) and so on

There are ${}^{n}C_t$ divisors sets of the type (t)

There are ${}^{n}C_{n}$ divisors sets of the type (n)

Let $Np_1p_2...p_n = M$ .Then

$F^*(M) = {}^{n}C_0$[sum of the factor partitions of all the divisors of row (0)]

+ ${}^{n}C_1$[sum of the factor partitions of all the divisors of row (1)]

+ ${}^{n}C_2$[sum of the factor partitions of all the divisors of row (2)]

+ . . .

+ ${}^{n}C_t$[sum of the factor partitions of all the divisors of row (t)]

+ . . .

+ ${}^{n}C_n$[sum of the factor partitions of all the divisors of row (n)]

Let us consider the contributions of divisor sets one by one.

Row (0) or type (0) contributes

$$F'(d_1) + F'(d_2) + F'(d_3) + ... + F'(d_n) = F'^*(N)$$

Row (1) or type (1) contributes

$$[F'(d_1p_1) + F'(d_2p_1) + ... F'(d_kp_1)]$$

$$= [F'^*(d_1) + F'^*(d_2) + ... + F'^*(d_k)]$$

$$= F'^{2*}(N)$$

Row (2) or type (2) contributes

$$[F'(d_1p_1p_2) + F'(d_2p_1p_2) + ... + F'(d_kp_1p_2)$$

Applying theorem (5) on each of the terms

$$F'(d_1p1p2) = F'^*(d_1) + F'^{**}(d_1) \quad ----(1)$$

$$F'(d_2p_1p_2) = F'^*(d_2) + F'^{**}(d_2) \quad ----(2)$$

.

.

.

$$F'(d_kp1p2) = F'^*(d_k) + F'^{**}(d_k) \quad ----(k)$$

on summing up (1) , (2) ... upto (n) we get

$$F'^{2*}(N) + F'^{3*}(N)$$

At this stage let us denote the coefficients as $a_{(n,r)}$ etc. say

$F'(N@1\#r) = a_{(r,1)}F'^{*}(N) + a_{(r,2)}F'^{2*}(N) + \ldots + a_{(r,t)}F'^{t*}(N) + \ldots +$

$a_{(r,r)}F'^{r*}(N)$

Consider row (t) , one divisor set is

$d_1p_1p_2 \ldots p_t$, $d_2p_1p_2 \ldots p_t$, . . . $d_kp_1p_2 \ldots p_t$,

and we have

$F'(d_1@1\#t) = a_{(t,1)}F'^{*}(d_1) + a_{(t,2)}F'^{2*}(d_1) + \ldots + a_{(t,t)}F'^{t*}(d_1)$

$F'(d_2@1\#t) = a_{(t,1)}F'^{*}(d_2) + a_{(t,2)}F'^{2*}(d_2) + \ldots + a_{(t,t)}F'^{t*}(d_2)$

.

.

.

$F'(d_k@1\#t) = a_{(t,1)}F'^{*}(d_k) + a_{(t,2)}F'^{2*}(d_k) + \ldots + a_{(t,t)}F'^{t*}(d_k)$

summing up both the sides columnwise we get for row (t) or divisors of type (t) one of the ${}^nC_t$ divisor sets contributes

$a_{(t,1)}F'^{2*}(N) + a_{(t,2)}F'^{3*}(N) + \ldots + a_{(t,t)}F'^{(t+1)*}(N)$

similarly for row (n) we get

$a_{(n,1)}F'^{2*}(N) + a_{(n,2)}F'^{3*}(N) + \ldots + a_{(n,n)}F'^{(n+1)*}(N)$

All the divisor sets of type (0) contribute

${}^nC_0\ a_{(0,0)}F'^{*}(N)$ factor partitions.

All the divisor sets of type (1) contribute

${}^nC_1\ a_{(1,1)}F'^{2*}(N)$ factor partitions.

All the divisor sets of type (2) contribute

${}^nC_2\ \{a_{(2,1)}F'^{2*}(N) + a_{(2,2)}F'^{3*}(N)\}$ factor partitions.

All the divisor sets of type (3) contribute

${}^nC_3\{a_{(3,1)}F'^{2*}(N) + a_{(3,2)}F'^{3*}(N) + a_{(3,3)}F'^{4*}(N\ \}$ factor partitions.

All the divisor sets of row (t) or type (t) contribute

$^nC_t \{ a_{(t,1)}F'^{2*}(N) + a_{(t,2)}F'^{3*}(N) + \ldots + a_{(t,t)}F'^{(t+1)*}(N)\}$

.

.

.

All the divisor sets of row (n) or type (n) contribute

$^nC_n\{a_{(n,1)}F'^{2*}(N) + a_{(n,2)}F'^{3*}(N) + \ldots + a_{(n,n)}F'^{(n+1)*}(N)\}$

Summing up the contributions from the divisor sets of all the types and considering the coefficient of $F'^{m*}(N)$ for $m = 1$ to $(n+1)$ we get, coefficient of $F'^*(N) = a_{(0,0)} = 1 = \mathbf{a_{(n+1,1)}}$

coefficient of $F'^{2*}(N)$

$= {}^nC_1\, a_{(1,1)} + {}^nC_2\, a_{(2,1)} + {}^nC_3\, a_{(3,1)} + \ldots\ {}^nC_t\, a_{(t,1)} + \ldots + {}^nC_n\, a_{(n,1)}$

$= \mathbf{a_{(n+1,2)}}$

coefficient of $F'^{3*}(N)$

$= {}^nC_2\, a_{(2,2)} + {}^nC_3\, a_{(3,2)} + {}^nC_4\, a_{(4,2)} + \ldots\ {}^nC_t\, a_{(t,2)} + \ldots + {}^nC_n\, a_{(n,2)}$

$= \mathbf{a_{(n+1,3)}}$

coefficient of $F'^{m*}(N) =$

$a_{(n+1,m)} = {}^nC_{m-1}.a_{(m-1,m-1)} + {}^nC_m.a_{(m,m-1)} + \ldots + {}^nC_n.a_{(n,m-1)}$

coefficient of $F'^{(n+1)*}(N) =$

$a_{(n+1,n+1)} = {}^nC_n.a_{(n,n)} = {}^nC_n.{}^{n-1}C_{n-1}.a_{(n-1,n-1)} = {}^nC_n.{}^{n-1}C_{n-1}\ldots$

${}^2C_2.a_{(1,1)}$

$= 1$

Consider $a_{(n+1,2)}$

$= {}^nC_1\, a_{(1,1)} + {}^nC_2\, a_{(2,1)} + \ldots\ {}^nC_t\, a_{(t,1)} + \ldots + {}^nC_n\, a_{(n,1)}$

$= {}^nC_1 + {}^nC_2 + \ldots + {}^nC_n$

$= 2^n - 1$

$= (2^{n+1} - 2)/2$.

Consider $a_{(n+1,3)}$

$= {}^nC_2\, a_{(2,2)} + {}^nC_3\, a_{(3,2)} + {}^nC_4\, a_{(4,2)} + \ldots {}^nC_t\, a_{(t,2)} + \ldots + {}^nC_n\, a_{(n,2)}$

$= {}^nC_2(2^1-1) + {}^nC_3(2^2-1) + {}^nC_4(2^3-1) + \ldots + {}^nC_n(2^{n-1} - 1)$

$= {}^nC_2 2^1 + {}^nC_3 2^2 + \ldots + {}^nC_n\, 2^{n-1} - \{ {}^nC_2 + {}^nC_3 + \ldots + {}^nC_n \}$

$$=(1/2) \{ {}^nC_2 2^2 + {}^nC_3 2^3 + \ldots + {}^nC_n\, 2^n\} - \{ \sum_{r=0}^{n} {}^nC_r - {}^nC_1 - {}^nC_0 \}$$

$$= (1/2) \{ \sum_{r=0}^{n} {}^nC_r\, 2^r - {}^nC_1.2^1 - {}^nC_0.2^0 \} - \{ 2^n - n - 1 \}$$

$= (1/2) \{ 3^n -2n -1\} - 2^n + n + 1$

$= (1/2) \{ 3^n -2^{n+1} + 1\}$ ----------(3.1)

$= \{1/3!\} \{ (1).3^{n+1} - (3).\ 2^{n+1} + (3).\ (1)^{n+1} -(1)\ (0)^{n+1} \}$

Evaluating $a_{(n+1,4)}$

$a_{(n+1,4)} = {}^nC_3\, a_{(3,3)} + {}^nC_4\, a_{(4,3)} + \ldots + {}^nC_n\, a_{(n,3)}$

$= {}^nC_3\{3^2 +1 -2^3 \}/2 + {}^nC_4\{ 3^3 + 1 - 2^4 \}/2 + \ldots + {}^nC_n\{ 3^{n-1} + 1-2^n \}/2$

$= (1/2)[\{ 3^2.{}^nC_3 + 3^3\, {}^nC_4 + \ldots + 3^{n-1}.{}^nC_n \}+ \{ {}^nC_3 + {}^nC_4 + \ldots + {}^nC_n \}$

$- \{ {}^nC_3 2^3 + {}^nC_4 2^4 + \ldots + {}^nC_n\, 2^n\}]$

$$= (1/2)[\ (1/3)\ \{ \sum_{r=0}^{n} {}^nC_r\, 3^r - 3^2\, {}^nC_2 - 3\, {}^nC_1 - {}^nC_0\} + \{ \sum_{r=0}^{n} {}^nC_r - {}^nC_2 - {}^nC_1$$

$$- {}^nC_0 \} - \{ \sum_{r=0}^{n} {}^nC_r.2^r - 2^2\, {}^nC_2 - 2\, {}^nC_1 - {}^nC_0\}]$$

$= (1/2)\ [(1/3)\{ 4^n - 9n(n-1)/2 - 3n -1 \} + \{ 2^n - n(n-1)/2 - n - 1 \}$

$- \{ 3^n - 4n(n-1)/2 - 2n - 1\}$

$$a_{(n+1,4)} = (1/4!) [(1) 4^{n+1} - (4) 3^{n+1} + (6) 2^{n+1} - (4) 1^{n+1} + 1(0)^{n+1}]$$

Observing the pattern we can explore the possibility of

$$\mathbf{a_{(n,r)} = (1/r!) \sum_{k=0}^{r} (-1)^{r-k} . {}^rC_k . k^n} \quad \text{-------}\mathbf{(3.2)}$$

which is yet to be established. Now we shall apply induction.

Let the following proposition (3.3) be true for r and all n > r.

$$a_{(n+1,r)} = (1/r!) \sum_{k=1}^{r} (-1)^{r-k} . {}^rC_k . k^{n+1} \quad \text{------}(3.3)$$

Given (3.3) our aim is to prove that

$$a_{(n+1,r+1)} = (1/(r+1)!) \sum_{k=1}^{r+1} [(-1)^{(r+1)-k} . {}^{r+1}C_k (k)^{n+1}]$$

we have

$$a_{(n+1,r+1)} = {}^nC_r \, a_{(r,r)} + {}^nC_{r+1} \, a_{(r+1,r)} + {}^nC_{r+2} \, a_{(r+2,r)} + \ldots + {}^nC_n \, a_{(n,r)}$$

$$a_{(n+1,r+1)} = {}^nC_r \{(1/r!) \sum_{k=0}^{r} (-1)^{r-k} . {}^rC_k . k^r \} + {}^nC_{r+1} \{(1/r!) \sum_{k=0}^{r} (-1)^{r-k} . {}^rC_k \, k^{r+1}$$

$$+ \ldots + {}^nC_n \{(1/r!) \sum_{k=0}^{r} (-1)^{r-k} . {}^rC_k . k^n \}$$

$$= (1/r!) \sum_{k=0}^{r} [(-1)^{r-k} . {}^rC_k \{ {}^nC_r \, k^r + {}^nC_{r+1} \, k^{r+1} + \ldots + {}^nC_n \, k^n \}]$$

$$= (1/r!) \sum_{k=0}^{r} [(-1)^{r-k} . {}^rC_k \{ \sum_{q=0}^{n} {}^nC_q \, k^q - \sum_{q=0}^{r-1} {}^nC_q \, k^q \}]$$

$$= (1/r!) \sum_{k=0}^{r} [(-1)^{r-k} . {}^rC_k (1+k)^n] - (1/r!) \sum_{q=0}^{r-1} [(-1)^{r-k} . {}^rC_k \{\sum {}^nC_q \, k^q \}]$$

If we denote the I[st] and the second term as T_1 and T_2, we have

$$a_{(n+1,r+1)} = T_1 - T_2 \qquad \text{----------(3.4)}$$

consider $T_1 = (1/r!) \sum_{k=0}^{r} [(-1)^{r-k} . {}^{r}C_k (1+k)^n]$

$$= (1/r!) \sum_{k=0}^{r} [(-1)^{r-k} \{ r!/((k!)(r-k)!)\} (1+k)^n]$$

$$= (1/(r+1)!) \sum_{k=0}^{r} [(-1)^{r-k} \{(r+1)!/((k+1)!(r-k)!)\}(1+k)^{n+1}]$$

$$= (1/(r+1)!) \sum_{k=0}^{r} [(-1)^{r-k} . {}^{r+1}C_{k+1} (1+k)^{n+1}]$$

$$= (1/(r+1)!) \sum_{k=0}^{r} [(-1)^{(r+1)-(k+1)} . {}^{r+1}C_{k+1} (1+k)^{n+1}]$$

Let $k + 1 = s$, we get, $s = 1$ at $k = 0$ and $s = r + 1$ at $k = r$

$$= (1/(r+1)!) \sum_{s=1}^{r+1} [(-1)^{(r+1)-s} . {}^{r+1}C_s (s)^{n+1}].$$

replacing s by k we get

$$= (1/(r+1)!) \sum_{k=1}^{r+1} [(-1)^{(r+1)-k} . {}^{r+1}C_k (k)^{n+1}]$$

in this if we include $k = 0$ case we get

$$\mathbf{T_1 = (1/(r+1)!) \sum_{k=0}^{r+1} [(-1)^{(r+1)-k} . {}^{r+1}C_k (k)^{n+1}]} \qquad \text{----(3.5)}$$

T_1 is nothing but the right hand side member of (3.3).

To prove (3.3) we have to prove $a_{(n+1,r+1)} = T_1$

In view of (3.4) our next step is to prove that $T_2 = 0$

$$T_2 = (1/r!) \sum_{k=0}^{r} [(-1)^{r-k} . {}^rC_k \{ \sum_{q=0}^{r-1} {}^nC_q \, k^q \}]$$

$$= (1/r!) \sum_{k=0}^{r} [(-1)^{r-k} . {}^rC_k \{ {}^nC_0 \, k^0 + {}^nC_1 \, k^1 + {}^nC_2 \, k^2 + . . . + {}^nC_{r-1} \, k^{r-1} \}]$$

$$= (1/r!) \sum_{k=0}^{r} [(-1)^{r-k} . {}^rC_k] + {}^nC_1[(1/r!) \sum_{k=0}^{r} \{(-1)^{r-k} . {}^rC_k \, k \}] +$$

$${}^nC_2[(1/r!) \sum_{k=0}^{r}\{(-1)^{r-k} \, {}^rC_k \, k^2\}] + . . . + {}^nC_{r-1}[(1/r!) \sum_{k=0}^{r} \{(-1)^{r-k} . {}^rC_k \, k^{r-1} \}]$$

$$= (1/r!) \sum_{k=0}^{r} [(-1)^{r-k} . {}^rC_k] + {}^nC_1[(1/r!) \sum_{k=0}^{r} \{(-1)^{r-k} . {}^rC_k \, k \}] +$$

$$[{}^nC_2 \, .a_{(2,r)} + {}^nC_3 \, .a_{(3,r)} .+ . . . + {}^nC_{r-1} \, .a_{(r-1,r)}]$$

$= X + Y + Z$ say where

$$X = (1/r!) \sum_{k=0}^{r} [(-1)^{r-k} . {}^rC_k] \quad , \quad Y = {}^nC_1[(1/r!) \sum_{k=0}^{r} \{(-1)^{r-k} . {}^rC_k \, k \}]$$

$$Z = [{}^nC_2 \, .a_{(2,r)} + {}^nC_3 \, .a_{(3,r)} + . . . + {}^nC_{r-1} \, .a_{(r-1,r)}]$$

We shall prove that $X = 0$, $Y = 0$, $Z = 0$ seperately.

(1) $$X = (1/r!) \sum_{k=0}^{r} [(-1)^{r-k} . {}^rC_k]$$

$$= (1/r!) \sum_{k=0}^{r} [(-1)^{r-k} . {}^rC_{r-k}]$$

let $r - k = w$ then we get at $k = 0$ $w = r$ and at $k = r$ $w = 0$.

$$= (1/r!) \sum_{w=r}^{0} [(-1)^{w} . {}^rC_w]$$

$$= (1/r!) \sum_{w=0}^{r} [(-1)^w . {}^rC_w]$$

$$=(1 - 1)^r /r!$$

$$= 0$$

We have proved that X = 0

(2)

$$Y = {}^nC_1[(1/r!) \sum_{k=0}^{r} \{(-1)^{r-k} . {}^rC_k k \}]$$

$$= {}^nC_1[(1/(r-1)!) \sum_{k=1}^{r} \{(-1)^{r-1-(k-1)} . {}^{r-1}C_{k-1} \}]$$

$$= {}^nC_1[(1/(r-1)!) \sum_{k-1=0}^{r-1} \{(-1)^{r-1-(k-1)} . {}^{r-1}C_{k-1} \}]$$

$$= {}^nC_1[(1/(r-1)!)(1 - 1)^{r-1}$$

$$= 0$$

We have proved that Y = 0

(3) To prove

$$Z = [{}^nC_2 .a_{(2,r)} + {}^nC_3 .a_{(3,r)} + . . . + {}^nC_{r-1} .a_{(r-1,r)}] = 0 \quad \text{----(3.6)}$$

Proof:

Refer the matrix

$\underline{a_{(1,1)}}$	$a_{(1,2)}$	$a_{(1,3)}$	$a_{(1,4)}$	. . .	$a_{(1,r)}$
$a_{(2,1)}$	$\underline{a_{(2,2)}}$	$a_{(2,3)}$	$a_{(2,4)}$	. . .	$a_{(2,r)}$
$a_{(3,1)}$	$a_{(3,2)}$	$\underline{a_{(3,3)}}$	$a_{(3,4)}$	. . .	$a_{(3,r)}$
$a_{(4,1)}$	$a_{(4,2)}$	$a_{(4,3)}$	$\underline{a_{(4,4)}}$	$a_{(4,5)}$ . . .	$a_{(4,r)}$

.....

...	...	...	...	$\underline{a_{(r-1,r-1)}}$	$\mathbf{a_{(r-1,r)}}$	...
$a_{(r,1)}$	$a_{(r,2)}$	$a_{(r,3)}$	...	$a_{(r,r-1)}$	$\underline{a_{(r,r)}}$	

The Diagonal elements are underlined . And the the elements above the leading diagonal are shown with bold face.

We have

$$a_{(1,r)} = [(1/r!) \sum_{k=0}^{r} \{(-1)^{r-k} . {}^{r}C_k \, k \}] = Y/ {}^{n}C_1 = 0 \text{ for } r > 1$$

All the elements of the first row except $a_{(1,1)}$ (the one on the leading diagonal) are zero.
Also

$$a_{(n+1,r)} = a_{(n,r-1)} + r . a_{(n,r)} \qquad \text{--------(3.7)}$$

(This can be easily established by simplifying the right hand side.)

(7) gives us

$$a_{(2,r)} = a_{(1,r-1)} + r . a_{(1,r)} = 0 \text{ for } r > 2$$

i.e. $a_{(2,r)}$ can be expresssed as a linear combination of two elements of the first row (except the one on the leading diagonal)

$\Rightarrow a_{(2,r)} = 0 \quad r > 2$

Similarly $a_{(3,r)}$ can be expresssed as a linear combination of two elements of the second row of the type $a_{(2,r)}$ with $r > 3$

$\Rightarrow a_{(2,r)} = 0 \quad r > 3$

and so on $a_{(r-1,r)} = 0$

substituting

$a_{(2,r)} =.a_{(3,r)} = . . . = a_{(r-1,r)} = 0$ in (6)

we get $Z = 0$

With $X = Y = Z = 0$ we get $T_2 = 0$
or $a_{(n+1,r+1)} = T_1 - T_2 = T_1$

from (5) we have

$$T_1 = (1/(r+1)!) \sum_{k=0}^{r+1} [(-1)^{(r+1)-k} \; {}^{r+1}C_k \, (k)^{n+1}]$$

which gives

$$a_{(n+1,r+1)} = (1/(r+1)!) \sum_{k=0}^{r+1} [(-1)^{(r+1)-k} \; {}^{r+1}C_k \, (k)^{n+1}]$$

We have proved ,if the propposition (3.3) is true for r it is true for (r+1) as well .We have already verified it for 1, 2, 3 etc. Hence by induction (3.3) is true for all n.

This completes the proof of theorem (3.1) .

Remarks: This proof is quite lengthy , clumsy and heavy in algebra. The readers can try some analytic , combinatorial approach.

REFERENCES:

[1] V.Krishnamurthy, "COMBINATORICS Theory and Applications" East West Press Private Ltd. ,1985.

[2] "Amarnath Murthy" , 'Generalization Of Partition Function, Introducing 'Smarandache Factor Partition', SNJ, Vol. 11, No. 1-2-3, 2000.

[3] " The Florentine Smarandache " Special Collection, Archives of American Mathematics, Centre for American History, University of Texax at Austin, USA.

[4] 'Smarandache Notion Journal' Vol. 10 ,No. 1-2-3, Spring 1999. Number Theory Association of the UNIVERSITY OF CRAIOVA .