A GENERALIZED NET FOR MACHINE LEARNING OF THE PROCESS OF MATHEMATICAL PROBLEMS SOLVING

On an Example with a Smarandache Problem

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The authors of the present paper prepared a series of research related to the ways of representation by Generalized nets (GNs, see [1] and the Appendix) the process of machine learning of different objects, e.g., neural networks, genetic algorithms, GNs, expert systems, systems (abstract, statical, dynamical, stohastical and others), etc. Working on their research [2], where they gave a counterexample of the 62-nd Smarandache's problem (see [3]), they saw that the process of the machine learning of the process of the mathematical problems solving also can be described by a GN and by this reason the result form [2] was used as an example of the present research. After this, they saw that the process of solving of a lot of the Smarandache's problems can be represented by GNs in a similar way and this will be an object of next their research.

The GN (see [1] and the Appendix), which is described below have three types of tokens $\alpha-$, $\beta-$ and $\gamma-$ tokens. They interprete respectively the object which will be studied, its known property (properties) and the hypothesis, related to it, which must be checked. The tokens' initial characteristics correspond to these interpretations. The tokens enter the GN, respectively, through places

- l_1 with the initial characteristic "description of the object" (if we use the example from [2], this characteristic will be, e.g., "sequence of natural numbers"),
- l_2 with the initial characteristic "property (properties) of the object, described as an initial characteristic of $\alpha-$token corresponding to the present $\beta-$token" (in the case of the example mentioned above, it will be the following property "there are no three elements of the sequence, which are members of an arithmetic progression") and
- l_3 with the initial characteristic "description of an hypothesis about the object" (for the discussed example this characteristic will be, e.g., "the sum of the reciprocal values of the members of the sequence are smaller than 2").

We shall would like for the places' priorities to satisfy the following inequalities:

$$\pi_L(l_1) > \pi_L(l_2) > \pi_L(l_3),$$

$$\pi_L(l_7) > \pi_L(l_4) > \pi_L(l_6) > \pi_L(l_5),$$

The GN transitions (see [1] and the Appendix) have the following forms:

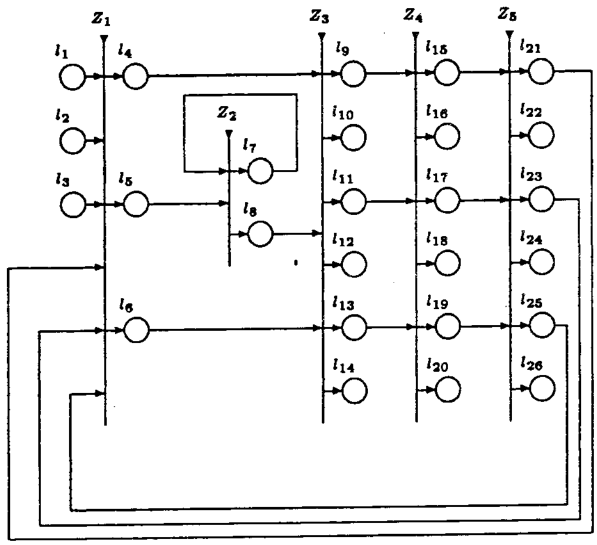
$$Z_1 = \langle \{l_1, l_2, l_3, l_{21}, l_{23}, l_{25}\}, \{l_4, l_5, l_6\}, r_1, M_1, \square_1 \rangle,$$

where

$$r_1 = \begin{array}{c|ccc} & l_4 & l_5 & l_6 \\ \hline l_1 & true & false & false \\ l_2 & false & true & false \\ l_3 & false & false & true \\ l_{21} & true & false & false \\ l_{23} & false & true & false \\ l_{25} & false & false & true \end{array},$$

$$M_1 = \begin{array}{c|ccc} & l_4 & l_5 & l_6 \\ \hline l_1 & 1 & 0 & 0 \\ l_2 & 0 & 1 & 0 \\ l_3 & 0 & 0 & 1 \\ l_{21} & 1 & 0 & 0 \\ l_{23} & 0 & 1 & 0 \\ l_{25} & 0 & 0 & 1 \end{array},$$

$$kv_1 = \wedge(\vee(l_1, l_{21}), \vee(l_2, l_{23}), \vee(l_3; l_{25})).$$



The $\alpha-$token obtains the characteristic "the initial status of the object, having in mind the current $\gamma-$characteristic" in place l_4, the $\beta-$token obtains the characteristic "a next state of the object, having in mind the current $\alpha-$ and $\gamma-$characteristics" in place l_5, and the $\gamma-$token obtains the characteristics "restrictions over the object, having in mind its property (properties) from the initial $\beta-$characteristic in place l_6. For the discussed example with the 62-nd Smarandache's problem, the last three characteristics have the following forms, respectively: "1, 2" (initial values of the sequence); "3" (next value of the sequence); e.g., "the members to be minimal possible".

$$Z_2 = \langle \{l_5, l_7\}, \{l_7, l_8\}, r_2, M_2, \vee(l_5, l_7)\rangle,$$

where

$$r_2 = \begin{array}{c|cc} & l_7 & l_8 \\ \hline l_5 & r_{5,7} & r_{5,8} \\ l_7 & r_{7,7} & r_{7,8} \end{array}$$

where
$r_{5,7} = r_{7,7} =$ "the new state of the object does not satisfy the property of the object determined by the initial $\beta-$characteristic"
$r_{5,8} = r_{7,8} = \neg r_{5,7}$. and

$$M_3 = \begin{array}{c|cc} & l_7 & l_8 \\ \hline l_5 & 1 & 1 \\ l_7 & 1 & 1 \end{array}.$$

The β-token obtains the characteristic "a next state of the object, having in mind the current α- and γ-characteristics" in place l_7 and it does not obtain any characteristic in place l_8. In the case of our example, on the first time, when the β-token enters place l_7 will obtain the characteristic "4".

$$Z_3 = \langle \{l_4, l_6, l_8\}, \{l_9, l_{10}, l_{11}, l_{12}, l_{13}, l_{14}\}, r_3, M_3, \Box_3,$$

where

$$r_3 = \begin{array}{c|cccccc} & l_9 & l_{10} & l_{11} & l_{12} & l_{13} & l_{14} \\ \hline l_4 & r_{4,9} & r_{4,10} & false & false & false & false \\ l_6 & false & false & r_{6,11} & r_{6,12} & false & false \\ l_8 & false & false & false & false & r_{8,13} & r_{8,14} \end{array},$$

where

$r_{4,9} = r_{6,11} = r_{8,13} =$ "the new state is not a final one",

$r_{4,10} = r_{6,12} = r_{8,14} = \neg r_{4,9}$,

$$M_3 = \begin{array}{c|cccccc} & l_9 & l_{10} & l_{11} & l_{12} & l_{13} & l_{14} \\ \hline l_4 & 1 & 1 & 0 & 0 & 0 & 0 \\ l_6 & 0 & 0 & 1 & 1 & 0 & 0 \\ l_8 & 0 & 0 & 0 & 0 & 1 & 1 \end{array},$$

$$\Box_3 = \wedge(l_4, l_6, l_8).$$

The α-token does not obtain any characteristic in place l_9, and it obtains the characteristic "the list of all states of the object" in place l_{10}; the β-token does not obtain any characteristic in places l_{11} and l_{12}; the γ-token does not obtain any characteristic in place l_{13}, and it obtains the characteristic

$$\begin{cases} \text{"the hypothesis is valid by the present step"}, & \text{if the last state of the object satisfies the hypothesis} \\ \text{"the hypothesis is not valid by the present step"}, & \text{if the last state of the object does not satisfy the hypothesis} \end{cases}$$

in place l_{14}. For the discussed example with the 62-nd Smarandache's problem, the tokens do not obtain any characteristics.

$$Z_4 = \langle \{l_9, l_{11}, l_{13}\}, \{l_{15}, l_{16}, l_{17}, l_{18}, l_{19}, l_{20}\}, r_4, M_4, \Box_4,$$

where

$$r_4 = \begin{array}{c|cccccc} & l_{15} & l_{16} & l_{17} & l_{18} & l_{19} & l_{20} \\ \hline l_9 & r_{9,15} & r_{9,16} & false & false & false & false \\ l_{11} & false & false & r_{11,17} & r_{11,18} & false & false \\ l_{13} & false & false & false & false & r_{13,19} & r_{13,20} \end{array},$$

where

$r_{9,15} = r_{11,17} = r_{13,19} =$ "the hypothesis is valid",

$r_{9,16} = r_{11,18} = r_{13,20} = \neg r_{9,15}$,

$$M_4 = \begin{array}{c|cccccc} & l_{15} & l_{16} & l_{17} & l_{18} & l_{19} & l_{20} \\ \hline l_9 & 1 & 1 & 0 & 0 & 0 & 0 \\ l_{11} & 0 & 0 & 1 & 1 & 0 & 0 \\ l_{13} & 0 & 0 & 0 & 0 & 1 & 1 \end{array},$$

$$\square_4 = \wedge(l_9, l_{11}, l_{13}).$$

The α–token does not obtain any characteristic in place l_{15}, and it obtains the characteristic "the final state, which violate the hypothesis" in place l_{16}; the β–token does not obtain any characteristic in places l_{17} and l_{18}; the γ–token does not obtain any characteristic in place l_{19}, and it obtains the characteristic "the hypothesis is not valid" in place l_{20}. For the discussed example with the 62-nd Smarandache's problem, the tokens do not obtain any characteristics.

$$Z_5 = \langle \{l_{15}, l_{17}, l_{19}\}, \{l_{21}, l_{22}, l_{23}, l_{24}, l_{25}, l_{26}\}, r_5, M_5, \square_5,$$

where

$$r_5 = \begin{array}{c|cccccc} & l_{21} & l_{22} & l_{23} & l_{24} & l_{25} & l_{26} \\ \hline l_{15} & r_{15,21} & r_{15,22} & false & false & false & false \\ l_{17} & false & false & r_{17,23} & r_{17,24} & false & false \\ l_{19} & false & false & false & false & r_{19,25} & r_{19,26} \end{array},$$

where
$r_{15,21} = r_{17,23} = r_{19,25} =$ "there is a possibility for a change of the restrictions over the object, which evolve from the hypothesis",
$r_{15,22} = r_{17,24} = r_{19,26} = \neg r_{15,21}$,

$$M_3 = \begin{array}{c|cccccc} & l_{21} & l_{22} & l_{23} & l_{24} & l_{25} & l_{26} \\ \hline l_{15} & 1 & 1 & 0 & 0 & 0 & 0 \\ l_{17} & 0 & 0 & 1 & 1 & 0 & 0 \\ l_{19} & 0 & 0 & 0 & 0 & 1 & 1 \end{array},$$

$$\square_3 = \wedge(l_{15}, l_{17}, l_{19}).$$

The α–token obtains as its current characteristic its initial characteristic in place l_{21}, and it does not obtain any characteristic in place l_{22}; the β–token obtains as its current characteristic its initial characteristic in place l_{23} and it does not obtain any characteristic in place l_{24}; the γ–token obtains the characteristic "new restrictions over the object" in place l_{25}, and it does not obtain any characteristic in place l_{26}. For the discussed example with the 62-nd Smarandache's problem, the tokens do not obtain any characteristics in places $l_{21}, l_{22}, l_{23}, l_{24}$ and l_{26}, and the γ–tokem will obtain as a characteristic "1,3" l_{25}. These two numbers will be initial for the next search of a sequence, which satisfy the hypothesis. In the next step they will be changed, e.g., by numbers 2 and 3, etc.

Using this scheme, it is possible to describe the process of solving of some of the other Smarandache's problems, too, e.g., problems ... from [3].

APPENDIX: Short remarks on Generalized Nets (GNs)

The concept of a *Generalized Net* (GN) is described in details in [1], see also

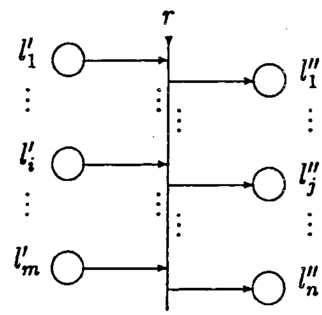
www.daimi.aau.dk/PetriNets/bibl/aboutpnbibl.html

They are essential extensions of the ordinary Petri nets. The GNs are defined in a way that is principly different from the ways of defining the other types of Petri nets. When some of the GN-conponents of a given model are not necessary, they can be ommited and the new nets are called reduced GNs. Here a reduced GN without temporal components is used.

Formally, every transition (or the used form of reduced GN) is described by a five-tuple:

$$Z = \langle L', L'', r, M, \square \rangle,$$

where:



(a) L' and L'' are finite, non-empty sets of places (the transition's input and output places, respectively); for the above transition these are

$$L' = \{l'_1, l'_2, \ldots, l'_m\}$$

and

$$L'' = \{l''_1, l''_2, \ldots, l''_n\};$$

(b) r is the transition's *condition* determining which tokens will pass (or *transfer*) from the transition's inputs to its outputs; it has the form of an Index Matrix (IM; see [1]):

$$r = \begin{array}{c|c} & l''_1 \ \ldots \ l''_j \ \ldots \ l''_n \\ \hline l'_1 & \\ \vdots & r_{i,j} \\ l'_i & (r_{i,j} \ - \ \text{predicate}) \\ \vdots & (1 \le i \le m, 1 \le j \le n) \\ l'_m & \end{array} \quad ;$$

$r_{i,j}$ is the predicate which corresponds to the i-th input and j-th output places. When its truth value is "*true*", a token from the i-th input place can be transferred to the j-th output place; otherwise, this is not possible;

(c) M is an IM of the capacities of transition's arcs:

$$M = \begin{array}{c|c} & l''_1 \ \ldots \ l''_j \ \ldots \ l''_n \\ \hline l'_1 & \\ \vdots & m_{i,j} \\ l'_i & (m_{i,j} \ge 0 - \ \text{natural number}) \\ \vdots & (1 \le i \le m, 1 \le j \le n) \\ l'_m & \end{array} \quad ;$$

(d) $\square$ is an object having a form similar to a Boolean expression. It may contain as variables the symbols which serve as labels for transition's input places, and is an expression built up from variables and the Boolean connectives $\wedge$ and $\vee$, with semantics defined as follows:

$\wedge(l_{i_1}, l_{i_2}, \ldots, l_{i_u})$ – every place $l_{i_1}, l_{i_2}, \ldots, l_{i_u}$ must contain at least one token,
$\vee(l_{i_1}, l_{i_2}, \ldots, l_{i_u})$ – there must be at least one token in all places$l_{i_1}, l_{i_2}, \ldots, l_{i_u}$, where $\{l_{i_1}, l_{i_2}, \ldots, l_{i_u}\} \subset L'$.

When the value of a type (calculated as a Boolean expression) is "*true*", the transition can become active, otherwise it cannot.

The object

$$E = \langle A, \pi_L, K, X, \Phi \rangle$$

is called a (reduced) GN, if

(a) A is a set of transitions;

(b) π_L is a function giving the priorities of the places, i.e., $\pi_L : L \to N$, where $L = pr_1 A \cup pr_2 A$, and $pr_i X$ is the i-th projection of the n-dimensional set, where $n \in N, n \geq 1$ and $1 \leq k \leq n$ (obviously, L is the set of all GN-places);

(c) K is the set of the GN's tokens;

(d) X is the set of all initial characteristics the tokens can receive when they enter the net;

(e) Φ is a characteristic function which assigns new characteristics to every token when it makes the transfer from an input to an output place of a given transition.

REFERENCES:

[1] Atanassov K., Generalized Nets, World Scientific, Singapore, New Jersey, London, 1991.
[2] Aladjov H., Atanassov K., Remark on the 62-th Smarandache's problem. Smarandache Notions Journal, Vol. 11, No. 1-2-3, 2000, 69-70.
[3] Smarandache F., Only Problems, Not Solutions!. Xiquan Publ. House, Chicago, 1993.