A LOWER BOUND FOR $S(2^{p-1}(2^p-1))$

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Abstract. Let p be a prime, and let $n=2^{p-1}(2^p-1)$. In this paper we prove that $S(n)\geqslant 2p+1$.

Key words . Smarandache function, function value, lower bound.

For any positive integer a, let $S(a)$ be the Smarandache function . In[2],Sandor showed that if

(1) $$n=2^{p-1}(2^p-1)$$

is an even perfect number ,then $S(n)=2^p-1$ .It is a well known fact that if n is an even perfect number ,then p must be a prime .But ,its inverse proposition is false (see [1,Theoerms 18 and 276]). In this paper we give a lower bound for $S(n)$ in the general cases . We prove the following result.

Theorem, If p is a prime and n can be expressed as (1), then $S(n)\geqslant 2p+1$.

Proof. Let

(2) $$2^p-1=q_1^{r1}q_2^{r2}\dots q_t^{rt}$$

be the factorization of 2^p-1,where $q_1,q_2,\dots,q_t$ are primes with $q_1<q_2<\dots<q_t$ and $r_1,r_2,\dots,r_t$ are positive integers . By (1) and (2),we get

(3) $$S(n)=\max(S(2^{p-1}),S(q_1^{r1}),S(q_2^{r2}),\dots S(q_t^{rt})).$$

It is a well known fact that $q_i\equiv 1(\mathrm{mod}\ 2p)$ for $i=1,2,\dots,t$. So we have

(4) $$2p+1\leqslant q_1<q_2<\dots<q_t.$$

Since $q_i=S(q_i)\leq S(q_i^{ri})$ for $i=1,2,\dots,t$, we get from (4) that

(5) $$2p+1 \leqslant \max(S(q_1^{r1}), S(q_2^{r2}), \ldots, S(q_t^{rt})).$$

On the other hand ,if m is the largest integer such that $(2p+1)!$ is a multiple of 2^m ,then

(6) $$m = \sum_{k=1}^{\infty} \left[\frac{2p+1}{2^k}\right] \geqslant \left[\frac{2p+1}{2}\right] = p.$$

It implies that $2^p \mid (2p+1)!$ .So we have

(7) $$S(2^{p-1}) \leqslant S(2^p) \leqslant 2p+1.$$

Thus,by (3),(5) and (7) ,we obtain $S(n) \geqslant 2p+1$ .The theorem is proved.

References

[1] G.H.Hardy and E.M. Wright , An introduction to the Theory of Numbers , Oxford University Press ,Oxford, 1937 .

[2] J.Sandor , A note on $S(n)$, where n is an even perfect number , Smarandache Notions J.11(2000),139.

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