A lucky derivative

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Question:

What is the value of the derivative of $f(x) = e^x$ when $x = e$?

Lucky answer:

We know that the derivative of $g(x) = x^n$ is $g'(x) = n.x^{n-1}$,

and when $x = n$ this is $g'(n) = n.n^{n-1} = n^n$,

so the derivative of $f(x) = e^x$ when $e = x$ is $f'(e) = x.e^{x-1} = x^x = e^e = 15.15426....$

As a check, note that $f(e) = e^e = f'(e)$ and $g(n) = n^n = g'(n)$.

Comments

This is in the tradition of other lucky mathematics. For example, when simplifying the fraction 16/64, canceling the 6s in the numerator and denominator leaves the correct result of 1/4.

In the smarandacheian lucky answer to the derivative, the only incorrect part is the word "so". The derivative of $f(x) = e^x$ with respect to x is $f'(x) = e^x$, not $x.e^{x-1}$ (unless $x = e$ in which case these are equal).

Conversely, $x.e^{x-1}$ has the indefinite integral $(x-1).e^{x-1}+C$ rather than e^x+C.

The derivative of $h(x) = c^x$ is $h'(x) = \log_e(c).c^x$ for a positive constant c, and so when $x = c$ it is $h'(c) = \log_e(c).c^c$, not c^c (unless $c = e$ in which case these are equal).

This lucky (i.e. wrong) derivative method can produce the correct answer to the more general question:

"What is the value of the derivative of $h(x) = c^x$ when $x = c.\log_e(c)$?"

(if c is a positive integer then x is close to the c^{th} prime number):

$h'(c.\log_e(c)) = c.\log_e(c).c^{c.\log_e(c)-1} = \log_e(c).c^{c.\log_e(c)}$.

References:

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