A NEW INEQUALITY FOR THE SMARANDACHE FUNCTION

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Theorem. Let $S(m)=\min\{k\in N: m | k!\}$ be the Smarandache Function, and $a_k, b_k \in N^*$ $(k=1,2,\dots,n)$, then we have the following inequality

$$S\left(\prod_{k=1}^{n}(a k!)^{b_k}\right) \le \sum_{k=1}^{n} a_k b_k$$

Proof:

$$\frac{\sum_{k=1}^{n}(a_k b_k)!}{\prod_{k=1}^{n}(a_k!)^{b_k}} = \frac{\sum_{k=1}^{n}(a_k b_k)!}{\prod_{k=1}^{n}(a_k b_k)!} * \frac{\prod_{k=1}^{n}(a_k b_k)!}{\prod_{k=1}^{n}(a_k!)^{b_k}} =$$

$$\binom{a_1 b_1 + a_2 b_2 + \dots + a_m b_m}{a_1 b_1}\binom{a_2 b_2 + \dots + a_m b_m}{a_2 b_2}\dots\binom{a_{m-1} b_{m-1} + a_m b_m}{a_{m-1} b_{m-1}}$$

$$\left(\prod_{k=1}^{n}\binom{a_k b_k}{a_k}\dots\binom{3a_k}{a_k}\binom{2a_k}{a_k}\right) \in N^*$$

From this result

$$S\left(\prod_{k=1}(a_k!)^{b_k}\right) \le \sum_{k=1}^{n} a_k b_k$$