A NOTE ON PRIMES IN THE SEQUENCE $\{a^n + b\}_{n=1}^{\infty}$

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Abstract. Let a,b be integers such that gcd(a,b)=1 and $a \neq -1$, 0 or 1. Let $U(a,b)=\{a^n + b\}_{n=1}^{\infty}$. In this note we discuss the primes in U(a,b).

Let a,b be integers such that gcd(a,b)=1 and $a \neq -1$, 0 or 1. Let $U(a,b)=\{a^n + b\}_{n=1}^{\infty}$. In [1, Problem 17], Smarandache posed the following questions:

Question. How many primes belong to U(a,b)?

It would seem that the answers of this questions is different from different pairs (a,b). We now give some observable examples as follows:

Example 1. If a,b are odd integers, then a^n+b is either an even integer or zero. It implies that U(a,b) contains at most one prime. In particular, U(3,-1) contains only the prime 2, U(3,1) does not contain any prime.

Example 2. If a>2 and b=-1, then we have

(1) $$a^n+b = a^n -1= (a-1)(a^{n-1} + a^{n-2} +...+1).$$

We see from(1)that $a^n + b$ is not a prime if n>1. It implies that U(a,b) contains at most one prime. In particular, U(4,-1) contains only the prime 3, U(10,-1) does not contain any prime.

Example 3. Under the assumption of Mersenne prime conjecture that there exist infinitely many primes with the form $2^n - 1$, then the sequence U(2,-1)contains infinitely many primes.

Reference

1. F.Smarandache, Only Problems, not Solutions!, Xiquan Pub. House, Phoenix, Chicago, 1990.