A NOTE ON THE 57-TH SMARANDACHE'S PROBLEM

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ABSTRACT. For any positive integer n, let r_1 be the positive integer such that: the set $\{1, 2, \cdots, r_1\}$ can be partitioned into n classes such that no class contains integers x, y, z with $x^y = z$, let r_2 be the positive integer such that: the set $\{1, 2, \cdots, r_2\}$ can be partitioned into n classes such that no class contains integers x, y, z with $x + y = z$. In this paper, we use the elementary methods to give two sharp lower bound estimates for r_1 and r_2.

1. INTRODUCTION

For any positive integer n, let r_1 be a positive integer such that: the set $\{1, 2, \cdots, r_1\}$ can be partitioned into n classes such that no class contains integers x, y, z with $x^y = z$. In [1], Schur asks us to find the maximum r_1, and there is the same question when no integer can be the sum of another integer of its class. About these problems, it appears that no one had studied them yet, at least, we have not seen such a paper before. These problems are interesting because it can help us to study some important partition problem. In this paper, we use the elementary methods to study Schur's problem and give two sharp lower bound estimates for r_1 and r_2. That is, we shall prove the following:

Theorem 1. *For sufficiently large integer n, let r_1 be a positive integer such that: the set $\{1, 2, \cdots, r_1\}$ can be partitioned into n classes such that no class contains integers x, y, z with $x^y = z$. For any integer m with $m \leq n + 1$, we have the estimate*

$$r_1 \geq n^{m+1}.$$

Theorem 2. *For sufficiently large integer n with $n \geq 3$, let r_2 be a positive integer such that: the set $\{1, 2, \cdots, r_2\}$ can be partitioned into n classes such that no class contains integers x, y, z with $x + y = z$. We have the estimate*

$$r_2 \geq 2^{n+1}.$$

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2. Proof of the theorems

In this section, we complete the proof of the Theorems.

First let $r_1 = n^{m+1}$ and partition the set $\{1, 2, \cdots, r_1\}$ into n classes as follows:

$$\begin{cases} \text{Class 1:} & 1, & n+1, & n+2, & \cdots, & n^m. \\ \text{Class 2:} & 2, & n^m+1, & n^m+2, & \cdots, & 2n^m. \\ \text{Class 3:} & 3, & 2n^m+1, & 2n^m+2, & \cdots, & 3n^m. \\ \vdots & & & & & \\ \text{Class k:} & k, & (k-1)n^m+1, & (k-1)n^m+2, & \cdots, & kn^m. \\ \vdots & & & & & \\ \text{Class n:} & n, & (n-1)n^m+1, & (n-1)n^m+2, & \cdots, & n^{m+1}. \end{cases}$$

It is clear that Class k contains no integers x, y, z with $x^y = z$ for $k = 2, 3, 4, \cdots, n$. In fact for any integers $x, y, z \in$ Class k, $k = 2, 3, 4, \cdots, n$, we have

$$x^y \geq ((k-1)n^m+1)^k > k(k-1)^{k-1}n^{m(k-1)} \geq kn^m \geq z,$$

or

$$x^y \geq k^{(k-1)n^m+1} > kn^m \geq z.$$

On the other hand, when $n \geq m-1$, we have $(n+2)^{(n+1)} > n^m$ and $(n+1)^{(n+2)} > n^m$. So Class 1 contains no integers x, y, z with $x^y = z$, if $n \geq m-1$.

This completes the proof of the Theorem 1.

Then let $r_2 = 2^{n+1}$ and partition the set $\{1, 2, \cdots, r_2\}$ into n classes as follows:

$$\begin{cases} \text{Class 1:} & 1, & 2, & 2^n+2^{n-1}+\cdots+2+1. & & \\ \text{Class 2:} & 2+1, & 2^2, & 2^2+1, & 2^2+2, & 2^{n+1}. \\ \text{Class 3:} & 2^2+2+1, & 2^3, & 2^3+1, & \cdots, & 2^3+2^2+2. \\ \text{Class 4:} & 2^3+2^2+2+1, & 2^4, & 2^4+1, & \cdots, & 2^4+2^3+2^2+2. \\ \vdots & & & & & \\ \text{Class k:} & 2^{k-1}+2^{k-2}+\cdots+2+1, & 2^k, & 2^k+1, & \cdots, & 2^k+2^{k-1}+\cdots+2^2+2. \\ \vdots & & & & & \\ \text{Class n:} & 2^{n-1}+2^{n-2}+\cdots+2+1, & 2^n, & 2^n+1, & \cdots, & 2^n+2^{n-1}+\cdots+2^2+2. \end{cases}$$

It is clear that Class k contains no integers x, y, z with $x+y = z$ for $k = 3, 4, \cdots, n$. In fact for any integers $x, y, z \in$ Class k, $k = 3, 4, \cdots, n$, we have

$$(2^{k-1}+2^{k-2}+\cdots+2+1)+2^k > 2^k+2^{k-1}+\cdots+2^2+2.$$

On the other hand, when $n \geq 3$, we have $(2^2+1)+(2^2+2) < 2^{n+1}$ and $1+2 < 2^n+2^{n-1}+\cdots+2+1$. So Class 1 and Class 2 contain no integers x, y, z with $x+y=z$, if $n \geq 3$.

This completes the proof of the Theorem 2.

References

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Diverse Algorithms To Obtain Prime numbers Based on the Prime Function of Smarandache

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Abstract: In this article one gives seven formulas, six of the author S. M. Ruiz, and one of Azmy Ariff. One also gives their corresponding algorithms programmed in MATHEMATICA.

In the first four formulas all the divisions are integer divisions.

FORMULA 1: Formula to obtain the nth prime [1], [3]:

$$p(n) = 1 + \sum_{k=1}^{2(\lfloor n\log n \rfloor + 1)} \left[1 - \left[\sum_{j=2}^{k} \left[1 + \left(2 + 2.\sum_{s=1}^{\sqrt{j}} ((j-1)/s - j/s) \right) \Big/ j \right] \right] \Big/ n \right]$$

ALGORITHM 1: (G is the Smarandache Prime Function in all Algorithms)

```
DD[i_]:=Sum[Quotient[i,k]-Quotient[(i-1),k],{k,1,Floor[Sqrt[i]]}]
G[n_]:=Sum[1+Quotient[(2-2*DD[j]),j],{j,2,n}]
P[n_]:=1+Sum[1-Quotient[G[k],n],{k,1,2*(Floor[n*Log[n]]+1)}]
Do[Print[P[n]," ",Prime[n]],{n,1,50}]
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FORMULA 2: Formula to obtain the next prime [2], [3].

$$nxt(p) = 1 + p + \sum_{k=p+1}^{2p} \prod_{j=p+1}^{k} \left[-\left(\left(2 + 2.\sum_{s=1}^{\sqrt{j}} ((j-1)/s - j/s) \right) \Big/ j \right) \right]$$

ALGORITHM 2:

```
p=Input["Input a positive integer number:"]
DD[i_]:=Sum[Quotient[i,j]-Quotient[(i-1),j], {j,1,Floor[Sqrt[i]]}]
G[i_]:=-Quotient[(2-2*DD[i]),i]
F[m_]:=Product[G[i],{i,p+1,m}]
S[n_]:=Sum[F[m],{m,n+1,2*n}]
Print["nxt(",p,")=",p+1+S[p]]
```