A NOTE ON THE SMARANDACHE DIVISORS OF DIVISORS SEQUENCE AND TWO SIMILAR SEQUENCES

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In [1] Murthy defined the Smarandache Divisors of Divisors sequence as $T_1 = 3$, and $T_{n-1} = d(T_n)$, the number of divisors of T_n, where T_n is smallest such number:

3, 4, 6, 12, 72, 559872, 2^2186 * 3^255,...

For example, 12 is the smallest number having 6 divisors.

Also in [1] Murthy conjectured that after incrementing the above sequence by 1:

4, 5, 7, 13, 73, 559873,...

it will contain all primes from the second term onward.

The purpose of this short note is to show that Murthy's Divisors of Divisors sequence contained errors from the 5th term onward, and based on this fact we give two counterexamples to Murthy's conjecture.

A program was written in PARI/GP [2] to compute the Smarandache Divisors of Divisors sequence and the terms 3,4,6,12,60,5040, were given. The value 72 was listed in the original sequence and while 72 does have 12 divisors, 60 is the least such number and therefore should be the 5th term. Seeing that our computed sequence differed from Murthy's sequence, we looked these 6 terms up at OEIS [3] and the correct version of the Smarandache Divisors of Divisors sequence was found (A009287) [3]--

3, 4, 6, 12, 60, 5040, 293318625600,
6700591682045851683714764389274211129/
33837297640990904154667968000000000000

which when incremented by 1 becomes--

4, 5, 7, 13, 61, 5041, 293318625601,
6700591682045851683714764389274211129/
33837297640990904154667968000000000001

Concerning Murthy's conjecture, that all of the terms in the incremented sequence will be prime from the second term onward, notice that 5041 = 71^2; and the factors of

6700591682045851683714764389274211129/
33837297640990904154667968000000000001

are:

127 * 25624431359 *
205899454650832422686209906658432939349460496699031746583235457

which are both counterexamples to the conjecture.

Open question: What is the next term of the Smarandache Divisors of Divisors sequence?

TWO NEW SMARANDACHE SEQUENCES

1. Let sopfr(n) denote the sum of primes dividing n (with repetition) (A001414) [3].

n	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
sopfr(n)	0	2	3	4	5	5	7	6	6	7	11	7	13	9	8	8	17	8	19	9

Let s(1)=2, s(n+1) = least k with sum of prime factors (with repetition) = s(n)+1 (A075721) [3]:

2,3,4,5,8,14,26,92,356,1412,5636,185559,556671,

Example: 92 is a term because it's the smallest number such that its sum of prime factors is equal to the previous term + 1;
92 = 2^2*23 and 2+2+23 = 26+1.

Conjecture: This sequence is infinite.

2. Let t(1)=2, t(n+1)= least k with sum of squares of digits = t(n).

2,11,113,78,257,18888,

For example, 113 is a term because the sum of its digits after being squared is equal to 11, the previous term;
11 = 1^2+1^2+3^2.

Problem: What is the next term of this sequence?

REFERENCES

[1] A. Murthy, "Smarandache Function of a Function and Other Sequences, Smarandache Notions Journal, http://www.gallup.unm.edu/~smarandache/amarnath/smarfofs.htm

[2] G. Niklasch, PARI/GP Homepage, http://www.parigp-home.de/

[3] N. J. A. Sloane, On-line Encyclopedia of Integer Sequences, http://www.research.att.com/~njas/sequences