A PROBLEM OF MAXIMUM (8)

by T. Yau, student, Pima Community College

Let $S(n)$ be defined as the smallest integer such that $(S(n))!$ is divisible by n (Smarandache Function). Find:

$$\max\{ S(n)/n \},$$

over all composite integers $n \neq 4$.

Solution:

Let $n = p_1^{r_1} \cdots p_s^{r_s}$, its canonical factorial decomposition.

Because $S(n) = \max_{1\leq i\leq s}\{ S(p_i^{r_i}) \} = S(p_j^{r_j}) \leq p_j r_j$,

it's easy to see that n should have only a prime divisor for $S(n)/n$ to become maximum. Therefore $s = 1$.

Then

$n = p^r$, where: p, r are integers, and p is prime.

$S(n)/n \leq pr/p^r$. Hence p and r should be as small as possible, i.e.

$p = 2$ or 3 or 5, and $r = 2$ or 3.

By checking these combinations, we find

$n = 3^2 = 9$, whence $\max\{ S(n)/n \} = 2/3$ over all composite integers $n \neq 4$.

Reference:

M. Mudge, "Mike Mudge pays a return visit to the Florentin Smarandache Function", in <Personal Computer World>, London, February 1993, p. 403.