A SET OF CONJECTURES ON SMARANDACHE SEQUENCES*

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ABSTRACT

Searching through the Archives of the Arizona State University, I found interesting sequences of numbers and problems related to them. I display some of them, and the readers are welcome to contribute with solutions or ideas.

Key words: Smarandache P-digital subsequences, Smarandache P-partial subsequences, Smarandache type partition, Smarandache S-sequences, Smarandache uniform sequences, Smarandache operation sequences.

Let { a_n }, n > 1 be a sequence defined by a property (or a relationship involving its terms P.)

Now, we screen this sequence, selecting only its terms whose digits hold the property (or relationship involving the digits) P.

The new sequence obtained is called:

(1) *Smarandache P-digital subsequences.*

For example:

(a) *Smarandache square-digital subsequence:*

0, 1, 4, 9, 49, 100, 144, 400, 441, . . .

i.e. from 0, 1, 4, 9, 16, 25, 36, ..., n^2, ... we choose only the terms whose digits are all perfect squares (therefore only 0, 1, 4, and 9).

Disregarding the square numbers of the form $\overline{N0\ldots0}$, where N is also a perfect
2k zeros
square, how many other numbers belong to this sequence?

(b) *Smarandache cube-digital subsequence:*

0, 1, 8, 1000, 8000, . . .

i.e. from 0, 1, 8, 27, 64, 125, 216, . . . , n^3, . . . we choose only the terms whose digits are all perfect cubes (therefore only 0, 1 and 8).

Similar question, disregarding the cube numbers of the form $\overline{M0\ldots0}$
3k zeros
where M is a perfect cube.

(c) *Smarandache prime digital subsequence*:

2, 3, 5, 7, 23, 37, 53, 73, . . .

i.e. the prime numbers whose digits are all primes.

Conjecture: this sequence is infinite.

In the same general conditions of a given sequence, we screen it selecting only its terms whose groups of digits hold the property (or relationship involving the groups of digits) P.

[A group of digits may contain one or more digits, but not the whole term.]

The new sequence obtained is called:

(2) *Smarandache P-partial digital subsequence.*

Similar examples:

(a) *Smarandache square-partial-digital subsequence:*

49, 100, 144, 169, 361, 400, 441, . . .

i.e. the square members that is to be partitioned into groups of digits which are also perfect squares. (169 can be partitioned as $16 = 4^2$ and $9 = 3^2$, etc.)

Disregarding the square numbers of the form

$\overline{N0 \;.\;.\;.\; 0}$, where N is also a perfect square,
2k zeros
how many other numbers belong to this sequence?

(b) *Smarandache cube-partial digital subsequence*:

1000, 8000, 10648, 27000, . . .

i.e. the cube numbers that can be partitioned into groups of digits which are also perfect cubes.

(10648 can be partitioned as $1 = 1^3$, $0 = 0^3$, $64 = 4^3$, and $8 = 2^3$).

Same question: disregarding the cube numbers of the form:

$\overline{M0 \;.\;.\;.\; 0}$ where M is also a perfect cube, how many other numbers belong
3k zeros
to this sequence.

(c) *Smarandache prime-partial digital subsequence*:

23, 37, 53, 73, 113, 137, 173, 193, 197, . . .

i.e. prime numbers, that can be partitioned into groups of digits which are also prime,

(113 can be partitioned as 11 and 3, both primes).

Conjecture: this sequence is infinite.

(d) *Smarandache Lucas-partial digital sunsequence*

123, . . .

i.e. the sum of the two first groups of digits is equal to the last group of digits, and the whole number belongs to Lucas numbers:

2, 1, 3, 4, 7, 11, 18, 29, 47, 76, 123, 199, . . .

(beginning at 2 and $L(n+2) = L(n+1) + L(n)$, $n > 1$) (123 is partitioned as 1, 2 and 3, then $3 = 2 + 1$). Is 123 the only Lucas number that verifies a *Smarandache type partition*?

Study some Smarandache P - (partial) - digital subsequences associated to:

- Fibonacci numbers (we were not able to find any Fibonacci number verifying a Smarandache type partition, but we could not investigate large numbers; can you? Do you think none of them would belong to a Smarandache F - partial-digital subsequence?

- Smith numbers, Eulerian numbers, Bernoulli numbers, Mock theta numbers, Smarandache type sequences etc.

Remark: Some sequences may not be smarandachely partitioned (i.e. their associated Smarandache type subsequences are empty).

If a sequence $\{a_n\}$, $n \geq 1$ is defined by $a_n = f(n)$ (a function of n), then

Smarandache f-digital subsequence is obtained by screening the sequence and selecting only its terms that can be partitioned in two groups of digits g_1 and g_2 such that $g_2 = f(g_1)$.

For example:

(a) If $a_n = 2n$, $n \geq 1$, then

Smarandache even-digital subsequence is:

12, 24, 36, 48, 510, 612, 714, 816, 918, 1020, 1122, 1224, . . .

(i.e. 714 can be partitioned as $g_1 = 7$, $g_2 = 14$, such that $14 = 2*7$, etc.)

(b) *Smarandache lucky-digital subsequence*

37, 49, . . .

(i.e. 37 can be partitioned as 3 and 7, and $L_3 = 7$; the lucky numbers are

1, 3, 7, 9, 13, 15, 21, 25, 31, 33, 37, 43, 49, 51, 63, . . .

How many other numbers belong to this subsequence? Study the Smarandache f-digital subsequence associated to other well-known sequences.

(3) *Smarandache odd sequence*:

1, 3, 135, 1357, 13579, 1357911, 135791113, 13579111315, 1357911131517, ...

How many of them are prime?

(4) *Smarandache even sequence*:

2, 24, 246, 2468, 246810, 24681012, 2468101214, 246810121416, . . .

Conjecture: None of them is a perfect power!

(5) *Smarandache prime sequence*:

2, 23, 235, 2357, 235711, 23571113, 2357111317, 235711131719, 23571113171923, . . .

How many of them are prime?

(Conjecture: a finite number).

(6) *Smarandache S-sequence*:

General definition:

Let $S_1, S_2, S_3, \ldots, S_n, \ldots$ be an infinite integer sequence (noted by S). Then

$$S_1, \overline{S_1 S_2}, \overline{S_1 S_2 S_3}, \ldots, \overline{S_1 S_2 S_3 \ldots S_n}, \ldots$$

is called the Smarandache S-sequence.

Question:

(a) How many of the Smarandache S-sequence belong to the initial S sequence?

(b) Or, how many of the Smarandache S-sequence verify the relation of other given sequences?

For example:

If S is the sequence of odd numbers 1, 3, 5, 7, 9, . . . then the Smarandache S-sequence is 1, 13, 135, 1357, . . . [(i.e.1)] and all the other terms are odd;

Same if S is the sequence of even numbers [(i.e. 2)]

The question (a) is trivial in this case.

But,when S is the sequence of primes [i.e. 3], the question becomes much harder.

Study the case when S (replaced by F) is the Fibonacci sequence (for one example):

1, 1, 2, 3, 5, 8, 13, 21, ...

Then the Smarandache F - sequence

1, 11, 112, 1123, 11235, 112358, . . .

How many primes does it contain?

(7) *Smarandache uniform sequences*:

General definition:

Let n be an integer not equal to zero and $d_1, d_2, \ldots, d_r$ digits in a base B (of course $r < B$).

Then: multiples of n, written with digits $d_1, d_2, \ldots, d_r$ only (but all r of them), in base B, increasingly ordered, are called the Smarandache uniform sequence.

As a particular case it's important to study the multiples written with one digit only (when $r = 1$).

Some examples (in base 10):

(a) Multiples of 7 written with digit 1 only:

111111, 111111,111111, 111111,111111,111111, 111111,111111,111111,111111, ...

(b) Multiples of 7 written with digit 2 only:

222222, 222222222222, 222222222222222222, 222222222222222222222222, ...

(c) Multiples of 79365 written with digit 5 only:

555555, 555555555555, 555555555555555555, 555555555555555555555555, ...

For some cases, the Smarandache uniform sequence may be empty (impossible):

(d) Multiples of 79365 written with digit 6 only (because any multiple of 79365 will end in 0 or 5.

Remark: If there exists at least a multiple m of n, written with digits $d_1, d_2, \ldots, d_r$ only, in base B, then there exists an infinite number of multiples of n (they have the form:

$m, \overline{mm}, \overline{mmm}, \overline{mmmm}, \ldots$).

With a computer program it's easy to select all multiples (written with certain digits) of a given number - up to some limit.

Exercise: Find the general term expression for multiples of 7 written with digits 1, 3, 5 only in base 10.

(8) *Smarandache operation sequences*:

General definition:

Let E be an ordered set of elements, $E = \{ e_1, e_2, \ldots \}$ and θ a set of binary

operations well-defined for these elements. Then:

a_1 is an element of $\{ e_1, e_2, \ldots \}$.

$a_{n+1} = \min \{ e_1 \theta_1 e_2 \theta_2 \ldots \theta_n e_n \} > a_n$, for $n > 1$.

where all θ_i are operations belonging to θ, is called the *Smarandache operation sequence*.

Some examples:

(a) When E is the natural number set, and θ is formed by the four arithmetic operations: +, -, *, /.

Then: $a_1 = 1$

$$a_{n-1} = \min \{ 1 \, \theta_1 \, 2 \, \theta_2 \ldots \theta_n \, (n+1) \} > a_n, \text{ for } n > 1,$$

(therefore, all θ_i may be chosen among addition, subtraction, multiplication or division in a convenient way).

Questions: Find this *Smarandache arithmetics operation infinite sequence*. Is it possible to get a general expression formula for this sequence (which starts with 1, 2, 3, 5, 4,?

(b) A finite sequence

$a_1 = 1$

$$a_{n+1} = \min \{ 1 \, \theta_1 \, 2 \, \theta_2 \ldots \theta_{98} \, 99 \} > a_n$$

for $n > 1$, where all θ_i are elements of { +, -, *, / }.

Same questions for this *Smarandache arithmetics operation finite sequence.*

(c) Similarly for *Smarandache algebraic operation infinite sequence*

$a_1 = 1$

$$a_{n+1} = \min \{ 1 \, \theta_1 \, 2 \, \theta_2 \ldots \theta_n \, (n+1) \} > a_n \text{ for } n > 1,$$

where all θ_i are elements of { +, -, *, /, **, $^y\sqrt{}$ }

(X**Y means X^Y and $^y\sqrt{x}$ means the yth root of x).

The same questions become harder but more exciting.

(d) Similarly for *Smarandache algebraic operation finite sequence*:

$a_1 = 1$

$$a_{n+1} = \min \{ 1 \, \theta_1 \, 2 \, \theta_2 \ldots \theta_{98} \, 99 \} > a, \text{ for } n > 1,$$

where all θ_i are elements of { +, -, *, /, **, $^y\sqrt{}$ }

(X**Y means X^Y and $^y\sqrt{x}$ means the yth root of x).

Same questions.

More generally: one replaces "binary operations" by "K_i -ary operations" where all K_i are integers ≥ 2). Therefore,

$a_i \in \{ e_1, e_2, \ldots \}$,

$$a_{n+1} = \min\{ 1\, \theta_1^{(K_1)} \theta_1^{(K_1)} \ldots \theta_1^{(K_1)} K_1$$
$\theta_1^{(K_1)}$ is K_1 - ary

$$\theta_2^{(K_2)} (K_2 + 1) \theta_2^{(K_2)} \ldots \theta_2^{(K_2)} (K_1 + K_2 - 1) \ldots$$
$\theta_2^{(K_2)}$ is K_2 - ary

$$(n + 2 - K_r)\, \theta_r^{(K_r)} \ldots \theta_r^{(K_r)} (n + 1) \} > a_n, \text{ for } n \geq 1.$$

Of course $K_1 + (K_2 - 1) + \ldots (K_r - 1) = n+1$.

Remark: The questions are much easier when $\theta = \{ +,- \}$; study the Smarandache operation type sequences in this case.

(9) *Smarandache operation sequences at random*:

Same definitions and questions as for the previous sequences, except that

$$a_{n+1} = \{ e_1 \theta_1 e_2 \theta_2 \ldots \theta_n e_{n+1} \} > a_n, \text{ for } n > 1,$$

(i.e. it's no "min" any more, therefore a_{n+1} will be chosen at random, but greater than a_n, for any $n > 1$). Study these sequences with a computer program for random variables (under weak conditions).

REFERENCES

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