

## About The $S(n) = S(n - S(n))$ Equation

Mihaly Bencze  
Str. Harmanului 6, 2212 Sacele 3  
Jud. Brasov, Romania

**Theorem 1:** (M. Bencze, 1997) There exists infinitely many  $n \in \mathbb{N}$  such that  $S(n) = S(n - S(n))$ , where  $S$  is the Smarandache function.

**Proof:** Let  $r$  be a positive integer and  $p > r$  a prime number. Then

$$S(pr) = S(p) = S((r-1)p) = S(pr - p) = S(pr - S(pr)).$$

**Remark 1.1** There exists infinitely many  $n \in \mathbb{N}$  such that

$$S(n) = S(n - S(n)) = S(n - S(n - S(n))) = \dots$$

**Theorem 2:** There exists infinitely many  $n \in \mathbb{N}$  such that

$$S(n) = S(n + S(n)).$$

**Proof:**

$$S(pr) = S(p) = S((r+1)p) = S(pr+p) = S(pr + S(pr)).$$

**Remark 2.1** There exists infinitely many  $n \in \mathbb{N}$  such that

$$S(n) = S(n + S(n)) = S(n + S(n + S(n))) = \dots$$

**Theorem 3** There exists infinitely many  $n \in \mathbb{N}$  such that

$$S(n) = S(n \pm kS(n)).$$

**Proof:** See theorems 1 and 2.