An experimental evidence on the validity of third Smarandache conjecture on primes

Felice Russo

Micron Technology Italy

Avezzano (Aq) Italy

Abstract

In this note we report the results regarding the check of the third Smarandache conjecture on primes [1],[2] for $p_n \leq 2^{25}$ and $2 \leq k \leq 10$. In the range analysed the conjecture is true. Moreover, according to experimental data obtained, it seems likely that the conjecture is true for all primes and for all positive values of k..

Introduction

In [1] and [2] the following function has been defined:

$$C(n,k) = p_{n+1}^{\frac{1}{k}} - p_n^{\frac{1}{k}}$$

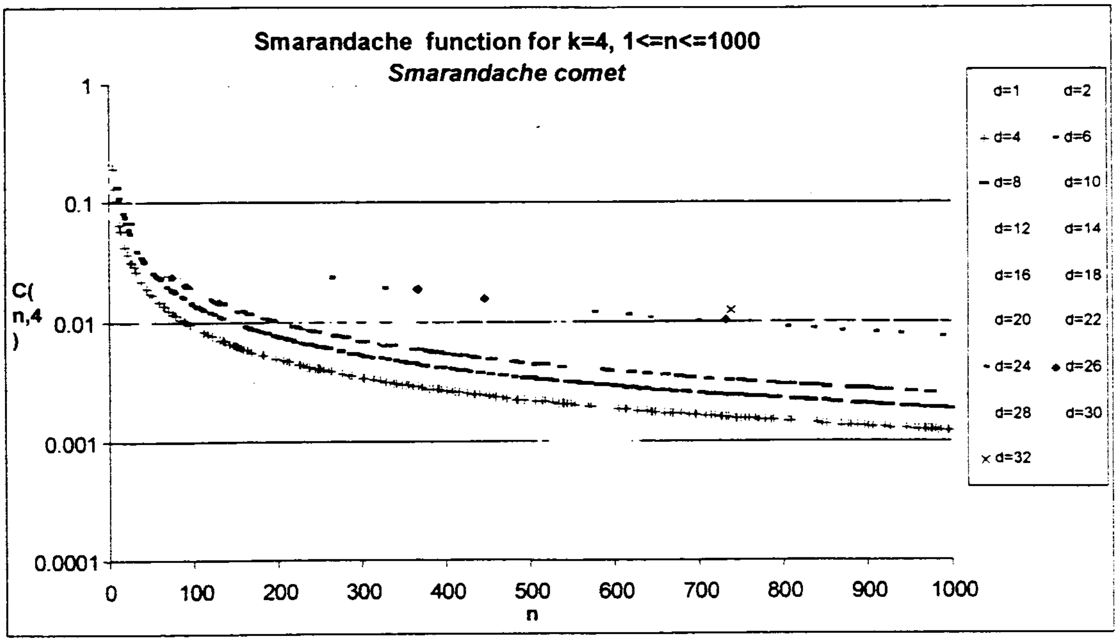
where p_n is the n-th prime and k is a positive integer. Moreover in the above mentioned papers the following conjecture has been formulated by F. Smarandache:

$$C(n,k) < \frac{2}{k} \text{ for } k \geq 2$$

This conjecture is the generalization of the Andrica conjecture (k=2) [3] that has not yet been proven. This third Smarandache conjecture has been tested for $p_n \leq 2^{25}$, $2 \leq k \leq 10$ and in this note the result of this search is reported. The computer code has been written utilizing the Ubasic software package.

Experimental Results

In the following graph the Smarandache function for k=4 and n<1000 is reported. As we can see the value of C(k,n) is modulated by the prime's gap indicated by $d_n = p_{n+1} - p_n$.
We call this graph the Smarandache "comet".



In the following table, instead, we report:

- the largest value Max_C(n,k) of Smarandache function for $2 \le k \le 10$ and $p_n \le 2^{25}$
- the difference $\Delta(k)$ between 2/k and Max_C(n,k)
- the value of p_n that maximize C(n,k)
- the value of 2/k

k	2	3	4	5	6	7	8	9	10
Max C(n,k)	0.67087	0.31105	0.19458	0.13962	0.10821	0.08857	0.07564	0.06598	0.05850
Δ	0.32913	0.35562	0.30542	0.26038	0.22512	0.19715	0.17436	0.15624	0.14150
p_n	7	7	7	7	7	3	3	3	3
2/k	1	0.666..	0.5	0.4	0.333..	0.2857..	0.25	0.222..	0.20

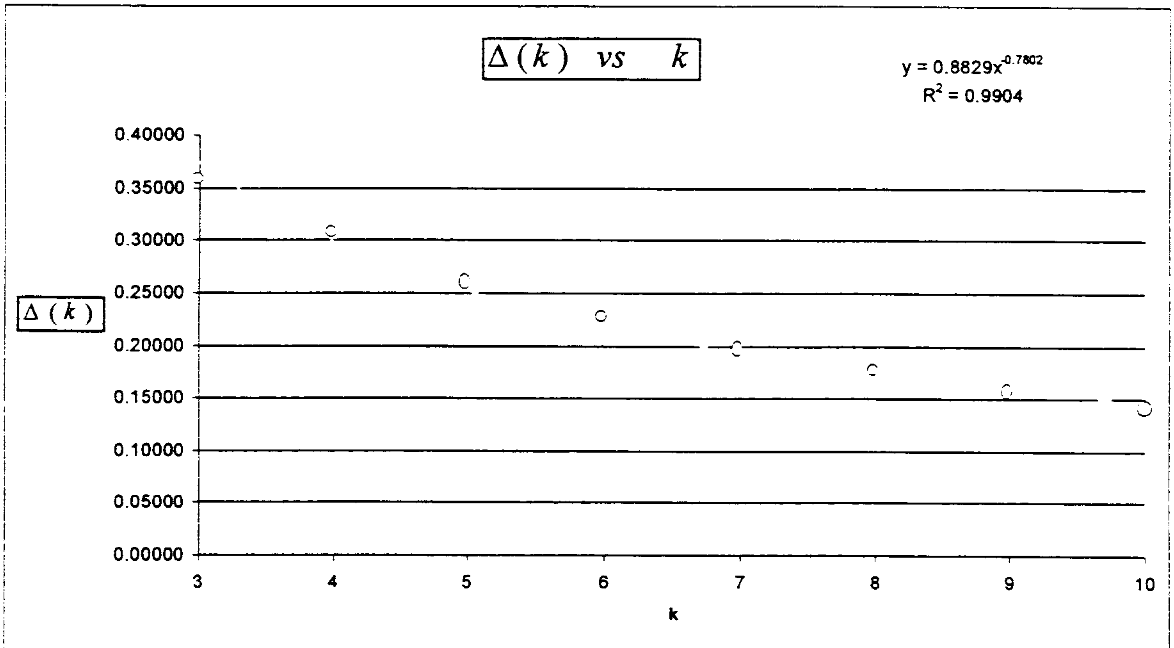
According to previous data the third Smarandache conjecture is verified in the range of k and p_n analysed due to the fact that Δ is always positive.
Moreover since the Smarandache C(n,k) function falls asymptotically as n increases it is likely that the estimated maximum is valid also for $p_n > 2^{25}$.

We can also analyse the behaviour of difference $\Delta(k)$ versus the k parameter that in the following graph is showed with white dots. We have estimated an interpolating function:

$$\Delta(k) \approx 0.88 \cdot \frac{1}{k^{0.78}} \quad \text{for } k>2$$

with a very good R^2 value (see the continuous curve). This result reinforces the validity of the third Smarandache conjecture since:

$$\Delta(k) \to 0 \quad for\ k \to \infty$$



New Question

According to previous experimental data can we reformulate the third Smarandache conjecture with a tighter limit as showed below?

$$C(n,k) < \frac{2}{k^{2 \cdot a_0}}$$

where $k \geq 2$ and a_0 is the Smarandache constant [4],[1].

References:

[1] See http://www.gallup.unm.edu/~smarandache/ConjPrim.txt

[2] Smarandache, Florentin, "Conjectures which Generalize Andrica's Conjecture", Octogon, Braşov, Vol. 7, No. 1, 1999, 173-176.

[3] Weisstein, Eric W., "Andrica's conjecture" in CRC Concise Encyclopedia of Mathematics, CRC Press, Florida, 1998.

[4] Sloane, Neil, Sequence A038458 ("Smarandache Constant" = 0.5671481302020177146468468755...) in <An On-Line Version of the Encyclopedia of Integer Sequences>, http://www.research.att.com/cgi-bin/access.cgi/as/njas/sequences/eishis.cgi.