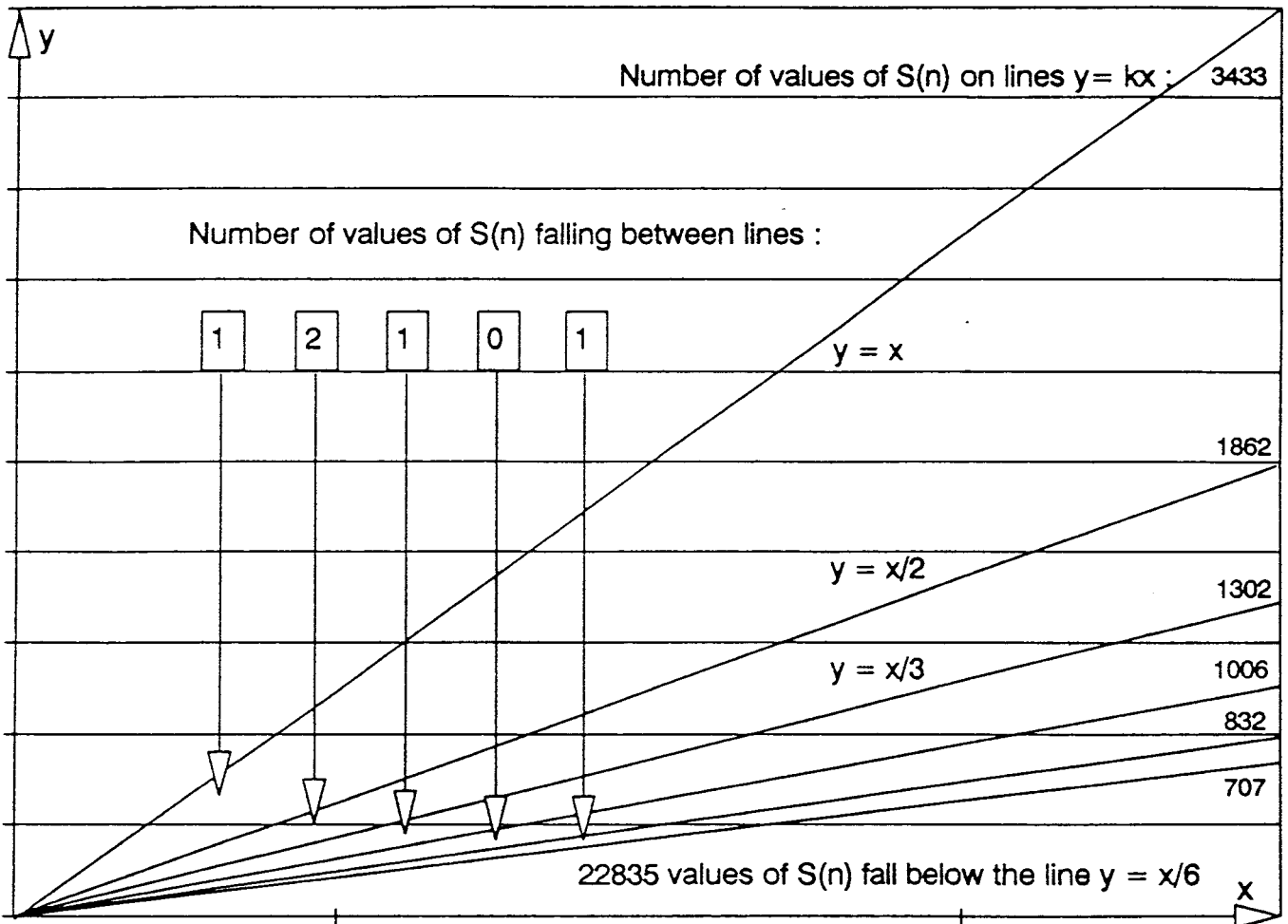


# An Illustration of the Distribution of the Smarandache Function

by Henry Ibstedt

The cover illustration is a representation of the values of the Smarandache function for  $n \leq 53$ . The group at the back of the diagram essentially corresponds to  $S(p) = p$ , the middle group to  $S(2p) = p$  ( $p \neq 2$ ) while the front group represents all the other values of  $S(n)$  for  $n \leq 53$ .

Diagram 1. Distribution of  $S(n)$  up to  $n = 32000$  (not to scale)



It may be interesting to take this graphical presentation a bit further. All the values of  $S(n)$  for  $n \leq 32000$  (conveniently chosen in order to use short integers only) have been sorted as shown in table 1. Of the 19114 points  $(n, S(n))$  situated above the line  $y = x/50$  only 61 points fall between lines. All of these of course correspond to cases where  $n$  is not square free. Diagram 1 illustrates this for the lines  $y = x$ ,  $y = x/2$ ,  $y = x/3$ ,  $y = x/4$ ,  $y = x/5$  and  $y = x/6$ . The top line contains 3433 points  $(n, S(n))$  although there are only 3432 primes less than 32000. This is because  $(4, S(4))$  belongs to this line.

TABLE 1. On the distribution of the Smarandache Function  $S(n)$  for  $n \leq 32000$ .

$N$  = number of values of  $S(n)$  on the line  $y=x/k$ , i.e.  $S(n)=n/k$ . The points  $(n,S(n))$  are the only ones between lines  $y=x/k$  and  $y=x/(k+1)$  for  $k < 50$ .

k	N	Points $(n,S(n))$ between lines:
1	3433	( 9, 6)
2	1862	( 16, 6) ( 25, 10)
3	1302	( 49, 14)
4	1006	
5	832	( 121, 22)
6	707	( 169, 26)
7	616	( 45, 6) ( 75, 10)
8	550	( 125, 15) ( 289, 34)
9	495	( 361, 38)
10	450	( 147, 14)
11	417	( 529, 46)
12	387	
13	359	( 80, 6)
14	336	( 841, 58)
15	321	( 961, 62)
16	301	( 250, 15) ( 343, 21) ( 363, 22)
17	283	( 175, 10) ( 245, 14)
18	273	(1369, 74)
19	256	( 507, 26)
20	250	( 243, 12) (1681, 82)
21	239	(1849, 86)
22	227	( 225, 10)
23	213	(2209, 94)
24	218	
25	204	( 256, 10) ( 867, 34)
26	196	(2809,106)
27	190	( 605, 22)
28	187	(1083, 38)
29	176	(3481,118)
30	179	(3721,122)
31	163	( 441, 14) ( 625, 20)
32	164	( 686, 21) ( 845, 26)
33	159	( 500, 15) (4489,134)
34	154	(1587, 46)
35	154	(5041,142)
36	153	(5329,146)
37	139	
38	139	( 539, 14) ( 847, 22)
39	136	(6241,158)
40	139	( 486, 12) (1331, 33)
41	125	(6889,166)
42	133	( 512, 12) (1445, 34)
43	119	(2523, 58)
44	125	(7921,178)
45	126	( 637, 14) (1183, 26)
46	117	(2883, 62)
47	109	(1805, 38)
48	120	( 729, 15) (9409,194)
49	114	(1089, 22)
50	112	

Number of elements below  $y = x/50$ : 12774 .