# AN IMPROVEMENT ON THE SMARANDACHE DIVISIBILITY THEOREM

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Abstract. Let a, n be positive integers. In this paper we prove that $n \mid (a^n - a)[n/2]!$

For any positive integer a and n, Smarandache [3] proved that

(1) $$n \mid (a^n - a)(n-1)!.$$

The above division relation is the Smarandache divisibility theorem (see [1, Notions 126]). In this paper we give an improvement on (1) as follows:

Theorem. For any positive integers a and n, we have

(2) $$n \mid (a^n - a)[n/2]!,$$

where $[n/2]$ is the largest integer which does not exceed $n/2$.

Proof. The division relation (2) holds for $n \le 9$, we may assume that $n > 9$. By Fermat's theorem (see [ 2, Theorem 71]), if n is a prime, then we have

(3) $$n \mid (a^n - a),$$

for any $a$. We see from (3) that (2) is true.

If $n$ is a composite number, then we have $n = pd$, where $p,d$ are integers satisfying $p \geq q \geq 2$. Further, if $p \neq q$, then we have $n|p!$ It implies that $n|(n / q)!$ Since $q \geq 2$, we get

(4) $n \mid [n/2]!$

If $p = q$, Then $n = p^2$ and

(5) $n \mid (2p)!$

Since $n > 9$, we have $n \geq 4^2$, $p \geq 4$ and $2p \leq n/2$. Hence, we see from (5) that (4) is also true in this case. The combination of (3) and (4), the theorem is proved.

References


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