AN INEQUALITY CONCERNING THE SMARANDACHE FUNCTION

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Abstract. For any positive integer n, let S(n) denote the Smarandache function of n. In this paper we prove that $S(mn) \le S(m)+S(n)$.

Let N be the set of all positive integers. For any positive integer n, let S(n) denote the Smarandache function of n. By [2], we have

(1) $S(n)= \min\{k \mid k \in N, n \mid k!\}$.

Recently, Jozsef[1] proved that

(2) $S(mn) \le mS(n)$, $m,n \in N$.

In this paper we give a considerable improvement for the upper bound (2). We prove the following result.

Theorem. For any positive integers m,n, we have
$$S(mn) \le S(m)+S(n).$$

Proof. Let $a=S(m)$ and $b=S(n)$. Then we have

(3) $n \mid b!$,

by (1). Let x be a positive integer with $x \ge a$, and let

(4) $\binom{x}{a} = \dfrac{x(x-1)\ldots(x-a+1)}{a!}$

be a binomial coeficient. It is a well known fact that $\binom{x}{a}$ is a positive integer. So we have

(5) $a! \mid x(x-1)\ldots(x-a+1)$,

by (4). Further, since $m \mid a!$, we get from (5) that

(6) $m \mid x(x-1)\ldots(x-a+1)$,

for any positive integer x with $x \ge a$. Put $x=a+b$. We see from (3) and (6) that

(7) $mn \mid b!(b+1)\ldots(b+a)=(a+b)!$.

Thus we get from (7) that $S(mn) \le a+b=S(m)+S(n)$. The theorem is proved.

References

1. S.Jozsef,On certain inequalites involving the Smarandache function, Smarandache Notce J. 7(1996), No.1-3, 3-6.
2. F.Smarandache "A function in the number theory"," Smarandache Function J". 1(1990), No.1, 3-17.