AN INEQUALITY FOR THE SMARANDACHE FUNCTION

by

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Let $S(m) = \min\{k \mid k \in N: m \mid k!\}$ be the Smarandache Function. In this paper we prove the following

THEOREM: $S(\prod_{k=1}^{m} m_k) \le \sum_{k=1}^{m} S(m_k)$.

We prove by induction. For m=1 it's true.
Let m=2, then we prove $S(m_1 m_2) \le S(m_1) + S(m_2)$.
We have $m_2 \mid S(m_2)!$ and if $r \ge S(m_1)$ then
$S(m_1)! \mid r(r-1)\ldots(r-S(m_1)+1)$.
If $t \mid S(n_1)!$ then $t \mid r(r-1)\ldots(r-S(n_1)+1)$ so
$m_1 m_2 \mid S(m_2)!(S(m_2)+1)\ldots(S(m_2)+S(m_1)) = (S(m_1)+S(m_2))!$
From this it results $S(m_1 m_2) \le S(m_1)+S(m_2)$.
We suppose they are true for m, and we prove for m+1.

$$S(\prod_{k=1}^{m+1} m_k) = S(m_1 \prod_{k=2}^{m+1} m_k) \le S(m_1) + S(\prod_{k=2}^{m+1} m_k) \le S(m_1 + \sum_{k=1}^{m+1} S(m_k) = \sum_{k=1}^{m+1} S(m_k).$$

REFERENCE:

[1] Smarandache, Florentin, "A function in the Number Theory", <Smarandache Function Journal> (1990), No. 1, pp. 3-17.