Base Solution
(The Smarandache Function)

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Definition of the Smarandache function S(n)

S(n) = the smallest positive integer such that S(n)! is divisible by n.

Problem A: Ashbacher's problem

For what triplets n, n-1, n-2 does the Smarandache function satisfy the Fibonacci reccurrence: S(n)=S(n-1)+S(n-2). Solutions have been found for n=11, 121, 4902, 26245, 32112, 64010, 368140 and 415664. Is there a pattern that would lead to the proof that there is an infinite family of solutions?

The next three triplets n, n-1, n-2 for which the Smarandache funtion S(n) satisfies the relation S(n)=S(n-1)+S(n-2) occur for n=2091206, n=2519648 and n=4573053. Apart from the triplet obtained from n=26245 the triplets have in common that one member is 2 times a prime and the other two members are primes.

This leads to a search for triplets restricted to integers which meet the following requirements:

$$n = xp^a \text{ with } a \le p+1 \text{ and } S(x) < ap \qquad (1)$$

$$n-1 = yq^b \text{ with } b \le q+1 \text{ and } S(y) < bq \qquad (2)$$

$$n-2 = zr^c \text{ with } c \le r+1 \text{ and } S(z) < cr \qquad (3)$$

p,q and r are primes. With then have S(n)=ap, S(n-1)=bq and S(n-2)=cr. From this and by subtracting (2) from (1) and (3) from (2) we get

$$ap = bq + cr \quad (4)$$

$$xp^a - yq^b = 1 \quad (5)$$

$$yq^b - zr^c = 1 \quad (6)$$

Each solution to (4) generates infinitely many solutions to (5) which can be written in the form:

$$x = x_0 + q^b t, \qquad y = y_0 - p^a t \quad (5')$$

where t is an integer and (x_0, y_0) is the principal solution, which can be otained using Euclid's algorithm.

Solutions to (5') are substituted in (6') in order to obtain integer solutions for z.

$$z = (yq^b - 1)/r^c \quad (6')$$

Implementation:
Solutions were generated for (a,b,c)=(2,1,1), (a,b,c)=(1,2,1) and (a,b,c)=(1,1,2) with the parameter t restricted to the interval $-9 \leq t \leq 10$. The output is presented on page 5. Since the correctness of these calculations are easily verfied from factorisations of S(n), S(n-1), and S(n-2) some of these are given in an annex. This study strongly indicates that the set of solutions is infinite.

Problem B: Radu's problem

Show that, except for a finite set of numbers, there exists at least one prime number between S(n) and S(n+1).

The immediate question is <u>what would be this finite set?</u> I order to examine this the following more stringent problem (which replaces "between" with the requirement that S(n) and S(n+1) must also be composite) will be considered.

Find the set of consecutive integers n and n+1 for which two consecutive primes p_k and p_{k+1} exists so that $p_k < Min(S(n),S(n+1))$ and $p_{k+1} > Max(S(n),S(n+1))$.

Consider

$$n+1 = xp_r^s$$

$$n = yp_{r+1}^s$$

where p_r and p_{r+1} are consecutive primes. Subtract

$$xp_r^s - yp_{r+1}^s = 1 \qquad (1)$$

The greatest common divisor $(p_r^s, p_{r+1}^s) = 1$ divides the right hand side of (1) which is the condition for this diophantine equation to have infinitely many integer solutions. We are interested in positive integer solutions (x,y) such that the following conditions are met.

I. $S(n+1) = sp_r$, i.e $S(x) < sp_r$

II. $S(n) = sp_{r+1}$, i.e $S(y) < sp_{r+1}$

in addition we require that the interval

III. $sp_r^s < q < sp_{r+1}^s$ is prime free, i.e. q is not a prime.

Euclid's algorithm has been used to obtain principal solutions (x_0, y_0) to (1). The general set of solutions to (1) are then given by

$$x = x_0 + p_{r+1}^s t, \qquad y = y_0 - p_r^s t$$

with t an integer.

Implementation:
The above algorithms have been implemented for various values of the parameters $d = p_{r+1} - p_r$, s and t. A very large set of solutions was obtained. There is no indication that the set would be finite. A pair of primes may produce several solutions. Within the limits set by the design of the program the largest prime difference for which a solution was found is d=42 and the largest exponent which produced solutions is 4. Some numerically large examples illustrating the above facts are given on page 6.

Problem C: Stuparu's problem

Consider numbers written in Smarandache Prime Base 1,2,3,5,7,11,.... given the example that 101 in Smarandache base means $1 \cdot 3 + 0 \cdot 2 + 1 \cdot 1 = 4_{10}$.

As this leads to several ways to translate a base 10 number into a Base Smarandache number it seems that further precisions are needed. Example

$$111_{Smarandache} = 1\cdot 3+1\cdot 2+1\cdot 1 = 6_{10}$$

$$1001_{Smarandache} = 1\cdot 5+0\cdot 3+0\cdot 2+1\cdot 1 = 6_{10}$$

Equipment and programs

Computer programs for this study were written in UBASIC ver. 8.77. Extensive use was made of NXTPRM(x) and PRMDIV(n) which are very convenient although they also set an upper limit for the search routines designed in the main program. Programs were run on a dtk 486/33 computer. Further numerical outputs and program codes are available on request.

Smarandache - Ashbacher's problem.

#	N	S(N)	S(N-1)	S(N-2)	t
1	11	11	5	6	0
2	121	22	5	17	0
3	4902	43	29	14	-4
4	32112	223	197	26	-1
5	64010	173	46	127	-1
6	368140	233	82	151	-1
7	415664	313	167	146	-8
8	2091206	269	202	67	-1
9	2519648	1109	202	907	0
10	4573053	569	106	463	-3
11	7783364	2591	202	2389	0
12	79269727	2861	2719	142	10
13	136193976	3433	554	2879	-1
14	321022289	7589	178	7411	5
15	445810543	1714	761	953	-1
16	559199345	1129	662	467	-5
17	670994143	6491	838	5653	-1
18	836250239	9859	482	9377	1
19	893950202	2213	2062	151	0
20	1041478032	2647	1286	1361	-1
21	1148788154	2467	746	1721	3
22	1305978672	5653	1514	4139	0
23	1834527185	3671	634	3037	-5
24	2390706171	6661	2642	4019	0
25	2502250627	2861	2578	283	-1
26	3969415464	5801	1198	4603	-2
27	3970638169	2066	643	1423	-6
28	6493607750	3049	1262	1787	5
29	6964546435	2161	1814	347	-4
30	11329931930	3023	2026	997	-4
31	13429326313	4778	1597	3181	1
32	13849559620	6883	2474	4409	1
33	14988125477	3209	2986	223	2
34	17560225226	4241	3118	1123	-2
35	25184038673	5582	1951	3631	-2
36	69481145903	6301	3722	2579	3
37	155205225351	8317	4034	4283	-5
38	196209376292	7246	3257	3989	-5
39	344645609138	7226	2803	4423	9
40	401379101876	32122	653	31469	2
41	484400122414	16811	12658	4153	-1
42	533671822944	21089	18118	2971	0
43	561967733244	21722	7159	14563	-1
44	703403257356	13147	10874	2273	-2
45	859525157632	14158	3557	10601	-5
46	898606860813	19973	13402	6571	1
47	1185892343342	18251	12022	6229	-2
48	1188795217601	29242	13049	16193	0
49	1294530625810	17614	5807	11807	-3
50	1517767218627	11617	8318	3299	-8
51	2677290337914	33494	3631	29863	-3
52	3043063820555	14951	12202	2749	5
53	6344309623744	22978	7451	15527	6
54	16738688950356	30538	6977	23561	10
55	19448047080036	34186	17027	17159	-4

Ashbacher's problem (ASHEDIT.UB), 951206, Henry Ibstedt

Smarandache - Reed's problem.

q=p(j), p=p(j+1), d=p-q, N=x*q^s or y*p^s, k solutions to x*q^s - y*p^s = +/-1 will examined.

Parameters for this run: d = 2 , s = 2 , k = 15 .

x	y	q	p	x*q^s	y*p^s
13039	12198	59	61	45388759	45388758
1876	1755	59	61	6530356	6530355
7975	7544	71	73	40201975	40201976
26	25	101	103	265226	265225
5913	5698	107	109	67697937	67697938
113967	110968	149	151	2530181367	2530181368
38	37	149	151	843638	843637
49063	48438	311	313	4745422423	4745422422
636720	628609	311	313	61584195120	61584195121
60988	60291	347	349	7343504092	7343504091
182614	180527	347	349	21988369126	21988369127
1071729	1062490	461	463	227764918809	227764918810
116	115	461	463	24652436	24652435
214485	212636	461	463	45582566685	45582566684
1071961	1062720	461	463	227814223681	227814223680
131	130	521	523	35558771	35558770
1914834	1900217	521	523	519764455794	519764455793
143	142	569	571	46297823	46297822
3386439	3370000	821	823	2282598729999	2282598730000
206	205	821	823	138852446	138852445
2709522	2696369	821	823	1826328918402	1826328918401
215	214	857	859	157906535	157906534
1475977	1469112	857	859	1084029831673	1084029831672
3689620	3672459	857	859	2709837719380	2709837719379
221	220	881	883	171531581	171531580
2339288	2328703	881	883	1815664113368	1815664113367
5649579	5628340	1061	1063	6359849721459	6359849721460
266	265	1061	1063	299441786	299441785
5650111	5628870	1061	1063	6330448605031	6360448605030
597051	594868	1091	1093	710658461331	710658461332
664416	662113	1151	1153	880218981216	880218981217
1993825	1986914	1151	1153	2641421353825	2641421353826
7311461	7286118	1151	1153	9686230844261	9686230844262
9970279	9935720	1151	1153	13208635589479	13208635589480
8488719	8462680	1301	1303	14368014268119	14368014268120
5093101	5077478	1301	1303	8620587845701	8620587845702
326	325	1301	1303	551787926	551787925
5093753	5078128	1301	1303	8621691421553	8621691421552
8489371	8463330	1301	1303	14369117843971	14369117843970
2617231	2609312	1319	1321	4553356421791	4553356421792
1393198	1389861	1667	1669	3871542597022	3871542597021
9749046	9725695	1667	1669	27091516689894	27091516689895
14432580	14398621	1697	1699	41563073777220	41563073777221
2886176	2879385	1697	1699	8311635620384	8311635620385
425	424	1697	1699	1223918825	1223918824
431	430	1721	1723	1276553471	1276553470
2969160	2962271	1721	1723	8794179823560	8794179823559
1600708	1597131	1787	1789	5111651305252	5111651305251
1599813	1596238	1787	1789	5108793239997	5108793239998
24003460	23949821	1787	1789	76651905056740	76651905056741
19295178	19253993	1871	1873	67545491209098	67545491209097
1753596	1749853	1871	1873	6138710055036	6138710055037
470	469	1877	1879	1655870630	1655870629
14123034	14092985	1877	1879	49757270653386	49757270653385
7612314	7596715	1949	1951	28916143572714	28916143572715
3805913	3798114	1949	1951	14457144927713	14457144927714
488	487	1949	1951	1853717288	1853717287
19032493	18993492	1949	1951	72296846942293	72296846942292
11987503	11963528	1997	1999	47806269851527	47806269851528
500	499	1997	1999	1994004500	1994004499
521	520	2081	2083	2256222281	2256222280

Smarandache - Radu's problem.
q=p(j), p=p(j+1), d=p-q, N=x*q^s or y*p^s, k solutions to x*q^s - y*p^s = +/-1 will examined.

Parameters for this run: d = 2 , s = 2 , k = 15 .

x	y	q	p	x*q^s	y*p^s
4339410	4331081	2081	2083	18792079709010	18792079709009
11162451	11141330	2111	2113	49743464802771	49743464802770
2232913	2228688	2111	2113	9950577093073	9950577093072
2231856	2227633	2111	2113	9945866761776	9945866761777
15626163	15596596	2111	2113	69635198326323	69635198326324
33485239	33421880	2111	2113	149220973745719	149220973745720
35091287	35028624	2237	2239	175602730575503	175602730575504
10025682	10007779	2237	2239	50170207068258	50170207068259
560	559	2237	2239	2802334640	2802334639
2574748	2570211	2267	2269	13232374074172	13232374074171
38612140	38544101	2267	2269	198438946368460	198438946368461
22714160	22676049	2381	2383	128770230019760	128770230019761
596	595	2381	2383	3378819956	3378819955
17036663	17008078	2381	2383	96583585449743	96583585449742
16809771	16783850	2591	2593	112848716268651	112848716268650
21210178	21178283	2657	2659	149736411907522	149736411907523
665	664	2657	2659	4694666585	4694666584
14141227	14119962	2657	2659	99832099049323	99832099049322
10846754	10830625	2687	2689	78313227630626	78313227630625
40482708	40423043	2711	2713	297528512582868	297528512582867
11041232	11024959	2711	2713	81147766449872	81147766449871
683	682	2729	2731	5086602203	5086602202
52209210	52132769	2729	2731	388825011131610	388825011131609
39283344	39227305	2801	2803	308201442969744	308201442969745
701	700	2801	2803	5499766301	5499766300
23571128	23537503	2801	2803	184929665407928	184929665407927
31427937	31383104	2801	2803	246571053955137	246571053955136
4871101	4864860	3119	3121	47386854775261	47386854775260
68783872	68699319	3251	3253	726976811951872	726976811951871
5291818	5285313	3251	3253	55929229733818	55929229733817
5290191	5283688	3251	3253	55912033969191	55912033969192
79364254	79266695	3251	3253	838800879890254	838800879890255
815	814	3257	3259	8645559935	8645559934
53106220	53041059	3257	3259	563353383964780	563353383964779
5687721	5680978	3371	3373	64633219552161	64633219552162
866	865	3461	3463	10373399186	10373399185
890	889	3557	3559	11260501610	11260501609
64188549	64116910	3581	3583	823125773602989	823125773602990
38512771	38469788	3581	3583	493870868197531	493870868197532
896	895	3581	3583	11489910656	11489910655
6744546	6737203	3671	3673	90891127331586	90891127331587
46074703	46027688	3917	3919	706919053836967	706919053836968
980	979	3917	3919	15036031220	15036031219
983	982	3929	3931	15174611303	15174611302
15453744	15438023	3929	3931	238560079731504	238560079731503
30906505	30875064	3929	3931	477104984851705	477104984851704
48071026	48023003	4001	4003	769521032279026	769521032279027
1001	1000	4001	4003	16024009001	16024009000
48073028	48025003	4001	4003	769553080297028	769553080297027
25129997	25105444	4091	4093	420582691321157	420582691321156
25127950	25103399	4091	4093	420548432153950	420548432153951
125643844	125521085	4091	4093	2102810679104164	2102810679104165
8525353	8517096	4127	4129	145204912066537	145204912066536
8523288	8515033	4127	4129	145169740720152	145169740720153
76717852	76643549	4127	4129	1306668351866908	1306668351866909
1055	1054	4217	4219	18761158895	18761158894
89000860	88916499	4217	4219	1582710214456540	1582710214456539
54008086	53957183	4241	4243	971393809450966	971393809450967
1061	1060	4241	4243	19083231941	19083231940
97813539	97725100	4421	4423	1911789192817899	1911789192817900
19561823	19544136	4421	4423	382340544934343	382340544934344
1106	1105	4421	4423	21617036546	21617036545

Smarandache - Radu's problem.
q=p(j), p=p(j+1), d=p-q, N=x*q^s or y*p^s, k solutions to x*q^s - y*p^s = +/-1 will examined.

Parameters for this run: d = 2 , s = 2 , k = 15 .

x	y	q	p	x*q^s	y*p^s
19564035	19546346	4421	4423	382383779007435	382383779007434
97815751	97727310	4421	4423	1911832426890991	1911832426890990
1130	1129	4517	4519	23055716570	23055716569
113814843	113714786	4547	4549	2353145666327187	2353145666327186
10347838	10338741	4547	4549	213943713348142	213943713348141
10345563	10336468	4547	4549	213896677247667	213896677247668
113812568	113712513	4547	4549	2353098630226712	2353098630226713
43262439	43225240	4649	4651	935039789857239	935039789857240
1163	1162	4649	4651	25136152763	25136152762
86528367	86453966	4649	4651	1870154988172767	1870154988172766
1181	1180	4721	4723	26321940221	26321940220
12168478	12158613	4931	4933	295873634303758	295873634303757
36500500	36470909	4931	4933	887500933880500	887500933880501
158172945	158044714	4931	4933	3845937354341145	3845937354341146
86419606	86350053	4967	4969	2132065790970934	2132065790970933
37035199	37005392	4967	4969	913698690661711	913698690661712
125549352	125449153	5009	5011	3150043411177512	3150043411177513
100439231	100359072	5009	5011	2520028441367711	2520028441367712
1253	1252	5009	5011	31437871493	31437871492
75331616	75271495	5009	5011	1890076347300896	1890076347300895
176612447	176471832	5021	5023	4452477674959127	4452477674959128
1256	1255	5021	5023	31664313896	31664313895
117092180	117000379	5099	5101	3044373378656180	3044373378656179
13011376	13001175	5099	5101	338293186736176	338293186736175
13008825	12998626	5099	5101	338226861243825	338226861243826
15979618	15968313	5651	5653	510289941268018	510289941268017
111846018	111766891	5651	5653	3571668481454418	3571668481454419
155004692	154899067	5867	5869	5335523301564788	5335523301564787
51666274	51631067	5867	5869	1778440415416786	1778440415416787
258337240	258161201	5867	5869	8892404132398360	8892404132398361
51880712	51845431	5879	5881	1793134423680392	1793134423680391
17294551	17282790	5879	5881	597745357469191	597745357469190
17291610	17279851	5879	5881	597643708742010	597643708742011
51877771	51842492	5879	5881	1793032774953211	1793032774953212
190222415	190093056	5879	5881	6574589039798015	6574589039798016
224808576	224655697	5879	5881	7769978106009216	7769978106009217
1523	1522	6089	6091	56466627683	56466627682
185502928	185381127	6089	6091	6877691903796688	6877691903796687
1550	1549	6197	6199	59524353950	59524353949
76856752	76807167	6197	6199	2951515167416368	2951515167416367
115284353	115209976	6197	6199	4427242988947577	4427242988947576
1643	1642	6569	6571	70898343323	70898343322
86357725	86305164	6569	6571	3726487909703725	3726487909703724
66555047	66515086	6659	6661	2951202596042207	2951202596042206
1676	1675	6701	6703	75258100076	75258100075
161508670	161413581	6791	6793	7448405321794270	7448405321794269
116589810	116521529	6827	6829	5434009586603490	5434009586603489
69954569	69913600	6827	6829	3260437585177601	3260437585177600
1718	1717	6869	6871	81060670598	81060670597
120719765	120650286	6947	6949	5826033521189885	5826033521189886
127058385	126987104	7127	7129	6453819998221665	6453819998221664
347241454	347051445	7307	7309	18540002175090046	18540002175090045
133551875	133478796	7307	7309	7130634964416875	7130634964416876
80661167	80617174	7331	7333	4335018348995687	4335018348995686
26888278	26873613	7331	7333	1445071808877958	1445071808877957
26884611	26869948	7331	7333	1444874731239771	1444874731239772

Radu's problem, PCW Oct. 95, 951128, Henry Ibstedt

Smarandache - Radu's problem.

q=p(j), p=p(j+1), d=p-q, N=x*q^s or y*p^s. Principal solution to x*q^s - y*p^s = +/-1: x0,y0.
General solutions: x = x0 + t*p^s, y = y0 + t*q^s.

N,N+1	S(N),S(N+1)	d	s	t	q,p
11822936664715339578483018	3225562	42	2	-2	1612781
11822936664715339578483017	3225646				1612823
11157906497858100263738683634	165999	4	3	0	55333
11157906497858100263738683635	166011				55337
17549865213221162413502236227	165999	4	3	-1	55333
17549865213221162413502236226	166011				55337
270329975921205253634707051822848570391314	669764	2	4	0	167441
270329975921205253634707051822848570391313	669772				167443

Radu's problem (RADUpres.UB), 951129, Henry Ibstedt

Factorisations:

11822936664715339578483018 = 2 * 3 * 89 * 193 * 431 * 1612781 ^ 2
11822936664715339578483017 = 509 * 3253 * 1612823 ^ 2

11157906497858100263738683634 = 2 * 7 * 37 ^ 2 * 56671 * 55333 ^ 3
11157906497858100263738683635 = 3 * 5 * 11 * 19 ^ 2 * 16433 * 55337 ^ 3

17549865213221162413502236227 = 3 * 11 ^ 2 * 307 * 12671 * 55333 ^ 3
17549865213221162413502236226 = 2 * 23 * 37 * 71 * 419 * 743 * 55337 ^ 3

270329975921205253634707051822848570391314 = 2 * 3 ^ 3 * 47 * 1289 * 2017 * 119983 * 167441 ^ 4
270329975921205253634707051822848570391313 = 37 * 23117 * 24517 * 38303 * 167443 ^ 4

Radufact, 951129, Henry Ibstedt

Adjacent primes:

Smarandache function values in the above examples: S1 and S2.
P1 and P2 are consecutive primes below and above S1 and S2 respectively. Prime gap = G.

P1	S1	S2	P2	G
3225539	3225562	3225646	3225647	108
165983	165999	166011	166013	30
669763	669764	669772	669787	24

Raduadj, 951130, Henry Ibstedt

Factorisations: Ashbacher - Fibonacci

N = 1185892343342 = 2 * 7 ^ 2 * 47 * 14107 * 18251 ^ 1
N-1 = 1185892343341 = 23 * 1427 * 6011 ^ 2
N-2 = 1185892343340 = 2 ^ 2 * 3 * 5 * 523 * 6067 * 6229 ^ 1
S(N) = 18251 = 18251 ^ 1
S(N-1) = 12022 = 2 * 6011 ^ 1
S(N-2) = 6229 = 6229 ^ 1

N = 1188795217601 = 67 * 83 * 14621 ^ 2
N-1 = 1188795217600 = 2 ^ 6 * 5 ^ 2 * 97 * 587 * 13049 ^ 1
N-2 = 1188795217599 = 3 ^ 2 * 11 * 17 * 181 * 241 * 16193 ^ 1
S(N) = 29242 = 2 * 14621 ^ 1
S(N-1) = 13049 = 13049 ^ 1
S(N-2) = 16193 = 16193 ^ 1

N = 1294530625810 = 2 * 5 * 1669 * 8807 ^ 2
N-1 = 1294530625809 = 3 ^ 2 * 101 * 103 * 2381 * 5807 ^ 1
N-2 = 1294530625808 = 2 ^ 4 * 7 * 19 * 67 * 769 * 11807 ^ 1
S(N) = 17614 = 2 * 8807 ^ 1
S(N-1) = 5807 = 5807 ^ 1
S(N-2) = 11807 = 11807 ^ 1

N = 1517767218627 = 3 * 11 * 107 * 163 * 227 * 11617 ^ 1
N-1 = 1517767218626 = 2 * 73 * 601 * 4159 ^ 2
N-2 = 1517767218625 = 5 ^ 3 * 7 * 17 * 157 * 197 * 3299 ^ 1
S(N) = 11617 = 11617 ^ 1
S(N-1) = 8318 = 2 * 4159 ^ 1
S(N-2) = 3299 = 3299 ^ 1

N = 2677290337914 = 2 * 3 * 37 * 43 * 16747 ^ 2
N-1 = 2677290337913 = 479 * 739 * 2083 * 3631 ^ 1
N-2 = 2677290337912 = 2 ^ 3 * 17 ^ 3 * 2281 * 29863 ^ 1
S(N) = 33494 = 2 * 16747 ^ 1
S(N-1) = 3631 = 3631 ^ 1
S(N-2) = 29863 = 29863 ^ 1

N = 3043063820555 = 5 * 11 * 571 * 6481 * 14951 ^ 1
N-1 = 3043063820554 = 2 * 41 * 997 * 6101 ^ 2
N-2 = 3043063820553 = 3 * 53 * 73 * 283 * 337 * 2749 ^ 1
S(N) = 14951 = 14951 ^ 1
S(N-1) = 12202 = 2 * 6101 ^ 1
S(N-2) = 2749 = 2749 ^ 1

N = 6344309623744 = 2 ^ 6 * 751 * 11489 ^ 2
N-1 = 6344309623743 = 3 ^ 3 * 7 ^ 2 * 13 * 31 * 1597 * 7451 ^ 1
N-2 = 6344309623742 = 2 * 107 * 211 * 9049 * 15527 ^ 1
S(N) = 22978 = 2 * 11489 ^ 1
S(N-1) = 7451 = 7451 ^ 1
S(N-2) = 15527 = 15527 ^ 1

N = 16738688950356 = 2 ^ 2 * 3 * 31 * 193 * 15269 ^ 2
N-1 = 16738688950355 = 5 * 197 * 1399 * 1741 * 6977 ^ 1
N-2 = 16738688950354 = 2 * 7 ^ 2 * 19 * 23 * 53 * 313 * 23561 ^ 1
S(N) = 30538 = 2 * 15269 ^ 1
S(N-1) = 6977 = 6977 ^ 1
S(N-2) = 23561 = 23561 ^ 1

N = 19448047080036 = 2 ^ 2 * 3 ^ 2 * 43 ^ 2 * 17093 ^ 2
N-1 = 19448047080035 = 5 * 7 * 19 * 37 * 61 * 761 * 17027 ^ 1
N-2 = 19448047080034 = 2 * 97 * 1609 * 3631 * 17159 ^ 1
S(N) = 34186 = 2 * 17093 ^ 1
S(N-1) = 17027 = 17027 ^ 1
S(N-2) = 17159 = 17159 ^ 1