Chains of Smarandache Semifields

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Abstract

In this paper we have constructed two chains of semifields. All semifields in the chains are Smarandache semifields. Every member of the chain is an extension semifield of Ordered equilateral Integral triangles with Zero triangle such that it is a semivector space over $R^I_{e\Delta}$

Key words: Ordered integral triangle, Zero triangle, Equilateral integral triangle Smarandache semiring, Smarandache semifield, Smarandache semivector space.

1. Introduction

Recently there has been an increasing interest in the study of Smarandache semirings and associated structures. We propose to construct two chains of infinite Smarandache semifields by defining Equilateral triangles.

An ordered integral triangle as defined in [1] is a triplet (a,b,c) where (a,b,c) are positive integers satisfying $a \geq b \geq c, b+c > a$.

Let us consider a set $R^I_\Delta = \{(a,b,c)/a,b,c \in I^+, a \geq b \geq c, b+c > a\} \cup \{0\}$ where $0 = (0,0,0)$. We shall call 0 as a Zero triangle.

We define the sum + and the product $\cdot$ of triangles as

$$(a_1,b_1,c_1)+(a_2,b_2,c_2) = (a_1+a_2, b_1+b_2, c_1+c_2) \tag{1}$$

and

$$(a_1,b_1,c_1)(a_2,b_2,c_2) = (a,b,c) \tag{2}$$

where

$a = \Sigma a_1a_2 - (b_1c_2 + c_1b_2)$
$b = \Sigma a_1a_2 - (a_1c_2 + c_1a_2)$
$c = \Sigma a_1a_2 - (a_1b_2 + b_1a_2)$

where

$\Sigma a_1a_2 = a_1a_2 + b_1b_2 + c_1c_2$

It is not difficult to see that;

i) $(R^I_\Delta, +)$ is a commutative semigroup with identity $(0,0,0)$.

ii) $(R^I_\Delta, \cdot)$ is a semigroup (in fact a monoid)

iii) Multiplication distributes over addition.

iv) $(1,1,1)$ is the multiplicative identity.

v) Commutativity holds for multiplication.

Thus, $(R^I_\Delta, +, \cdot)$ is a commutative semiring.

Also,

vi) $(a_1,b_1,c_1)+(a_2,b_2,c_2)=(0,0,0) \Rightarrow a_1 = a_2 = b_1 = b_2 = c_1 = c_2 = 0$

Thus, $(R^I_\Delta, +, \cdot)$ is a strict commutative semiring with unity (1,1,1). See [2].

vii) Let $x.y = 0$ where $x, y \in R^I_\Delta$. Then $x = 0$ or $y = 0$

We conclude that

- **A1** $(R^I_\Delta, +, \cdot)$ is a semifield.

A triplet (a,b,c) where (a,b,c) are positive rational numbers satisfying $a \geq b \geq c, b+c > a$ is called an ordered rational triangle.

Consider the following set

$R^Q_\Delta = \{(a,b,c)/a,b,c \in Q^+, a \geq b \geq c, b+c > a\} \cup \{0\}$ where 0 is a zero triangle. Then, it can be verified that $(R^Q_\Delta, +, .)$ is a strict commutative semiring with unity $(1,1,1)$.

Also, R^Q_Δ is without zero divisors.

Thus,

- **A2** $(R^Q_\Delta, +, .)$ is a semifield.

A triplet (a,b,c) where (a,b,c) are positive real numbers satisfying $a \geq b \geq c, b+c > a$ is called an ordered real triangle.

Consider the set

$R^R_\Delta = \{(a,b,c)/a,b,c \in R^+, a \geq b \geq c, b+c > a\} \cup \{0\}$ where 0 is a zero triangle. Then, it can be verified that $(R^R_\Delta, +, .)$ is a strict commutative semiring with unity $(1,1,1)$.

Also, R^R_Δ is without zero divisors.

Thus,

- **A3** $(R^R_\Delta, +, .)$ is a semifield.

Consider the set,

$R_\Delta = \{(a,b,c)/a,b,c \in R^+, a \geq b \geq c\} \cup \{0\}$ where 0 is a zero triangle. Then, it can be verified that $(R_\Delta, +, .)$ is a strict commutative semiring with unity $(1,1,1)$.

Also, R_Δ is without zero divisors.

Thus,

- **A4** $(R_\Delta, +, .)$ is a semifield.

Result: From **A1, A2, A3** and **A4** we obtain a chain of semifields as

- **(A)** $R_\Delta \supset R^R_\Delta \supset R^Q_\Delta \supset R^I_\Delta \supset R^I_{e\Delta}$

Where $R^I_{e\Delta}$ is a real equilateral triangle defined in (A_7)

Ordered equilateral triangles lead us to a new chain of semifields. A triplet (a,a,a) where $a \in R^+$ is called an ordered equilateral real triangle.

Consider the following set
$R^{R}_{e_\Delta} = \{(a,a,a)/a \in R^{+}\} \cup \{0\}$ where $0 = (0,0,0)$.

Then, $(R^{R}_{e_\Delta}, +, .)$ is a strict commutative semiring with unity $(1,1,1)$ and is without zero divisors.
Thus,

- **A5** $(R^{R}_{e_\Delta}, +, .)$ is a semifield.

Similarly, triplet (a,a,a) where $a \in Q^{+}$ is called an ordered equilateral rational triangle.
Consider the following set
$R^{Q}_{e_\Delta} = \{(a,a,a)/a \in Q^{+}\} \cup \{0\}$ where $0 = (0,0,0)$.

Then, $(R^{Q}_{e_\Delta}, +, .)$ is a strict commutative semiring with unity $(1,1,1)$ and is without zero divisors.
Thus,

- **A6** $(R^{Q}_{e_\Delta}, +, .)$ is a semifield.

Similarly, a triplet (a,a,a) where $a \in I^{+}$ is called an ordered equilateral Integral triangle.
Consider the following set
$R^{I}_{e_\Delta} = \{(a,a,a)/a \in I^{+}\} \cup \{0\}$ where $0 = (0,0,0)$.

Then, $(R^{I}_{e_\Delta}, +, .)$ is a strict commutative semiring with unity $(1,1,1)$ and is without zero divisors.
Thus,

- **A7** $(R^{I}_{e_\Delta}, +, .)$ is a semifield.

Result: From **A1, A2, A5, A6** and **A7** we obtain a chain of semifields as

- **(B)** $R_\Delta \supset R^{R}_{\Delta} \supset R^{R}_{e_\Delta} \supset R^{Q}_{e_\Delta} \supset R^{I}_{e_\Delta}$

2. Some Observations

1. Members of ordered equilateral triangles act as scalar multiples for every semifield in the chain.
   E.g. let $(a,a,a) \in R^{R}_{e_\Delta}$ and $(x,y,z) \in R_\Delta$. Then
   $(a,a,a)(x,y,z) = (ax, ay, az) = a(x,y,z)$.
   Thus, multiplication by $(a,a,a) \in R^{R}_{e_\Delta}$ amounts to component wise multiplication. Hence, we call (a,a,a) a magnifier.
2. There is a chain of magnifiers
   $R^{R}_{e_\Delta} \supset R^{Q}_{e_\Delta} \supset R^{I}_{e_\Delta}$
3. Every semifield in the chains **(A)** and **(B)** is of characteristic 0.
4. Every semiring except $R^{I}_{e_\Delta}$ in chains **(A)** and **(B)** is a Smarandache semiring.
5. Every semifield in the chains **(A)** and **(B)** is an extension semifield of $R^{I}_{e_\Delta}$
6. $R^{I}_{e_\Delta}$ is a prime semifield as it has no proper subsemifield.
7. All the members in the chains are semivector spaces over the semifield $R^{I}_{e_\Delta}$
8. All the semifields in the chains **(A)** and **(B)** are Smarandache semi fields because they contain A as a proper subset where A is
   a. $A = \{(0,0,0), (p,p,p), (2p,2p,2p), \ldots (rp,rp,rp)\}$ which is isomorphic

with $A' = \{0, p, 2p \ldots rp \ldots\}$ which is a k-semi algebra [2].

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