

**DEPASCALISATION OF SMARANDACHE PASCAL DERIVED SEQUENCES
AND BACKWARD EXTENDED FIBONACCI SEQUENCE**

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Given a sequence S_b (called the base sequence).

$$b_1, b_2, b_3, b_4, \dots$$

Then the Smarandache Pascal derived Sequence S_d

$d_1, d_2, d_3, d_4, \dots$ is defined as follows: **Ref [1]**

$$d_1 = b_1$$

$$d_2 = b_1 + b_2$$

$$d_3 = b_1 + 2b_2 + b_3$$

$$d_4 = b_1 + 3b_2 + 3b_3 + b_4$$

...

$$d_{n+1} = \sum_{k=0}^n {}^n C_k \cdot b_{k+1}$$

Now Given S_d the task ahead is to find out the base sequence S_b . We call the process of extracting the base sequence from the Pascal derived sequence as **Depascalisation**. The interesting observation is that this again involves the Pascal's triangle though with a difference.

We see that

$$b_1 = d_1$$

$$b_2 = -d_1 + d_2$$

$$b_3 = d_1 - 2d_2 + d_3$$

$$b_4 = -d_1 + 3d_2 - 3d_3 + d_4$$

...

which suggests the possibility of

$$b_{n+1} = \sum_{k=0}^n (-1)^{n+k} \cdot {}^n C_k \cdot d_{k+1}$$

This can be established by induction.

We shall see that the depascalised sequences also exhibit interesting patterns.

To begin with we define The **Backward Extended Fibonacci Sequence (BEFS)** as Follows:

The Fibonacci sequence is

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, ...

In which $T_1 = 1$, $T_2 = 1$, and $T_{n-2} = T_n - T_{n-1}$, $n > 2$ (A)

Now If we allow n to take values $0, -1, -2, \dots$ also, we get

$T_0 = T_2 - T_1 = 0$, $T_{-1} = T_1 - T_0 = 1$, $T_{-2} = T_0 - T_{-1} = -1$, etc. and we get the Fibonacci sequence extended backwards as follows { T_r is the r^{th} term }

... T_{-6} , T_{-5} , T_{-4} , T_{-3} , T_{-2} , T_{-1} , T_0 , T_1 , T_2 , T_3 , T_4 , T_5 , T_6 , T_7 , T_8 , T_9 , ...

... -8, 5, -3, 2, -1, 1, 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, ...

1. Depascalisation of the Fibonacci sequence:

The Fibonacci sequence is

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, ...

The corresponding depascalised sequence $S_{d(-1)}$ comes out to be

$S_{d(-1)} \rightarrow 1, 0, 1, -1, 2, -3, 5, -8, \dots$

It can be noticed that, The resulting sequence is nothing but the **BEFS** rotated by 180° about T_1 . and then the terms to the left of T_1 omitted. { This has been generalised in the Proposition 2 below. }

It is not over here. If we further depascalise the above sequence we get the following sequence $S_{d(-2)}$ as

1, -1, 2, -5, 13, -34, 89, -233

This can be obtained alternately from the Fibonacci Sequence by:

- (a) Removing even numbered terms.
- (b) Multiplying alternate terms with (-1) in the thus obtained sequence.

Propositions:

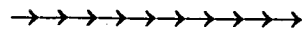
Following two propositions are conjectured on Pascalisation and Depascalisation of Fibonacci Sequence.

(1) If the first r terms of the Fibonacci Sequence are removed and the remaining sequence is Pascalised, the resulting Derived Sequence is $F_{2r+2}, F_{2r+4}, F_{2r+6}, F_{2r+8}, \dots$ where F_r is the r^{th} term of the Fibonacci Sequence.

(2) In the FEBS If we take T_r as the first term and Depascalise the Right side of it then we get the resulting sequence as the left side of it (looking rightwards) with T_r as the first term.

As an example let $r = 7, T_7 = 13$

$\dots T_{-6} T_{-5} T_{-4} T_{-3} T_{-2} T_{-1} T_0 T_1 T_2 T_3 T_4 T_5 T_6 \underline{T_7} T_8 T_9 \dots$
 $\dots -8, 5, -3, 2, -1, 1, 0, 1, 1, 2, 3, 5, 8, \underline{13}, 21, 34, 55, 89, \dots$



depascalisation

The Depascalised sequence is

13, 8, 5, 3, 2, 1, 1, 0, 1, -1, 2, -3, 5, -8 . . .

which is obtained by rotating the FEBS around 13 (T_7) by 180° and then removing the terms on the left side of 13.

One can explore for more fascinating results.

References:

[1] "Amarnath Murthy", 'Smarandache Pascal derived Sequences', SNJ, 2000.