Diverse Algorithms To Obtain Prime numbers Based on the Prime Function of Smarandache

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Abstract: In this article one gives seven formulas, six of the author S. M. Ruiz, and one of Azmy Ariff. One also gives their corresponding algorithms programmed in MATHEMATICA.

In the first four formulas all the divisions are integer divisions.

FORMULA 1: Formula to obtain the nth prime [1], [3]:

$$p(n) = 1 + \sum_{k=1}^{2(\lfloor n \log n \rfloor + 1)} \left[1 - \left[\sum_{j=2}^{k} \left[1 + \left(2 + 2.\sum_{s=1}^{\sqrt{j}} ((j-1)/s - j/s) \right) \Big/ j \right] \right] \Big/ n \right]$$

ALGORITHM 1: (G is the Smarandache Prime Function in all Algorithms)

```
DD[i_]:=Sum[Quotient[i,k]-Quotient[(i-1),k],{k,1,Floor[Sqrt[i]]}]
G[n_]:=Sum[1+Quotient[(2-2*DD[j]),j],{j,2,n}]
P[n_]:=1+Sum[1-Quotient[G[k],n],{k,1,2*(Floor[n*Log[n]]+1)}]
Do[Print[P[n]," ",Prime[n]],{n,1,50}]
```

FORMULA 2: Formula to obtain the next prime [2], [3].

$$nxt(p) = 1 + p + \sum_{k=p+1}^{2p} \prod_{j=p+1}^{k} \left[- \left(\left(2 + 2.\sum_{s=1}^{\sqrt{j}} ((j-1)/s - j/s) \right) \Big/ j \right) \right]$$

ALGORITHM 2:

```
p=Input["Input a positive integer number:"]
DD[i_]:=Sum[Quotient[i,j]-Quotient[(i-1),j], {j,1,Floor[Sqrt[i]]}]
G[i_]:=-Quotient[(2-2*DD[i]),i]
F[m_]:=Product[G[i],{i,p+1,m}]
S[n_]:=Sum[F[m],{m,n+1,2*n}]
Print["nxt(",p,")=",p+1+S[p]]
```

FORMULA 3: Formula to obtain the next prime in an arithmetic progression a+dn [4]:

$$nxt(a,d)(p) = p + d + d \cdot \sum_{k=1+(p-a)/d}^{M} \prod_{j=1+(p-a)/d}^{k} \left[-\left(\left(2 + 2\sum_{s=1}^{\sqrt{a+jd}} ((a+jd-1)/s - (a+jd)/s) \right) / (a+jd) \right) \right]$$

ALGORITHM 3: Example for the arithmetic progression 5+4n

```
a=5
5
dd=4
4
M=20
20
p=5
5
DD[i_]:=Sum[Quotient[(a+i*dd),j]-Quotient[a+i*dd-1,j],
{j,1,Sqrt[a+i*dd]}]
G[i_]:=-Quotient[(2-2*DD[i]),(a+i*dd)]
F[m_]:=Product[G[i],{i,(p-a)/dd+1,m}]
S[n_]:=Sum[F[m],{m,(p-a)/dd+1,M}]
While[p<a+(M-1)*dd+1,Print["nxt(",p,")=",p+dd+dd*S[p]];
p=p+dd+dd*S[p]]
```

```
nxt(5)=13
nxt(13)=17
nxt(17)=29
nxt(29)=37
nxt(37)=41
nxt(41)=53
nxt(53)=61
nxt(61)=73
nxt(73)=89
```

FORMULA 4: Formula to obtain the next prime in all positive increasing integer sequence $\{a_n\}_{n\geq 1} = \{f(n)\}_{n\geq 1}$.

$$NXT_f(p) = f\left[f^{-1}(p) + 1 + \sum_{k \geq f^{-1}(p)+1} \prod_{j=f^{-1}(p)+1}^{k} G(f(j)) \right]$$

(G is the same of the previous algorithm 2)

ALGORITHM 4:

Example 1: For $a_n = n^3 + 4$

```
M=40
40
f[n_]:=n^3+4
f 1[p_]:=(p-4)^(1/3)
G[x_]:=-Quotient[(2+2*Sum[Quotient[(x-1) , s]-Quotient[x , s], {s, 1, Sqrt[x]}]), x]
NXT[p_]:=f[f 1[p]+1+Sum[Product[G[f[j]],{j , f 1[p]+1 , k}], {k , f 1[p]+1,M}]]
p=f[1]
5
While[p < f[M], (Print[ NXT[p]," ", PrimeQ[NXT[p]]]; p = NXT[p])]
```

31 True
347 True
733 True
6863 True
15629 True
19687 True

(It is necessary that f(M) > NXT(p) so that the result is correct.)

Example 2: For $a_n = n^2 + 1$

```
M=125
125
f[n_]:=n^2+1
f 1[p_]:=Sqrt[p-1]
G[x_]:=-Quotient[(2+2*Sum[Quotient[(x-1) , s]-Quotient[x , s], {s, 1, Sqrt[x]}]), x]
NXT[p_]:=f[f 1[p]+1+Sum[Product[G[f[j]],{j , f 1[p]+1 , k}], {k , f 1[p]+1,M}]]
p=f[1]
2
While[p < f[M], (Print[ NXT[p]," ", PrimeQ[NXT[p]]]; p = NXT[p])]
```

5 True
17 True
37 True
101 True
197 True
257 True
401 True
577 True
677 True
1297 True
1601 True

FORMULA 5: Algorithm to obtain the prime numbers based on Newton's method applied to the function gamma [3].

```
(*NEWTON'S METHOD APPLIED TO THE CALCULATION OF PRIME
NUMBERS  *)

ndiez[s_]:=N[s,10]
$Post=ndiez
ndiez
P={}
{}
er=10.^(-5)
0.00001
B[x_,i_,j_]:=(x-1.)/P[[i]]^j
EB[x_,i_,j_]:=Floor[B[x,i,j]+er]
LL[x_,i_]:=Log[P[[i]],x-1.]
EE[x_,i_]:=Floor[LL[x,i]+er]
S[x_,i_]:=Sum[EB[x,i,j],{j,1,EE[x,i]}]
F[x_,n_]:=Gamma[x]-Product[(P[[i]])^S[x,i],{i,1,n-1}]
xx=0.
0.
Do[{xx=xx+25.,
Do[xx=xx-F[xx,i]/ (Gamma[xx]*PolyGamma[0.,xx])
,{175}],P=Join[P,{xx}],Print[xx," ",Prime[i]]},{i,1,50}]
```

FORMULA 6: Formula to obtain twin primes:

For odd $n > 7$, the pair $(n, n+2)$ of integers are twin primes if and only if

$$\sum_{i\ \mathrm{odd}}^{j}\left(\left\lfloor\frac{n+2}{i}\right\rfloor-\left\lfloor\frac{n+1}{i}\right\rfloor+\left\lfloor\frac{n}{i}\right\rfloor-\left\lfloor\frac{n-1}{i}\right\rfloor\right)=2$$

where the summation is over odd values of i through $j = \left\lfloor\frac{n}{3}\right\rfloor$.

AGORITHM 6: Algorithm to check if a given number is part of a couple of twin primes (Ruiz-Ariff):

```
In[1]:=   n=2000081; If[Sum[Floor[(n+2)/i]-Floor[(n+1)/i]
            +Floor[n/i]-Floor[(n-1)/i],{i,1,Floor[n/3],2}]
            ==2, "True", "False"]

Out[1]=   True
```

FORMULA 7: (Azmy Ariff): If $a \geq 0$, $e_0 = 0$ and $\{e_1, e_2, \ldots, e_k\}$ is an admissible set of positive integers in the open interval $(0, n-2)$, then $(n, n+e_1, n+e_2, \ldots, n+e_k)$ is a sequence of primes if and only if

$$\sum_{i=1}^{n} i^a\left(\sum_{j=0}^{k}\left\lfloor\frac{n+e_j}{i}\right\rfloor\right)=1+k+n^a+\sum_{i=1}^{n} i^a\left(\sum_{j=0}^{k}\left\lfloor\frac{n+e_j-1}{i}\right\rfloor\right)$$

ALGORITHM 7:

The following example is a non-optimum implementation with $a = 3$ to search for prime quadruplets *(n, n+2, n+6, n+8)* below *10000.*

```
In[2]:=   a=3; n=10000; e={0, 2, 6, 8};
          Do[If[Sum[i^a Floor[(j + e[[k]])/i], {k, Length[e]},{i,
          j}]
            == Length[e] + j^a + Sum[i^a Floor[(j + e[[k]] - 1)/i],
            {k, Length[e]},{i, j}], Print[Table[j + e[[k]],
            {k, Length[e]}]]],{j, n}]

          {5, 7, 11, 13}
          {11, 13, 17, 19}
          {101, 103, 107, 109}
          {191, 193, 197, 199}
          {821, 823, 827, 829}
          {1481, 1483, 1487, 1489}
          {1871, 1873, 1877, 1879}
          {2081, 2083, 2087, 2089}
          {3251, 3253, 3257, 3259}
          {3461, 3463, 3467, 3469}
          {5651, 5653, 5657, 5659}
          {9431, 9433, 9437, 9439}
```

REFERENCES:

[1] ***S M Ruiz***. The General Term of the Prime Number Sequence and the Smarandache prime function. SMARANDACHE NOTIONS JOURNAL vol 11 n° 1-2-3 Spring 2000 page 59

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[3] ***Carlos Rivera***. The Prime Puzzles & Problems Connection. Problem 38 and 39. www.primepuzzles.net

[4] ***S M Ruiz***. Formula to obtain the next prime in an arithmetic progression. http://www.gallup.unm.edu/~Smarandache/SMRuiz-nextprime.pdf