## EXAMPLES OF SMARANDACHE MAGIC SQUARES

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For $\mathrm{n} \geq 2$, let A be a set of $\mathrm{n}^{2}$ elements, and l a n -ary law defined on A .
As a generalization of the XVI-th - XVII-th centuries magic squares, we present the Smarandache magic square of order $n$, which is: 2 square array of rows of elements of A arranged so that the law 1 applied to each horizontal and vertical row and diagonal give the same result.

If A is an arithmetical progression and 1 the addition of n numbers, then many magic squares have been found. Look at Durer's 1514 engraving "Melancholia" 's one:

| 16 | 3 | 2 | 13 |
| :---: | ---: | ---: | ---: |
| 5 | 10 | 11 | 8 |
| 9 | 6 | 7 | 12 |
| 4 | 15 | 14 | 1 |

1. Can you find a such magic square of order at least 3 or 4 , when $A$ is a set of prime numbers and 1 the addition?
2. Same question when $A$ is a set of square numbers, or cube numbers, or special numbers [for example: Fibonacci or Lucas numbers, triangular numbers, Smarandache quotients (i.e. $q(m)$ is the smallest $k$ such that $m k$ is a factorial), etc.].

A similar definition for the Smarandache magic cube of order $n$, where the elements of $A$ are arranged in the form of a cube of lenth $n$ :
a. either each element inside of a unitary cube (that the initial cube is divided in)
b. either each element on a surface of a unitary cube
c. either each element on a vertex of a unitary cube.
3. Study similar questions for this case, which is much more complex.

An interesting law may be $l\left(a_{1}, a_{2}, \ldots, a_{n}\right)=a_{1}+a_{2}-a_{3}+a_{4}-a_{5}+\ldots$
Now some examples of Smarandache Magic Squares: if A is a set of PRIME NUMBERS and 1 is the operation of addition, for orders at least 3 or 4 .
Some examples, with the constant in brackets, elements drawn from the first hundred PRIME NUMBERS are :
$\left|\begin{array}{ccc}83 & 89 & 41 \\ 29 & 71 & 113 \\ 101 & 53 & 59 \\ (213)\end{array}\right| \quad\left|\begin{array}{ccc}101 & 491 & 251 \\ 431 & 281 & 131 \\ 311 & 71 & 461 \\ & (843)\end{array}\right| \quad\left|\begin{array}{ccc}71 & 461 & 311 \\ 521 & 281 & 41 \\ 251 & 101 & 491 \\ (843)\end{array}\right| \quad\left|\begin{array}{ccc}113 & 149 & 257 \\ 317 & 173 & 29 \\ 89 & 197 & 233 \\ & (519)\end{array}\right|$

| 97 | 907 | 557 | 397 | 197 |
| :---: | :---: | :---: | :---: | :---: |
| 367 | 167 | 67 | 877 | 677 |
| 997 | 647 | 337 | 137 | 37 |
| 107 | 157 | 967 | 617 | 307 |
| 587 | 277 | 227 | 127 | 937 |$|$

Now recall the year A.D. 1987 and consider the following .. all elements are primes congruent to seven modulo ten ...

| 967 | 1987 | 2017 |
| :---: | :---: | :---: |
| 2707 | 1657 | 607 |
| 1297 | 1327 | 2347 |$|$ (4971) \(\left|\begin{array}{cccc|}\hline 1987 \& 9907 \& 11677 \& 5237 <br>

4877 \& 12037 \& 9547 \& 2347 <br>
10627 \& 2707 \& 4517 \& 10957 <br>
11317 \& 4157 \& 3067 \& 10267\end{array}\right|\)

| 7 | 2707 | 5237 | 937 | 947 |
| :---: | :---: | :---: | :---: | :---: |
| 4157 | 1297 | 227 | 1087 | 3067 |
| 1307 | 1447 | $\underline{1987}$ | 4517 | 577 |
| 2347 | 3797 | 1657 | 1667 | 367 |
| 2017 | 587 | 727 | 1627 | 4877 |$|$

What about the years 1993, 1997, \& 1999 ?
In Personal Computer World, May 1991, page 288, I examine:
A multiplication magic square such as:
$\left|\begin{array}{ccc}18 & 1 & 12 \\ 4 & 6 & 9 \\ 3 & 36 & 2\end{array}\right|$
with constant 216 obtained by multiplication of the elements in any row/column/principal diagonal.

A geometric magic square is obtained using elements which are a given base raised to the powers of the corresponding elements of a magic square .. it is clearly a multiplication magic square.
e.g. from
$\left|\begin{array}{lll}8 & 1 & 6 \\ 3 & 5 & 7 \\ 4 & 9 & 2\end{array}\right| \quad \mathrm{C}=15$
and base 2 obtain
$\left|\begin{array}{ccc}256 & 2 & 64 \\ 8 & 32 & 128 \\ 16 & 512 & 4\end{array}\right| \quad$ where $\mathrm{M}=2^{15}=32768$

Note that Henry Nelson of Califormia has found an order three magic square consisting of consecutive ten-digit prime numbers. But "How did he do that" ???

## A particular case:

TALISMAN MAGIC SQUARES are a relatively new concept, contain the integers from 1 to $\mathrm{n}^{2}$ in such a way that the difference between any integer and its neighbours (either row-, column- or diagonal-wise) is greater than some given constant, D say.
e.g.
$\left|\begin{array}{cccc}5 & 15 & 9 & 12 \\ 10 & 1 & 6 & 3 \\ 13 & 16 & 11 & 14 \\ 2 & 8 & 4 & 7\end{array}\right|$ illustrates $D=2$.

## References

1. Smarandache, Florentin, "Properties of Numbers", University of Craiova Archives, 1975;
(see also Arizona State University, Special Collections, Tempe, AZ, USA)
2. Mudge, Mike, England, Letter to R. Muller, Arizona, August 8, 1995.

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