EXPLORING SOME NEW IDEAS ON SMARANDACHE TYPE SETS, FUNCTIONS AND SEQUENCES

(Amarnath Murthy ,Superintending Engineer(E&T),Well Logging Section ,Oil And Natural Gas Corporation Ltd., Chandkheda, Ahmedabad, India-380005)

ABSTRACT: In this article I have defined a number of SMARANDACHE type sets ,sequences which I found very interesting. The problems and conjectures proposed would give food for thought and would pave ways for more work in this field.

(1)SMARANDACHE PATTERNED PERFECT SQUARE SEQUENCES.

Consider following sequence of numbers

13, 133, 1333, 13333, . . .--------(1)

The sequence formed by the square of the numbers is

169, 17689, 1776889, 177768889, . . . -------(2)

We define (1) as the root sequence

It is evident that the above sequence (2) follows a pattern.

i.e. The square of one followed by n three's is ,one followed by (n-1) seven's, followed by a six, followed by (n-1) eight's followed by a nine.

There are a finite number of such patterned perfect square sequences. Here we list the root sequences.

(I) 13, 133, 1333, 13333, . . .
(2) 16, 166, 1666, 16666, . . .
(3) 19, 199, 1999, 19999, . . .
(4) 23, 233, 2333, 23333, . . .
(5) 26, 266, 2666, 26666, . . .
(6) 29, 299, 2999, 29999, . . .

on similar lines we have the root sequences with the first terms as (7) 33, (8) 36, (9) 39,(10) 43,(11) 46,(12) 49, (13) 53,(14) 66, (15) 73, (16) 93,(17) 96,(18) 99.

There are some root sequences which start with a three digit number , like

799, 7999, 79999, . . .

The patterned perfect square sequence is

638401 , 63984001 , 6399840001 , 639998400001, . . .

(the nine's and zero's inserted are shown in darker print to identify the pattern.)

Open Problem : (1) Are there any patterned perfect cube sequences ?
(2) Are there any patterned higher perfect power sequences ?

(2) SMARANDACHE BREAKUP SQUARE SEQUENCES

4, 9, 284, 61209, . . .

the terms are such that we have
$4 = 2^2$
$49 = 7^2$
$49284 = 222^2$
$4928461209 = 70203^2$
T_n = the smallest number whose digits when placed adjacent to other terms of the sequence in the following manner

$T_1T_2 \ldots T_{n-1}T_n$ yields a perfect square.

$$\operatorname{Lt}_{n\to\infty} \frac{(T_1T_2\ldots T_{n-1}T_n)^{1/2}}{10^k}$$ where k is the number of digits in the numerator for this kind of sequence can be analyzed. As it is evident that for large values of n the value of $\frac{(T_1T_2\ldots T_{n-1}T_n)^{1/2}}{10^k}$ is

close to either 2.22.... or to 7.0203. . .

(3) SMARANDACHE BREAKUP CUBE SEQUENCES
On similar lines SMARANDACHE BREAKUP CUBE SEQUENCES can be defined. The same idea can be extended to define SMARANDACHE BREAKUP PERFECT POWER SEQUENCES

(4) SMARANDACHE BREAKUP INCREMENTED PERFECT POWER SEQUENCES

1, 6, 6375,

$1 = 1^1$, $16 = 4^2$, $166375 = 55^3$, etc.

T_n = the smallest number whose digits when placed adjacent to other terms of the sequence in the following manner

$T_1T_2 \ldots T_{n-1}T_n$ yields a perfect n^{th} power.

(5) SMARANDACHE BREAKUP PRIME SEQUENCE

2 , 3, 3, . . .

2, 23, 233 etc. are primes.

$T_1T_2...T_{n-1}T_n$ is a prime

(6) SMARANDACHE SYMMETRIC PERFECT SQUARE SEQUENCE

1, 4, 9, 121, 484, 14641, . . .

(7) SMARANDACHE SYMMETRIC PERFECT CUBE SEQUENCE

1, 8, 343, 1331 . . .

This can be extended to define

(8) SMARANDACHE SYMMETRIC PERFECT POWER SEQUENCE

(9) SMARANDACHE DIVISIBLE BY n SEQUENCE

1, 2, 3, 2, 5, 2, 5, 6, 1, 0, 8, 4. . .

the terms are the smallest numbers such that n divides $T_1T_2...T_{n-1}T_n$ the terms placed adjacent digit wise.
e.g. 1 divides 1 , 2 divides 12 , 3 divides 123 , 4 divides 1232 , 5 divides 12325 , 6 divides 123252 , 7 divides 1232535 , 8 divides 12325256 9 divides 123252561 , 10 divides 1232525610 , 11 divides 12325256108 ,
12 divides 123252561084 , etc.

(9) SMARANDACHE SEQUENCE OF NUMBERS WITH SUM OF THE DIGIT'S = PRIME

2,3,5,7,11,12,14,16,20,21,23,25,29, . . .

(10) SMARANDACHE SEQUENCE OF PRIMES WITH SUM OF THE DIGIT'S = PRIME

2,3,5,7,11,23, 29, 41,43, 47, 61, 67, 83, 89,. . .

(11) SMARANDACHE SEQUENCE OF PRIMES SUCH THAT 2P + 1 IS ALSO A PRIME

2, 3, 5, 11, 23, 29, 41, 53, . . .

(11) SMARANDACHE SEQUENCE OF PRIMES SUCH THAT 2P - 1 IS ALSO A PRIME

3, 7,19, 31, . . .

(13) SMARANDACHE SEQUENCE OF PRIMES SUCH THAT $P^2 + 2$ IS ALSO A PRIME

3, 17, . . .

(14) SMARANDACHE SEQUENCE OF SMALLEST PRIME WHICH DIFFER BY 2n FROM ITS PREDECESSOR

5, 17 , 29, 97, . . .

(T_1 = 5 = 3+ 2, T_2 = 17 = 13 + 4 , T_3 = 29 = 23 + 6 ,T_4 = 97 = 89 + 8 etc.)

(15)) SMARANDACHE SEQUENCE OF SMALLEST PRIME p FOR WHICH p + 2r IS A PRIME

3 , 13, 23, 89 ,. . .

3 + 2 X 1 = 5 is a prime , 13 + 2 X 2 = 17 is a prime , 23 + 2 X 3 = 29 , 89 + 2 X 4 = 97 is a prime etc.

(16) SMARANDACHE SEQUENCE OF THE SMALLEST NUMBER WHOSE SUM OF DIGITS IS n .

1, 2, 3, 4, 5, 6, 7, 8, 9, 19, 29, 39, 49, 59, 69, 79, 89, 99, 199, 299, 399, 499, 599, 699 , . . .

It is a sequence of the only numbers which have the following property.

$$N + 1 = \prod_{r=1}^{k} (a_r +1)$$

PROOF:

Let N be a k-digit number with a_r the r^{th} digit (a_1 = LSB) such that

$$N + 1 = \prod_{r=1}^{k} (a_r +1) \qquad \text{---------------(1)}$$

to find all such k -digit numbers.

The largest k-digit number is $N = 10^k - 1$, with all the digits as 9 . It can be verified that this is a solution. Are there other solutions ?

Let the m^{th} digit be changed from 9 to a_m ($a_m < 9$) . Then the right member of (1) becomes $10^{(k-1)}(a_m + 1)$ .This amounts to the reduction in value by $10^{(k-1)}(9-a_m)$. The value of the k-digit number N goes down by $10^{(m-1)}(9-a_m)$. For the new number to be a solution these two values have to be equal which occurs only at m = k. This gives 8 more solutions. In all there are 9 solutions given by $a.10^k - 1$, for a = 1 to 9.
e.g. for k = 3 the solutions are

199 , 299 , 399 , 499, ,599 , 699 , 799 , 899 , 999 ,.

Are there infinitely many primes in this sequence.

(17) SMARANDACHE SEQUENCE OF NUMBERS SUCH THAT THE SUM OF THE DIGITS DIVIDES n

1,3,6,9,10,12,18,20,21,24,27,30,36,40,42,45,48,50,54,60,63,72,80,81,84,90, 100,102,108,110,112,114,120,126,132,133,135,140,144,150, . . .

(18)) SMARANDACHE SEQUENCE OF NUMBERS SUCH THAT EACH DIGIT DIVIDES n

1,2,3,4,5,6,7,8,9,10,11,12,15,20,22,24,30,33,36,40,44,50,55,60,66, . . .

(19)) SMARANDACHE POWER STACK SEQUENCE FOR n

SPSS(2)

1, 12, 124, 1248, 124816, 12481632 ,
The n^{th} term is obtained by placing the digits of the powers of 2 starting from 2^0 to 2^n from left to right.

SPSS(3)
1, 13, 139, 13927, 1392781, 1392781243, . . .

Problem : If n is an odd number not divisible by 5 how many of the above sequence SPSS(n) are prime ? (It is evident that n divides T_n iff $n \equiv 0 \mod (5)$).

(20) SMARANDACHE SELF POWER STACK SEQUENCE

SSPSS

1, 14, 1427, 1427256, 14272563125, 142725631257776, . . .

$T_r = T_{r-1}a_1a_2a_3...a_k$ where , $r^r = a_1a_2a_3...a_k$
(the digits are placed adjacent).

How many terms of the above sequence, SSPSS are prime ?

(21) SMARANDACHE PERFECT SQUARE COUNT PARTITION SEQUENCE

the rth term of SPSCPS (n) Is defined as

$T_r = O\{ x \mid x \text{ is a perfect square }, nr+1 \le x \le nr + n \}$

O stands for the order of the set

e.g. for n = 12 SPSCPS(12) is

3, 1, 2, 0, 1, 0 , 1 , 0, 0, 1, 0 , 1, 0, 1, 1 , 0, 0, 1, 0, 1

(number of perfect squares ≤ 12 is 3 (1, 4, and 9), number of perfect squares between 13 to 24 is 1 (only 16) etc.)

(21) SMARANDACHE PERFECT POWER COUNT PARTITION SEQUENCE

The r^{th} term of **SPPCPS (n,k)** is defined as

$T_r = O\{ x \mid x \text{ is a } k^{th} \text{ perfect power }, nr+1 \le x \le nr + n \}$
where O stands for the order of the set
By this definition we get

SPSCPS(12) = SPPCPS (12,2)

Another example, SPPCPS (100,3)is
4,1,1,1,0,1,0,1,0,1, 0,0,0,1, 0, 0, 0, 1, 0,0,0,1,0, 0, 0, 0, 0, 1, . . .

Problem: Does $\sum (T_r/(nr))$ converge as $n \to \infty$?

(22) SMARANDACHE BERTRAND PRIME SEQUENCE

According to Bertrand 's postulate there exists a prime between n and 2n. Starting from 2 let us form a sequence by taking the largest prime less than double of the previous prime in the sequence . We get

2, 3, 5, 7, 13, 23, 43, 83, 163, . . .

(23) SMARANDACHE SEMI- PERFECT NUMBER SEQUENCE

6, 12, 18, 20, 24, 30, 36, 40, . . .

A semi perfect number is defined as one which can be expressed as the sum of its (all or fewer) distinct divisors.

e.g. $12 = 2 + 4 + 6 = 1 + 2 + 3 + 6$
$20 = 1 + 4 + 5 + 10$
$30 = 2 + 3 + 10 + 15 = 5 + 10 + 15 = 1 + 3 + 5 + 6 + 15$ etc.

It is evident that every perfect number is also a semi perfect number.

THEOREM : There are infinitely many semi perfect numbers.
Proof: We shall prove that $N = 2^n p$ where p is a prime less than

$2^{n+1} - 1$, is a semi- perfect number.

The divisors of N are

row 1------ $1, 2, 2^2, 2^3, 2^4, \ldots 2^n$

row 2------ $p, 2p, 2^2p, 2^3p, 2^4p, \ldots 2^np$

we have $\sum_{r=0}^{n-1} 2^r p = p(1+2+2^2+2^3+\ldots 2^{n-1}) = p(2^n-1) = M$

M is short of N by p . The task ahead is to express p as the sum of divisors from the first row. It is an established fact that every number can be expressed as the sum of powers of 2 .i.e.

$p = \sum_{r=0}^{n} a_r . 2^r$, where $a_r = 0$ or $a_r = 1$. iff $p \le 2^{n+1} - 1$, the

equality giving a perfect number.

(note:$a_1a_2a_3 \ldots a_n$ is the binary representation of p).

N = M + p is expressible as the sum of its divisors.
Remark: This of-course is not exhaustive. There are many more such examples possible giving infinitely many semi perfect numbers. One can explore the possibility of more such expressions.

(24) SMARANDACHE CO-PRIME BUT NO PRIME SEQUENCE

4, 9, 10, 21, 22, 25, 26, 27, 28, 33,34 , 35 , 36 ,49, 50, 51, 52 , . . .

The n^{th} term T_n is defined as follows

$T_n = \{ x \mid (T_{n-1}, x) = 1$, x is not a prime and $(T_{n-1}, y) \neq 1$ for $T_{n-1} < y < x \}$

The smallest number which is not a prime but is relatively prime to the previous term in the sequence.

Open problem : Is it possible to as large as we want but finite increasing sequence k , k+1 , k+2 , k+3 , . . . included in the above sequence?

DEFINITION : We define a prime to be week , strong or balanced prime accordingly as p_r < = or > $(p_{r-1} + p_{r+1})/2$. where p_r is the r^{th} prime. e.g. 3 < (2+5)/2 3 is week prime . 5= (3 + 7)/2 is a balanced prime . 71 > (67 + 73)/2 is a strong prime .

(25) SMARANDACHE WEEK PRIME SEQUENCE :

3, 7 ,13, 19, 23,29, 31, 37, . . .

(26) SMARANDACHE STRONG PRIME SEQUENCE :

11 , 17, 41, . . .

(27) SMARANDACHE BALANCED PRIME SEQUENCE :

5 , 157, 173, 257, 263, 373, . . .

It is evident that for a balanced prime > 5 , $p_r = p_{r-1} + 6k$.

OPEN PROBLEM: Are there infinitly many terms in the SMARANDACHE BALANCED PRIME SEQUENCE ?

How big is N? One of the first estimates of its size was approximately [6]:

$$10^{6846168}$$

But this is a rather large number; to test all odd numbers up to this limit would take more time and computer power than we have. Recent work has improved the estimate of N. In 1989 J.R. Chen and T. Wang computed N to be approximately [7]:

$$10^{43000}$$

This new value for N is much smaller than the previous one, and suggests that some day soon we will be able to test all odd numbers up to this limit to see if they can be written as the sum of three primes.
Anyway assuming the truth of the generalized Riemann hypothesis [5], the number N has been reduced to 10^{20} by Zinoviev [9], Saouter [10] and Deshouillers. Effinger, te Riele and Zinoviev[11] have now successfully reduced N to 5.
Therefore the weak Goldbach conjecture is true, subject to the truth of the generalized Riemann hypothesis.
Let's now analyse the generalizations of Goldbach conjectures reported in [3] and [4]; six different conjectures for odd numbers and four conjectures for even numbers have been formulated. We will consider only the conjectures 1, 4 and 5 for the odd numbers and the conjectures 1, 2 and 3 for the even ones.

4.1 First Smarandache Goldbach conjecture on even numbers.

Every even integer n can be written as the difference of two odd primes, that is $n = p - q$ *with p and q two primes.*

This conjecture is equivalent to:

For each even integer n, we can find a prime q such that the sum of n and q is itself a prime p.

A program in Ubasic language to check this conjecture has been written.

for SLES and SLOS.

(2) SMARANDACHE DIVISOR SEQUENCES:

Define $A_n = \{ x | d(x) = n \}$
Then

$A_1 = \{ 1 \}$

$A_2 = \{ p \mid p \text{ is a prime} \}$

$A_3 = \{ x \mid x = p^2 , p \text{ is a prime} \}$

$A_4 = \{ x \mid x = p^3 \text{ or } x = p_1p_2 , p , p_1 ,p_2 \text{ are primes} \}$.

$A_4 \rightarrow$ 6 , 8 , 10 , 14 , 15 , 21 , 22 , 26 , 27 , . . .

We have

$\sum 1/T_n = 1$ for A_1

This limit does not exist for A_2

$\underset{n \to \infty}{\text{Lt}} \sum 1/T_n$ exists and is less than $\pi^2/6$ for A_3 as Lt $\sum 1/n^2 = \pi^2/6$.

The above limit does exist for A_p where p is a prime.

* Whether these limits exist for A_4 , A_6 etc is to be explored.

DIVISOR SUB SEQUENCES

The sub sequences for A_4 A_5 etc can be defined as follows:

$B(r_1, r_2, r_3, \ldots r_k)$ is the sequence of numbers $p_1^{r_1} p_2^{r_2} p_3^{r_3} \ldots p_k^{r_k}$

in increasing order ,where $p_1, p_2, p_3, \ldots p_k$ are primes . All the

numbers having the same unique factorization structure.

DIVISOR MULTIPLE SEQUENCE

SDMS = $\{ n \mid n = k.\, d(n) \}$.

SDMS $\rightarrow$ 1, 2, 8 ,9, 12 , . . .

(3) SMARANDACHE QUAD PRIME SEQUENCE GENERATOR :

SQPSG = $\{ r \mid 90r+11 , 90r+13 , 90r+17, 90r+19 \text{ are all primes} \}$

SQPSG $\rightarrow$ 0 , 1, 2 , . . .

Are there infintely many terms in the above sequence ?

(4) SMARANDACHE PRIME LOCATION SEQUENCES

Define P_0 = sequence of primes .

P_1 = sequence of primeth primes

$P_1 \rightarrow 3 , 5 , 11 , 17, \ldots$

P_2 = sequence of primeth , primeth prime .

$\downarrow$
$\downarrow$

P_r = sequence of primeth , primeth , . . . r times ,primes

* If T_n is the n^{th} term of P_r , then what is the minimum value of r for which

$$\lim_{n \to \infty} \sum 1/T_n \text{ exists ?.}$$

(5) SMARANDACHE PARTITION SEQUENCES

(i) PRIME PARTITION

Number of partitions into prime parts

$Sp_p(n) \rightarrow$ 0, 1, 1, 1 ,1 2, 2, 3, . . .

(ii) COMPOSITE PARTITION

Number of partitions into composite parts

$Sp_c(n) \rightarrow$ 1, 1, 1, 2, 1, 3, . . .

(iii) DIVISOR PARTITIONS

Number of partitions into parts which are the divisors of n.

$SP_d(n) \rightarrow$ 1, ,1 ,1 ,2 ,1,

On similar lines following two partition sequences can be defined.

(iv) CO-PRIME PARTITIONS : $SP_{cp}(n)$

Number of partitions into co-prime parts .

(v) NON- CO-PRIME PARTITIONS $SP_{ncp}(n)$

Number of partitions into non coprime parts.

(vi) PRIME SQUARE PARTITIONS

Partitions into prime square parts .

This idea could be generalised to define more such functions.

(6) SMARANDACHE COMBINATORIAL SEQUENCES.

(I) Let the first two terms of a sequence be 1 & 2. The $(n+1)^{th}$ term is defined as

T_{n+1} = sum of all the products of the previous terms of the sequence taking two at a time .

$T_1 = 1$, $T_2 = 2$, $\Rightarrow T_3 = 2$, and $T_4 = 8$,

SCS(2) = 1, 2, 2, 8 ,48, . . .

The above definition can be generalized as follows:

Let $T_k = k$ for $k = 1$ to n .

T_{n+1} = sum of all the products of the previous terms of the sequence taking r at a time. This defines SCS(r) .

Another generalization could be :

Let $T_k = k$ for $k = 1$ to n .

T_r = sum of all products of (r-1) terms of the sequence taking (r-2) at a time ($r > n$) . This defines SC_vS.

for n = 2 $T_1 = 1$, $T_2 = 2$, $T_3 = 3$, $T_4 = 17$ etc

$SC_vS \rightarrow 1 , 2 , 3, 17 , \ldots$

PROBLEM : (1) How many of the consecutive terms of SCS(r) are pairwise coprime ?

(2) How many of the terms of SC_vS are primes ?

(ii) SMARANDACHE PRIME PRODUCT SEQUENCES

SPPS(n)

T_n = sum of all the products of primes chosen from first n primes

taking (n-1) primes at a time.

SPPS(n) $\rightarrow$ 1, 5, 31, 247, 2927 . . .

$T_1 = 1$, $T_2 = 2 + 3$, $T_3 = 2*3 + 2*5 + 3*5 = 31$.

$T_4 = 2*3*5 + 2*3*7 + 2*5*7 + 3*5*7 = 247$ etc.

How many of these are primes ?

(7) SMARANDACHE ϕ -SEQUENCE

$(S\phi S) = \{ n \mid n = k * \phi(n) \}$

$S\phi S \rightarrow$ 1, 2 , 4 , 6 , 8 , 12 . . .

(8) SMARANDACHE PRIME DIVISIBILITY SEQUENCE

SPDS = $\{ n \mid n$ divides $p_n + 1$, p_n is the n^{th} prime. $\}$

SPDS$\rightarrow$ 1 , 2 , 3 , 4 , 10 , . . .