LENGTH / EXTENT OF SMARANDACHE FACTOR PARTITIONS

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ABSTRACT: In [1] we define SMARANDACHE FACTOR PARTITION FUNCTION (**SFP**) , as follows:

Let $\alpha_1, \alpha_2, \alpha_3, \ldots \alpha_r$ be a set of r natural numbers and $p_1, p_2, p_3, \ldots p_r$ be arbitrarily chosen distinct primes then $F(\alpha_1, \alpha_2, \alpha_3, \ldots \alpha_r)$ called the Smarandache Factor Partition of $(\alpha_1, \alpha_2, \alpha_3, \ldots \alpha_r)$ is defined as the number of ways in which the number

$$N = p_1^{\alpha 1} \; p_2^{\alpha 2} \; p_3^{\alpha 3} \ldots p_r^{\alpha r}$$

could be expressed as the product of its' divisors. For simplicity , we denote $F(\alpha_1, \alpha_2, \alpha_3, \ldots \alpha_r) = F'(N)$,where

$$N = p_1^{\alpha_1} \; p_2^{\alpha_2} \; p_3^{\alpha_3} \ldots p_r^{\alpha_r} \ldots p_n^{\alpha_n}$$

and p_r is the r^{th} prime. $p_1 = 2$, $p_2 = 3$ etc.

Also for the case

$$\alpha_1 = \alpha_2 = \alpha_3 = \ldots = \alpha_r = \ldots = \alpha_n = 1$$

we denote

$$F(\underbrace{1, 1, 1, 1, 1 \ldots}_{\leftarrow \text{ n - ones } \rightarrow}) = F(1\#n)$$

In the present note we define two interesting parameters the

length and **extent** of an **SFP** and study the interesting properties they exhibit for square free numbers.

DISCUSSION:

DEFINITION: Let $F'(N) = R$

LENGTH : If we denote each SFP of N , say like F_1 , F_2 , . . . F_R arbtrarily and let F_k be the SFP representation of N as the product of its divisors as follows:

F_k ---- $N = (h_1)(h_2)(h_3)(h_4)...(h_t)$, where each h_i ($1< i < t$) is an entity in the SFP ' F_k ' of N. Then $T(F_k) = t$ is defined as the '**length**' of the factor partition F_k.

e.g. say 60 = 15 **X** 2 **X** 2 , is a factor partition F_k of 60. Then

$$T(F_k) = 3.$$

$T(F_k)$ can also be defined as one more than the number of product signs in the factor partition.

EXTENT : The extent of a number is defined as the sum of the lengths of all the SFPs.

Consider F(1#3)

$N = p_1p_2p_3 = 2 X 3 X 5 = 30.$

SN	Factor Partition	length
1	30	1
2	15 X 2	2
3	10 X 3	2
4	6 X 5	2
5	5 X 3 X 2	3

Extent (30) = $\sum$ length = 10

We observe that

F(1#4) - F(1#3) = 10. = Extent { F(1#4)}

Consider F(1#4)

N = 2 X 3 X 5 X 7 = 210

SN	Factor Partition	Length
1	210	1
2	105 X 2	2
3	70 X 3	2
4	42 X 5	2
5	35 X 6	2
6	35 X 3 X 2	3
7	30 X 7	2
8	21 X 10	2
9	21 X 5 X 2	3
10	15 X 14	2
11	15 X 7 X 2	3
12	14 X 5 X 2	3
13	10 X 7 X 3	3
14	7 X 6 X 5	3
15	7 X 5 X 3 X 2	4

Extent(210) = $\sum$ length = 37

We observe that

F(1#5) - F(1#4) = 37. =Extent { F(1#4)}

Similarly it has been verified that

F(1#6) - F(1#5) = Extent { F(1#5)}

CONJECTURE (6.1)

F(1#(n+1)) - F(1#n) = Extent { F(1#n)}

CONJECTURE (6.2)

$$F(1\#(n+1)) = \sum_{r=0}^{n} \text{Extent } \{F(1\#r)\}$$

Motivation for this conjecture:

If conjecture (1) is true then we would have

F(1#2) - F(1#1) = Extent { F(1#1)}

F(1#3) - F(1#2) = Extent { F(1#2)}

F(1#4) - F(1#3) = Extent { F(1#3)}

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F(1#(n+1)) - F(1#n) = Extent { F(1#n)}

Summing up we would get

$$F(1\#(n+1)) - F(1\#1) = \sum_{r=1}^{n} \text{Extent } \{F(1\#r)$$

F(1#1) = 1 = Extent {F(1#0) can be taken , hence we get

$$\mathbf{F(1\#(n+1)) = \sum_{r=0}^{n} Extent\ \{F(1\#r)}$$

Another Interesting Observation:

Given below is the chart of r versus w where w is the number of

SFPs having same length r.

F(1#0) = 1 , Σ r. w = 1

r	1
w	1

F(1#1) = 1 , Σ r. w = 1

r	1
w	1

F(1#2) = 2 , Σ r. w = 3

r	1	2
w	1	1

F(1#3) = 5 , Σ r. w = 10

r	1	2	3
w	1	3	1

$F(1\#4) = 15$, $\sum r.w = 37$

r	1	2	3	4
w	1	7	6	1

$F(1\#5) = 52$, $\sum r.w = 151$

r	1	2	3	4	5
w	1	15	25	10	1

The interesting observation is the row of w is the same as the n^{th} row of the **SMARANDACHE STAR TRIANGLE. (ref.: [4])**

CONJECTURE (6.3)

$$w_r = a_{(n,r)} = (1/r!) \sum_{k=0}^{r} (-1)^{r-k} . {}^rC_k .k^n$$

where w_r is the number of SFPs of F(1#n) having length r.

Further Scope: One can study the length and contents of other cases (other than the square-free numbers.) explore for patterns if any.

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[3] "Amarnath Murthy" , 'More Results And Applications Of The Generalized Smarandache Star Function' SNJ,.1999.

[4] " The Florentine Smarandache " Special Collection, Archives of American Mathematics, Centre for American History, University of Texas at Austin, USA.