On a Concatenation Problem

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Abstract: This article has been inspired by questions asked by Charles Ashbacher in the *Journal of Recreational Mathematics, vol. 29.2.* It concerns the Smarandache Deconstructive Sequence. This sequence is a special case of a more general concatenation and sequencing procedure which is the subject of this study. Answers are given to the above questions. The properties of this kind of sequences are studied with particular emphasis on the divisibility of their terms by primes.

1. Introduction

In this article the concatenation of a and b is expressed by a_b or simply ab when there can be no misunderstanding. Multiple concatenations like abcabcabc will be expressed by 3(abc).

We consider n different elements (or n objects) arranged (concatenated) one after the other in the following way to form:

$A=a_1a_2\ldots a_n$.

Infinitely many objects A, which will be referred to as cycles, are concatenated to form the chain:

$B= a_1a_2\ldots a_n\ a_1a_2\ldots a_n\ a_1a_2\ldots a_n\ldots$

B contains identical elements which are at equidistant positions in the chain. Let's write B as

$B=b_1b_2b_3\ldots b_k\ldots$ where $b_k=b_j$ when $j\equiv k \pmod n$, $1\le j\le n$.

An infinte sequence $C_1, C_2, C_3, \ldots C_k, \ldots$ is formed by sequentially selecting 1,2,3, ...k, ... elements from the chain B:

$C_1=b_1=a_1$

$C2=b_2b_3=a_2a_3$

$C_3=b_4b_5b_6=a_4a_5a_6$ (if $n\le6$, if $n=5$ we would have $C_3=a_4a_5a_1$)

The number of elements from the chain B used to form the first k-1 terms of the sequence C is $1+2+3+\ldots+k-1=(k-1)k/2$. Hence

$$C_k=b_{\frac{(k-1)k}{2}+1}b_{\frac{(k-1)k}{2}+2}\ldots b_{\frac{k(k+1)}{2}}$$

However, what is interesting to see is how C_k is expressed in terms of $a_1, \ldots a_n$. For sufficiently large values of k C_k will be composed of three parts:

The first part:	$F(k)=a_ua_{u+1}\ldots a_n$
The middle part:	$M(k)=AA\ldots A$. The number of concatenated A's depends on k.
The last part:	$L(k)=a_1a_2\ldots a_w$

Hence $\qquad C_k=F(k)M(k)L(k) \qquad (1)$

The number of elements used to form $C_1,C_2, \ldots C_k$ is $(k-1)k/2$. Since the number of elements in A is finite there will be infinitely many terms C_k which have the same first element a_u. u can be determined from $\frac{(k-1)k}{2}+1\equiv u \pmod n$. There can be at most n^2 different combinations to form F(k) and L(k). Let C_j and C_i be two different terms for which F(i)=F(j) and L(i)=L(j). They will then be separated by a number m of complete cycles of length n, i.e.

$$\frac{(j-1)j}{2}-\frac{(i-1)i}{2}=mn$$

Let's write j=i+p and see if p exists so that there is a solution for p which is independent of i.

$$(i+p-1)(i+p)-(i-1)i=2mn$$
$$i^2+2ip+p^2-i-p-i^2+i=2mn$$
$$2ip+p^2-p=2mn$$
$$p^2+p(2i-1)=2mn$$

If n is odd we will put p=n to otain n+2i-1, or m=(n+2i-1)/2. If n is even we put p=2n to obtain m=2n+2i-1. From this we see that the terms C_k have a peculiar periodic behaviour. The periodicity is p=n for odd n and p=2n for even n. Let's illustrate this for n=4 and n=5 for which the periodicity will be p=8 and p=5 respectively.

Table 1. n=4. A=abcd. B=abcd abcd abcd abcd....

i	C_i	Period #	F(i)	M(i)	L(i)
1	a		a		
2	bc		bc		
3	dab	1	d		ab
4	cdab	1	cd		ab
5	cdabc	1	cd		abc
6	dabcda	1	d	abcd	a
7	bcdabcd	1	bcd	abcd	
8	abcdabcd	1		2(abcd)	
9	abcdabcda	1		2(abcd)	a
10	bcdabcdabc	1	bcd	abcd	abc
11	dabcdabcdab	2	d	2(abcd)	ab
12	cdabcdabcdab	2	cd	2(abcd)	ab
13	cdabcdabcdabc	2	cd	2(abcd)	abc
14	dabcdabcdabcda	2	d	3(abcd)	a
15	bcdabcdabcdabcd	2	bcd	3(abcd)	
16	abcdabcdabcdabcd	2		4(abcd)	
17	abcdabcdabcdabcda	2		4(abcd)	a
18	bcdabcdabcdabcdabc	2	bcd	3(abcd)	abc
19	dabcdabcdabcdabcdab	3	d	4(abcd)	ab
20	cdabcdabcdabcdabcdab	3	cdcd	4(abcd)	ab

Note that the periodicity starts for i=3.

Numerals are chosen as elements to illustrate the case n=5. Let's write i=s+k+pj, where s is the index of the term preceding the first periodical term, k=1,2,..., p is the index of members of the period and j is the number of the period (for convenience the first period is numbered 0). The first part of C_i is denoted B(k) and the last part E(k). C_i is now given by the expression below where q is the number of cycles concatenated between the first part B(k) and the last part E(k).

$$C_i=B(k)\_qA\_E(k), \text{ where k is determined from } i-s\equiv k \pmod p \qquad (2)$$

Table 2. n=5. A=12345. B=123451234512345......

i		C_i	k	q	F(i)⇔B(k)	M(i)	L(i)⇔E(k)
1		1			1		
s=2		23			2		
	j=0						
3		451	1	0	45		1
4		2345	2	0	2345		
5		12345	3	1		12345	
6		123451	4	1		12345	1
7		2345123	5	0	2345		123
	j=1						
3+5j		45123451	1	j	45	12345	1
4+5j		234512345	2	j	2345	12345	
5+5j		1234512345	3	j+1		2(12345)	
6+5j		12345123451	4	j+1		2(12345)	1
7+5j		234512345123	5	j	2345	12345	123
	j=2						
3+5j		4512345123451	1	j	45	2(12345)	1
4+5j		23451234512345	2	j	2345	2(12345)	
....							

2. The Smarandache Deconstructive Sequence

The Smarandache Deconstructive Sequence of integers [1] is constructed by sequentially repeating the digits 1 to 9 in the following way

1,23,456,7891,23456,789123,4567891,23456789,123456789,1234567891,...

The sequence was studied in a booklet by Kashihara [2] and a number of questions on this sequence were posed by Ashbacher [3]. In thinking about these questions two observations lead to this study.

1. Why did Smarandache exclude 0 from the integers used to create the sequence? After all 0 is indispensible in all arithmetics most of which can be done using 0 and 1 only.
2. The process used to create the Deconstructive Sequence is a process which applies to any set of objects as has been shown in the introduction.

The periodicity and the general expression for terms in the "generalized deconstructive sequence" shown in the introduction may be the most important results of this study. These results will now be used to examine the questions raised by Ashbacher. It is worth noting that these divisibility questions are dealt with in base10 although only nine digits 1,2,3,4,5,6,7,8,9 are used to express terms in the sequence. In the last part of this article questions on divisibility will be posed for a deconstructive sequence generated form A="0123456789".

For i>5 (s=5) any term C_i in the sequence is composed by concatenating a first part B(k), a number q of cycles A"123456789" and a last part E(k), where i=5+k+9j, k=1,2, ... ,9, j≥0, as expressed in (2) and q=j or j+1 as shown in table 3.

Members of the Smarandache Deconstructive Sequence are now interpreted as decimal integers. The factorization of B(k) and E(k) is shown in table 3. The last two columns of this table will be useful later in this article.

Table 3. Factorization of Smarandache Deconstructive Sequence
$i=5+k+9j$

i	k	B(k)	q	E(k)	Digit sum	$3 \mid C_i$?
6+9j	1	$789=3\cdot263$	j	$123=3\cdot41$	$30+j\cdot45$	3
7+9j	2	$456789=3\cdot43\cdot3541$	j	1	$40+j\cdot45$	No
8+9j	3	23456789	j		$44+j\cdot45$	No
9+9j	4		j+1		$(j+1)\cdot45$	$9\cdot3^z$ *
10+9j	5		j+1	1	$1+(j+1)\cdot45$	No
11+9j	6	23456789	j	$123=3\cdot41$	$50+j\cdot45$	No
12+9j	7	$456789=3\cdot43\cdot3541$	j	$123456=2^6\cdot3\cdot643$	$60+j\cdot45$	3
13+9j	8	$789=3\cdot263$	j+1	1	$25+(j+1)\cdot45$	No
14+9j	9	23456789	j	$123456=2^6\cdot3\cdot643$	$65+j\cdot45$	No

*) where z depends on j.

Together with the factoization of the cycle $A=1223456789=3^2\cdot3607\cdot3803$ it is now possible to study some divisibility properties of the sequence. We will first find a general expression for C_i in terms of j and k. For this purpose we introduce:

> $q(k)=0$ for $k=1,2,3,6,7,9$ and $q(k)=1$ for $k=4,5,8$
> $u(k)=1+[\log_{10}(E(k)]$ if E(k) exists otherwise $u(k)=0$, i.e. $u(3)=u(4)=0$
> $\delta(j,k)=0$ if $j=0$ and $q(k)=0$ otherwise $\delta(j,k)=1$

With the help of these functions we can now use table 3 to formulate the general expression

$$C_{5+k+9j} = E(k) + \delta(j,k)\cdot A\cdot 10^{u(k)}\cdot \sum_{r=0}^{j-1+q(k)} 10^{9r} + B(k)\cdot 10^{9(j+q(k))+u(k)} \qquad (3)$$

Before dealing with the questions posed by Ashbacher we recall the familiar rules: An even number is divisible by 2; a number whose last two digits form a number which is divisible by 4 is divisible by 4. In general we have the following:

Theorem. Let N be an n-digit integer such that $N>2^\alpha$ then N is divisible by 2^α if and only if the number formed by the α last digits of N is divisible by 2^α.
Proof. To begin wwith we note that

> If x divides a and x divides b then x divides (a+b).
> If x divides one but not the other of a and b the x does not divide (a+b).
> If neither a nor b is divisible by x then x may or may not divide (a+b).

Let's write the n-digit number in the form $a\cdot10^\alpha+b$. We then see from the following that $a\cdot10^\alpha$ is divisible by 2^α.

> $10\equiv0 \pmod 2$
> $100\equiv0 \pmod 4$
> $1000=2^3\cdot5^3\equiv0 \pmod{2^3}$

....

$10^\alpha \equiv 0 \pmod{2^\alpha}$

and then

$a \cdot 10^\alpha \equiv 0 \pmod{2^\alpha}$ independent of a.

Now let b be the number formed by the α last digits of N, we then see from the introductory remark that N is divisible by 2^α if and only if the number formed by the α last digits is divisible by 2^α.

Question 1. Does every even element of the Smarandache Deconstructive Sequence contain at least three instances of the prime 2 as a factor?

Question 2. If we form a sequence from the elements of the Smarandache Deconstructive Sequence that end in a 6, do the powers of 2 that divide them form a monotonically increasing sequence?

These two quetions are reelated and are dealt with together. From the previous analysis we know that all even elements of the Smarandache Deconstructive end in a 6. For $i \le 5$ they are:

$C_3=456=57 \cdot 2^3$

$C_5=23456=733 \cdot 2^5$

For $i>5$ they are of the forms:

C_{12+9j} and C_{14+9j} which both end in ...789123456.

Examining the numbers formed by the 6, 7 and 8 last digits for divisibility by 2^6, 2^7 and 2^8 respectively we have:

$123456=2^6 \cdot 3 \cdot 643$

$9123456=2^7 \cdot 149 \cdot 4673$

89123456 is not divisible by 2^8

From this we conclude that all even Smarandache Deconstructive Sequence elements for $i \ge 12$ are divisible by 2^7 and that no elements in the sequence are divisible by higher powers of 2 than 7.

Answer to Qn 1. Yes

Answer to Qn 2. The sequence is monotonically increasing for $i \le 12$. For $i \ge 12$ the powers of 2 that divide even elements remain constant = 2^7.

Question 3. Let x be the largest integer such that $3^x \mid i$ and y the largest integer such that $3^y \mid C_i$. It is true that x is always equal to y?

From table 3 we see that the only elements C_i of the Smarandache Deconstructive Sequence which are divisible by powers of 3 correspond to $i=6+9j$, $9+9j$ or $12+9j$. Furthermore, we see that $i=6+9j$ and C_{6+9j} are divisible by 3, no more no less. The same is true for $i=12+9j$ and C_{12+9j}. So the statement holds in these cases. From the congruences

$9+9j \equiv 0 \pmod{3^x}$ for the index of the element

and

$45(1+j) \equiv 0 \pmod{3^y}$ for the corresponding element

we conclude that $x=y$.

Answer: The statement is true. It is interesting to note that, for example, the 729 digit number C_{729} is divisible by 729.

Question 4. Are there other patterns of divisibility in this sequence?

A search for patterns would continue by examining divisibility by the next lower primes 5, 7, 11, ... It is obvious from table 3 and the periodicity of the sequence that there are no elements divisible by 5. Algorithm (3) will prove useful. For each value of k the value of C_i depends on j only. The divisibility by a prime p is therefore determined by finding out for which values of j and k the congruence $C_i \equiv 0 \pmod p$ holds. We evaluate $\sum_{r=0}^{j-1+q(k)} 10^{9r} = \frac{10^{9(j+q(k))}-1}{10^9-1}$ and introduce $G=10^9-1$. We note that $G=3^4 \cdot 37 \cdot 333667$. From (3) we now obtain:

$$G \cdot C_i = G \cdot E(k) + (\delta(j,k) \cdot A \cdot + G \cdot B(k)) 10^{9(j+q(k))+n(k)} - \delta(j,k) \cdot A \cdot 10^{n(k)} \qquad (3')$$

The divisibility of C_i by a prime p other than 3, 37 and 333667 is therefore determined by solutions for j to the congruences $G \cdot C_i \equiv 0 \pmod p$ which are of the form

$$a \cdot (10^9)^j + b \equiv 0 \pmod p \qquad (4)$$

Table 4 shows the results from computer implementation of the congruences $G \cdot C_i \equiv 0 \pmod p$ for k=1,2,...9 and p<100. The appearance of elements divisible by a prime p is periodic, the periodicity is given by $j = j_1 + m \cdot d$, m=1,2,3, ...The first element divisible by p appears for i_1 corresponding to j_1. In general the terms C_i divisible by p are $C_{5+k+9(j_1+md)}$ where d is specific to the prime p and m=1,2,3, ... We note from table 4 that d is either equal to p-1 or a divisor of p-1 except for the case p=37 which as we have noted is a factor of A. Indeed this periodicity follows from Euler's extension of Fermat's little theorem because we can write (mod p):

$$a \cdot (10^9)^j + b = a \cdot (10^9)^{j_1+md} + b \equiv a \cdot (10^9)^{j_1} + b \text{ for } d=p-1 \text{ or a divisor of } p-1.$$

Finally we note that the periodicity for p=37 is d=37, which is found by examining (3') modulus 37^2.

Table 4. Smarandache Deconstructive elements divisible by p.

p	k	i_1	j_1	d
7	4	18	1	2
11	4	18	1	2
13	4	18	1	2
13	8	22	1	2
13	9	14	0	2
17	1	6	0	16
17	2	43	4	16
17	3	44	4	16
17	4	144	15	16
17	5	100	10	16
17	6	101	10	16
17	7	138	14	16
17	8	49	4	16
17	9	95	9	16
19	1	15	1	2
19	4	18	1	2
19	7	21	1	2
23	1	186	20	22
23	2	196	21	22
23	3	80	8	22
23	4	198	21	22
23	5	118	12	22
23	6	200	21	22
23	7	12	0	22
23	8	184	19	22
23	9	14	0	22
29	1	24	2	28
29	2	115	12	28
29	3	197	21	28
29	4	252	27	28
29	5	55	5	28
29	6	137	14	28
29	7	228	24	28
29	8	139	14	28
29	9	113	11	28
31	3	26	2	5
31	4	45	4	5
31	5	19	1	5
37	1	222	24	37
37	2	124	13	37
37	3	98	10	37
37	4	333	36	37
37	5	235	25	37
37	6	209	22	37
37	7	111	11	37
37	8	13	0	37
37	9	320	34	37
41	4	45	4	5
43	1	33	3	7
43	4	63	6	7
43	7	30	2	7

p	k	i_1	j_1	d
47	1	150	16	46
47	2	250	27	46
47	3	368	40	46
47	4	414	45	46
47	5	46	4	46
47	6	164	17	46
47	7	264	28	46
47	8	400	43	46
47	9	14	0	46
53	1	24	2	13
53	4	117	12	13
53	7	93	9	13
59	1	267	29	58
59	2	511	56	58
59	3	413	45	58
59	4	522	57	58
59	5	109	11	58
59	6	11	0	58
59	7	255	27	58
59	8	256	27	58
59	9	266	28	58
61	2	79	8	20
61	4	180	19	20
61	6	101	10	20
67	4	99	10	11
67	8	67	6	11
67	9	32	2	11
71	1	114	12	35
71	3	53	5	35
71	4	315	34	35
71	5	262	28	35
71	7	201	21	35
73	4	72	7	8
79	4	117	12	13
83	1	348	38	41
83	2	133	14	41
83	4	369	40	41
83	6	236	25	41
83	7	21	1	41
83	8	112	11	41
83	9	257	27	41
89	2	97	10	44
89	4	396	43	44
89	6	299	32	44
97	1	87	9	32
97	2	115	12	32
97	3	107	11	32
97	4	288	31	32
97	5	181	19	32
97	6	173	18	32
97	7	201	21	32
97	8	202	21	32
97	9	86	8	32

Question: Table 4 indicates some interesting patterns. For instance, the primes 19, 43 and 53 only divides elements corresponding to k=1, 4 and 7 for j<250 which was set as an upper limit for this study. Simiarly, the primes 7, 11, 41, 73 and 79 only divides elements corresponding to k=4. Is 5 the only prime that cannot divide an element of the Smarandache Deconstructive Sequence?

3. A Deconstructive Sequence generated by the cycle A=0123456789

Instead of sequentially repeating the digits 1-9 as in the case of the Smarandache Deconstructive Sequence we will use the digits 0-9 to form the corresponding sequence:

0,12,345,6789,01234,567890,1234567,89012345,678901234,5678901234, 56789012345,678901234567, ...

In this case the cycle has n=10 elements. As we have seen in the introduction the sequence then has a period 2n=20. The periodicity starts for i=8. Table 5 shows how, for i>7, any term C_i in the sequence is composed by concatenating a first part B(k), a number q of cycles A="0123456789" and a last part E(k), where i=7+k+20j, k=1,2, ... 20, j≥0, as expressed in (2) and q=2j, 2j+1 or 2j+2. In the analysis of the sequence it is important to distinguish between the cases where E(k)=0, k=6,11,14,19 and cases where E(k) does not exist, i.e. k=8,12,13,14. In order to cope with this problem we introduce a function u(k) which will at the same time replace the functions δ(j,k) and $u=1+[\log_{10}E(k)]$ used previously. u(k) is defined as shown in table 5. It is now possible to express C_i in a single formula.

$$C_i = C_{7+k+20j} = E(k) + (A\cdot \sum_{r=0}^{q(k)+2j-1} (10^{10})^r + B(k)\cdot (10^{10})^{q(k)+2j})10^{u(k)} \qquad (5)$$

The formula for C_i was implemented modulus prime numbers less than 100. The result is shown in table 6. Again we note that the divisibility by a prime p is periodic with a period d which is equal to p-1 or a divisor of p-1, except for p=11 and p=41 which are factors of $10^{10}-1$. The cases p=3 and 5 have very simple answers and are not included in table 6.

Table 5. n=10, A=0123456789

i	k	B(k)	q	E(k)	u(k)
8+20j	1	89	2j	012345=3·5·823	6
9+20j	2	6789=3·31·73	2j	01234=2·617	5
10+20j	3	56789=109·521	2j	01234=2·617	5
11+20j	4	56789=109·521	2j	012345=3·5·823	6
12+20j	5	6789=3·31·73	2j	01234567=127·9721	8
13+20j	6	89	2j+1	0	1
14+20j	7	123456789=3^2·3607·3803	2j	01234=2·617	5
15+20j	8	56789=109·521	2j+1		0
16+20j	9		2j+1	012345=3·5·823	6
17+20j	10	6789=3·31·73	2j+1	012=2^2·3	3
18+20j	11	3456789=3·7·97·1697	2j+1	0	1
19+20j	12	123456789=3^2·3607·3803	2j+1		0
20+20j	13		2j+2		0
21+20j	14		2j+2	0	1
22+20j	15	123456789=3^2·3607·3803	2j+1	012=2^2·3	3
23+20j	16	3456789=3·7·97·1697	2j+1	012345=3·5·823	6
24+20j	17	6789=3·31·73	2j+2		0
25+20j	18		2j+2	01234=2·617	5
26+20j	19	56789=109·521	2j+2	0	1
27+20j	20	123456789=3^2·3607·3803	2j+1	01234567=127·9721	8

Table 6. Divisibility of the 10-cycle deconstructive sequence by primes p≤97

p	k	i_1	j_1	d	p	k	i_1	j_1	d
7	3	30	1	3	11	11	18	0	11
7	6	13	0	3	11	12	219	10	11
7	7	14	0	3	11	13	220	10	11
7	8	15	0	3	11	14	221	10	11
7	11	38	1	3	11	15	202	9	11
7	12	59	2	3	11	16	83	3	11
7	13	60	2	3	11	17	44	1	11
7	14	61	2	3	11	18	185	8	11
7	15	22	0	3	11	19	146	6	11
7	18	45	1	3	11	20	87	3	11
7	19	46	1	3	13	2	49	2	3
7	20	47	1	3	13	3	30	1	3
11	1	88	4	11	13	4	11	0	3
11	2	9	0	11	13	12	59	2	3
11	3	110	5	11	13	13	60	2	3
11	4	211	10	11	13	14	61	2	3
11	5	132	6	11	17	1	48	2	4
11	6	133	6	11	17	5	32	1	4
11	7	74	3	11	17	10	37	1	4
11	8	35	1	11	17	12	79	3	4
11	9	176	8	11	17	13	80	3	4
11	10	137	6	11	17	14	81	3	4

Table 6, cont. Divisibility of the 10-cycle deconstructive sequence by primes p≤97

p	k	i_1	j_1	d	p	k	i_1	j_1	d
17	16	43	1	4	41	11	678	33	41
19	1	128	6	9	41	12	819	40	41
19	2	149	7	9	41	13	820	40	41
19	3	90	4	9	41	14	821	40	41
19	4	31	1	9	41	15	142	6	41
19	5	52	2	9	41	16	703	34	41
19	10	117	5	9	41	17	384	18	41
19	12	179	8	9	41	18	205	9	41
19	13	180	8	9	41	19	206	9	41
19	14	181	8	9	41	20	467	22	41
19	16	63	2	9	43	2	109	5	21
23	1	168	8	11	43	3	210	10	21
23	2	149	7	11	43	4	311	15	21
23	3	110	5	11	43	6	173	8	21
23	4	71	3	11	43	10	217	10	21
23	5	52	2	11	43	12	419	20	21
23	10	217	10	11	43	13	420	20	21
23	12	219	10	11	43	14	421	20	21
23	13	220	10	11	43	16	203	9	21
23	14	221	10	11	43	20	247	11	21
23	16	223	10	11	47	1	28	1	23
29	2	129	6	7	47	2	69	3	23
29	4	11	0	7	47	3	230	11	23
29	10	97	4	7	47	4	391	19	23
29	12	139	6	7	47	5	432	21	23
29	13	140	6	7	47	6	113	5	23
29	14	141	6	7	47	7	214	10	23
29	16	43	1	7	47	8	15	0	23
31	3	30	1	3	47	9	376	18	23
31	9	56	2	3	47	12	459	22	23
31	12	59	2	3	47	13	460	22	23
31	13	60	2	3	47	14	461	22	23
31	14	61	2	3	47	17	84	3	23
31	17	64	2	3	47	18	445	21	23
37	2	9	0	3	47	19	246	11	23
37	3	30	1	3	47	20	347	16	23
37	4	51	2	3	53	3	130	6	13
37	12	59	2	3	53	12	259	12	13
37	13	60	2	3	53	13	260	12	13
37	14	61	2	3	53	14	261	12	13
41	1	788	39	41	59	2	269	13	29
41	2	589	29	41	59	3	290	14	29
41	3	410	20	41	59	4	311	15	29
41	4	231	11	41	59	7	474	23	29
41	5	32	1	41	59	8	395	19	29
41	6	353	17	41	59	9	496	24	29
41	7	614	30	41	59	10	297	14	29
41	8	615	30	41	59	11	78	3	29
41	9	436	21	41	59	12	579	28	29
41	10	117	5	41	59	13	580	28	29

Table 6, cont. Divisibility of the 10-cycle deconstructive sequence by primes p≤97

p	k	i_1	j_1	d	p	k	i_1	j_1	d
59	14	581	28	29	71	8	95	4	7
59	15	502	24	29	71	12	139	6	7
59	16	283	13	29	71	13	140	6	7
59	17	84	3	29	71	14	141	6	7
59	18	185	8	29	71	18	45	1	7
59	19	106	4	29	71	19	26	0	7
61	12	59	2	3	73	7	14	0	2
61	13	60	2	3	73	9	36	1	2
61	14	61	2	3	73	12	39	1	2
67	1	328	16	33	73	13	40	1	2
67	2	509	25	33	73	14	41	1	2
67	3	330	16	33	73	17	44	1	2
67	4	151	7	33	73	19	26	0	2
67	5	332	16	33	79	1	228	11	13
67	6	273	13	33	79	3	130	6	13
67	7	234	11	33	79	5	32	1	13
67	8	95	4	33	79	12	259	12	13
67	9	56	2	33	79	13	260	12	13
67	10	557	27	33	79	14	261	12	13
67	11	378	18	33	83	3	410	20	41
67	12	659	32	33	83	9	476	23	41
67	13	660	32	33	83	12	819	40	41
67	14	661	32	33	83	13	820	40	41
67	15	282	13	33	83	14	821	40	41
67	16	103	4	33	83	17	344	16	41
67	17	604	29	33	89	12	219	10	11
67	18	565	27	33	89	13	220	10	11
67	19	426	20	33	89	14	221	10	11
67	20	387	18	33	97	8	455	22	24
71	1	8	0	7	97	12	479	23	24
71	3	70	3	7	97	13	480	23	24
71	5	132	6	7	97	14	481	23	24
71	7	114	5	7	97	18	25	0	24

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