ON A CONJECTURE CONCERNING THE SMARANDACHE FUNCTION

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Let $S : Z^* \rightarrow N$, $S(n)$ is the smallest integer n such that n! is divisibil by m (Smarandache function), for any $m \in Z^*$.
Then the following Diophantine equation

$S(x) = S(x+1)$, where $x > 0$,

has no solution.

Some remarks:

$S(1) = 0$. Let $a \geq 2$, then $S(a) \neq 0$.

Anytime $S(a) \neq 1$, because $1! = 1 = 0!$ and $1 > 0$.

Lemma.

If $a \geq 2$ and $S(a) = b$, then $(a,b) \neq 1$.

Proof:

Let $a = p_1^{r_1} \ldots p_s^{r_s}$, with all p_i distinct prime numbers, its canonical factor decomposition.

Then $S(a) = \max\left\{ S\left(p_1^{r_1}\right), \ldots, S\left(p_s^{r_s}\right) \right\}$.

But $S\left(p_i^{r_i}\right)$ is a multiple of p_i, $\forall i \in \{1, \ldots, s\}$.

Therefore, $\exists q \in \{p_1, \ldots, p_s\}$ such that q divides S(a), but q divides a, too. Q.E.D.

These results do not solve the Conjecture 2068 proposed by Florentin Smarandache in 1986 (see [1]) and republished by Mike Mudge in 1992 as problem viii, a) (see [2]).

References:

[1] R.Muller, "Smarandache Function Journal", New York, Vol. 1., December 1990, 37.
[2] M.Mudge, "The Smarandache Function" in <Personal Computer Word>, London, July 1992, 420.

Remark:

Professor Lucian Tutescu considered that this conjecture may be extended for $S(\alpha x + \beta) = S(\gamma x + \delta)$ equations,
where $(\alpha x - \beta, \gamma x + \delta) = 1$ and $\alpha, \beta, \gamma, \delta \in Z$.