On a Conjecture of F. Smarandache

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Abstract: The main purpose of this paper is to solve a problem generated by Professor F.Smarandache.

Key word: Permutation sequence; k-power.

Let n be a positive integer, n is called a k–power if $n=m^k$, where k and m are positive integer, and $k \geq 2$. Obviously, if n is a k-power, p is a prime, then we have $p^k|n$, if $p|n$.

In his book "Only Problems, not Solutions", Professor F.Smarandache defined a permutation sequence: 12, 1342, 135642, 13578642, 13579108642, 135791112108642, 1357911131412108642,13579111315161412108642,135791113151718161412108642,..., and generated a conjecture: there is no any k-power among these numbers. The main purpose of this paper is to prove that this conjecture is true.

Suppose there is a k-power a(n) among the permutation sequence. Noting the fact: $12=2^2\times 3$, we may immediately get: $a(n)\geq 1342>10000$. For the last two digits of a(n) is 42, so we have $a(n)\equiv 42 \pmod{100}$

Noting that $4|100$, we may immediately deduce : $a(n)\equiv 42\equiv 2 \pmod 4$.

So we get $2|a(n)$, $4\nmid a(n)$. However, 2 is a prime, then $4|a(n)$ contradicts with $4\nmid a(n)$. So a(n) is not a k-power.

This complete the proof of the conjecture.

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