ON A CONJECTURE OF SMARANDACHE ON PRIME NUMBERS

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Let p_n denote the n-th prime number. One of Smarandache's conjectures in [3] is the following inequality:

$$p_{n+1}/p_n \leq 5/3, \text{ with equality for } n = 2. \tag{1}$$

Clearly, for n = 1, 2, 3, 4 this is true, and for n = 2 there is equality. Let n > 4. Then we prove that (1) holds true with strict inequality. Indeed, by a result of Dressler, Pigno and Young (see [1] or [2]) we have

$$p_{n+1}^2 \leq 2p_n^2. \tag{2}$$

Thus $p_{n+1}/p_n \leq \sqrt{2} \leq 5/3$, since $3\sqrt{2} < 5$ (i.e. 18 < 25). This finishes the proof of (1).

References:

[1] R. E. Dressler, L. Pigno and R. Young, *Sums of Squares of Primes*, Nordisk Mat. Tidskrift 24(1976), 39.

[2] D. S. Mitrinovi and J. Sándor (in coop. with B. Crstici), *Handbook of Number Theory*, Kluwer Acad. Publ., 1995.

[3] M. L. Perez, editor, *Five Smarandache Conjectures on Primes*, http://www.gallup.unm.edu/~smarandache/conjprim.txt.