On a Deconcatenation Problem

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Abstract: In a recent study of the *Primality of the Smarandache Symmetric Sequences* Sabin and Tatiana Tabirca [1] observed a very high frequency of the prime factor 333667 in the factorization of the terms of the second order sequence. The question if this prime factor occurs peridically was raised. The odd behaviour of this and a few other primefactors of this sequence will be explained and details of the periodic occurence of this and of several other prime factors will be given.

Definition: The n*th* term of the Smarandache symmetric sequence of the second order is defined by S(n)=123...n_n...321 which is to be understood as a concatenation[1] of the first n natural numbers concatenated with a concatenation in reverse order of the n first natural numbers.

Factorization and Patterns of Divisibility

The first five terms of the sequence are: 11, 1221, 123321, 12344321, 1234554321.
The number of digits D(n) of S(n) is growing rapidly. It can be found from the formula:

$$D(n) = 2k(n+1) - \frac{2(10^k - 1)}{9} \text{ for n in the interval } 10^{k-1} \le n < 10^k - 1 \qquad (1)$$

In order to study the repeated occurrance of certain prime factors the table of S(n) for n≤100 produced in [1] has been extended to n≤200. Tabirca's aim was to factorize the terms S(n) as far as possible which is more ambitious then the aim of the present calculation which is to find prime factors which are less than 10^8. The result is shown in table 1.

The computer file containing table 1 is analysed in various ways. Of the 664579 primes which are smaller than 10^7 only 192 occur in the prime factoriztions of S(n) for 1≤n≤200. Of these 192 primes 37 occur more than once. The record holder is 333667, the 28693th prime, which occurs 45 times for 1≤n≤200 while its neighbours 333647 and 333673 do not even occur once. Obviously there is something to be explained here. The frequency of the most frequently occurring primes is shown below..

Table 2. Most frequently occurring primes

p	3	33367	37	41	271	9091	11	43	73	53	97	31	47
Freq	132	45	41	41	41	29	25	24	14	8	7	6	6

[1] In this article the concatenation of a and b is written a_b. Multiplication ab is often made explicit by writing a.b. When there is no reason for misunderstanding the signs "_" and "." are omitted. Several tables contain prime factorizations. Prime factors are given in ascending order, multiplication is expressed by "." and the last factor is followed by ".." if the factorization is incomplete or by Fxxx indicating the number of digits of the last factor. To avoid typing errors all tables are electronically transferred from the calculation program, which is DOS-based, to the wordprocessor. All editing has been done either with a spreadsheet program or directly with the text editor. Full page tables have been placed at the end of the article. A non-proportional font has been used to illustrate the placement of digits when this has been found useful.

The distribution of the primes 11, 37, 41, 43, 271, 9091 and 333667 is shown in table 3. It is seen that the occurance patterns are different in the intervals $1 \le n \le 9$, $10 \le n \le 99$ and $100 \le n \le 200$. Indeed the last interval is part of the interval $100 \le n \le 999$. It would have been very interesting to include part of the interval $1000 \le n \le 9999$ but as we can see from (1) already S(1000) has 5786 digits. Partition lines are drawn in the table to highlight the different intervals. The less frequent primes are listed in table 4 where primes occurring more than once are partitioned.

From the patterns in table 3 we can formulate the occurance of these primes in the intervals $1 \le n \le 9$, $10 \le n \le 99$ and $100 \le n \le 200$, where the formulas for the last interval are indicative. We note, for example, that 11 is not a factor of any term in the interval $100 \le n \le 999$. This indicates that the divisibility patterns for the interval $1000 \le n \le 9999$ and further intervals is a completely open question.

Table 5 shows an analysis of the patterns of occurance of the primes in table 1 by interval. Note that we only have observations up to n=200. Nevertheless the interval $100 \le n \le 999$ is used. This will be justified in the further analysis.

Table 5. Divisibility patterns

Interval	p	n	Range for j
$1 \le n \le \ldots$	3	$2+3j$	$j=0,1,\ldots$
$1 \le n \le \ldots$		$3j$	$j=1,2,\ldots$
$1 \le n \le 9$	11	All values of n	
$10 \le n \le 99$		$12+11j$	$j=0,1,\ldots,7$
		$20+11j$	$j=0,1,\ldots,7$
$100 \le n \le 999$		None	
$1 \le n \le 9$	37	$2+3j$	$j=0,1,2$
		$3+3j$	$j=0,1,2$
$10 \le n \le 99$		$12+3j$	$j=0,1,\ldots,28,29$
$100 \le n \le 999$		$122+37j$	$j=0,1,\ldots,23$
		$136+37j$	$j=0,1,\ldots,23$
$1 \le n \le 9$	41	$4+5j$	$j=0,1$
		5	
$10 \le n \le 999$		$14+5j$	$j=0,1,\ldots,197$
$1 \le n \le 9$	43	None	
$10 \le n \le 99$		$11+21j$	$j=0,1,3,4$
		$24+21j$	$j=0,1,2,3$
$100 \le n \le 999$		100	
		$107+7j$	$j=0,1,\ldots,127$
$1 \le n \le 9$	271	$4+5j$	$j=0,1$
		5	
$10 \le n \le 999$		$14+5j$	$j=0,1,\ldots,197$
$1 \le n \le 999$	9091	$9+5j$	$j=0,1,\ldots,98$
$1 \le n \le 9$	333667	8,9	
$10 \le n \le 99$		$18+9j$	$j=0,1,\ldots,9$
$100 \le n \le 999$		$102+3j$	$j=0,1,\ldots,299$

We note that no terms are divisible by 11 for $n>100$ in the interval $100 \le n \le 200$ and that no term is divisible by 43 in the interval $1 \le n \le 9$. Another remarkable observation is that the sequence shows exactly the same behaviour for the primes 41 and 271 in the intervals included in the study. Will they show the same behaviour when $n \ge 1000$?

Consider

$S(n)=12\ldots n\_n\ldots 21$.

Let p be a divisor of S(n). We will construct a number

$N=12\ldots n\_0..0\_n\ldots 21$ (2)

so that p also divides N. What will be the number of zeros? Before discussing this let's consider the case p=3.

Case 1. p=3.

In the case p=3 we use the familiar rule that a number is divisible by 3 if and only if its digit sum is divisible by 3. In this case we can insert as many zeros as we like in (2) since this does not change the sum of digits. We also note that any integer formed by concatenation of three consecutive integers is divisible by 3, cf a_a+1_a+2, digit sum 3a+3. It follows that also a_a+1_a+2_a+2_a+1_a is divisible by 3. For a=n+1 we insert this instead of the appropriate number of zeros in (2). This means that if $S(n)\equiv 0 \pmod 3$ then $S(n+3)\equiv 0 \pmod 3$. We have seen that $S(2)\equiv 0 \pmod 3$ and $S(3)\equiv 0 \pmod 3$. By induction it follows that $S(2+3j)\equiv 0 \pmod 3$ for j=1,2,... and $S(3j)\equiv 0 \pmod 3$ for j=1,2,... .

We now return to the general case. S(n) is deconcatenated into two numbers 12...n and n... 21 from which we form the numbers

$A=12\ldots n\cdot 10^{1+[\log_{10}B]}$ and $B=n\ldots 21$

We note that this is a different way of writing S(n) since indeed A+B=S(n) and that $A+B\equiv 0 \pmod p$. We now form $M=A\cdot 10^s+B$ where we want to determine s so that $M\equiv 0 \pmod p$. We write M in the form $M=A(10^s-1)+A+B$ where A+B can be ignored mod p. We exclude the possibility $A\equiv 0 \pmod p$ which is not interesting. This leaves us with the congruence

$M\equiv A(10^s-1)\equiv 0 \pmod p$

or

$10^s-1\equiv 0 \pmod p$

We are particularly interested in solutions for which

$p\in\{11,37,41,43,271,9091,333667\}$

By the nature of the problem these solutions are periodic. Only the two first values of s are given for each prime.

Table 6. $10^s-1\equiv 0 \pmod p$

p	3	11	37	41	43	271	9091	33367
s	1,2	2,4	3,6	5,10	21,42	5,10	10,20	9,18

We note that the result is independent of n. This means that we can use n as a parameter when searching for a sequence C=n+1_n+2_...n+k_n+k_...n+2_n+1 such that this is also divisible by p and hence can be inserted in place of the zeros to form S(n+k) which then fills the condition $S(n+k)\equiv 0 \pmod p$. Here k is a multiple of s or s/2 in case s is even. This explains the results which we have already obtained in a different way as part of the factorization of S(n) for $n\leq 200$, see tables 3 and 5. It remains to explain the periodicity which as we have seen is different in different intervals $10^u\leq n\leq 10^u-1$.

This may be best done by using concrete examples. Let us use the sequences starting with n=12 for p=37, n=12 and n=20 for p=11 and n=102 for p=333667. At the same time we will illustrate what we have done above.

Case 2: n=12, p=37. Period=3. Interval: $10 \le n \le 99$.

```
S(n)=   123456789101112______________121110987654321
N=      123456789101112000000000000000121110987654321
C=                     131415151413
S(n+k)=123456789101112131415151413121110987654321
```

Let's look at C which carries the explanation to the periodicity. We write C in the form

$$C=101010101010+30405050403$$

We know that $C \equiv 0 \pmod{37}$. What about 101010101010? Let's write

$$101010101010=10+10^3+10^5+\ldots+10^{11}=(10^{12}-1)/9 \equiv 0 \pmod{37}$$

This congruence mod 37 has already been established in table 6. It follows that also

$$30405050403 \equiv 0 \pmod{37}$$

and that

$$x\cdot(101010101010) \equiv 0 \pmod{37} \quad \text{for } x = \text{any integer}$$

Combining these observations we se that

$$232425252423,\ 333435353433,\ \ldots\ 939495959493 \equiv 0 \pmod{37}$$

Hence the periodicity is explained.

Case 3a: n=12, p=11. Period=11. Interval: $10 \le n \le 99$.

```
S(12)=12_.._12_____________________________________________12_.._21
S(23)=12_.._1213141516171819202122232322212019181716151413 12_.._21
C=          13141516171819202122232322212019181716151413=
C1=         10101010101010101010101010101010101010101010+
C2=          3040506070809101112131312111009080706050403
```

From this we form

```
2·C1+C2=    23242526272829303132333332313029282726252423
```

which is NOT what we wanted, but $C1 \equiv 0 \pmod{11}$ and also $C1/10 \equiv 0 \pmod{11}$.
Hence we form

```
2·C1+C1/10+C2=24252627282930313233343433323130292827262524
```

which is exactly the C-term required to form the next term S(34) of the sequence. For the next term S(45) the C-term is formed by $3\cdot C1+2\cdot C1/10+C2$ The process is repeated adding $C1+C1/10$ to proceed from a C-term to the next until the last term <100, i.e. S(89) is reached.

Case 3b: n=20, p=11. Period=11. Interval: $10 \le n \le 99$.

This case does not differ much from the case n=12. We have

```
S(20)=12_.._20_____________________________________________20_.._21
S(31)=12_.._20212223242526272829303131302928272625242322212 0_.._21
C=          21222324252627282930313130292827262524232221=
C1=         10101010101010101010101010101010101010101010+
C2=          1020304050607080910111110090807060504030201
```

The C-term for S(42) is

```
3·C1+C1/10+C2=32333435363738394041424241403938373635343332
```

In general $C=x\cdot C1+(x-1)\cdot C1/10+C2$ for x=3,4,5, ..,8. For x=8 the last term S(97) of this sequence is reached.

Case 4: n=102, p=333667. Period=3. Interval: 100≤n≤999.

```
S(102)=12_.._101102__________________102101_.._21
S(105)=12_.._101102103104105105104103102101_.._21
C=                 103104105105104103             ≡0 (mod 333667)
C1=                100100100100100100             ≡0 (mod 333667)
C2=                  3004005005004003             ≡0 (mod 333667)
```

Removing 1 or 2 zeros at the end of C1 does not affect the congruence modulus 333667, we have:

```
C1'=                10010010010010010             ≡0 (mod 333667)
C1''=                1001001001001001             ≡0 (mod 333667)
```

We now form the combinations:

x·C1+y·C1'+z·C1''+C2≡0 (mod 333667)

This, in my mind, is quite remarkable: All 18-digit integers formed by the concatenation of three consecutive 3-digit integers followed by a concatenation of the same integers in descending order are divisible by 333667, example 376377378378377376≡0 (mod 333667). As far as the C-terms are concerned all S(n) in the range 100≤n≤999 could be divisible by 333667, but they are not. Why? It is because S(100) and S(101) are not divisible by 333667. Consequently n=100+3k and 101+3k can not be used for insertion of an appropriate C-value as we did in the case of S(102). This completes the explanation of the remarkable fact that every third term S(102+3j) in the range 100≤n≤999 is divisible by 333667.

These three cases have shown what causes the periodicity of the divisibility of the Smarandache symmetric sequence of the second order by primes. The mechanism is the same for the other periodic sequences.

Beyond 1000

We have seen that numbers of the type:

10101010…10, 100100100…100, 10001000…1000, etc

play an important role. Such numbers have been factorized and the occurrence of our favorite primes 11, 37, ..., 333667 have been listed in table 7. In this table a number like 100100100100 has been abbreviated 4(100) or q(E), where q and E are listed in separate columns.

Question 1. Does the sequence of terms S(n) divisible by 333667 continue beyond 1000?

Although S(n) was partially factorized only up n=200 we have been able to draw conclusions on divisibility up n=1000. The last term that we have found divisible by 333667 is S(999). Two conditions must be met for there to be a sequence of terms divisible by p=333667 in the interval 1000≤n≤9999.

Condition 1. There must exist a number 10001000...1000 divisible by 333667 to ensure the periodicity as we have seen in our case studies.
In table 7 we find q=9, E=1000. This means that the periodicity will be 9 – if it exists, i.e. condition 1 is met.

Condition 2. There must exist a term S(n) with n≥1000 divisible by 333667 which will constitute the first term of the sequence.
The last term for n<1000 which is divisible by 333667 is S(999) from which we build

S(108)=12_999_1000_..._1008_1008_...1000_999-...21

where we deconcatenate 100010011002...10081008...10011000 which is divisible by 333667 and provides the C-term (as introduced in the case studies) needed to generate the sequence, i.e. condition 2 is met.

We conclude that S(1008+9j)≡0 (mod 333667) for j=0,1,2, ... 999. The last term in this sequence is S(9999). From table 7 we see that there could be a sequence with the period 9 in the interval 10000≤n≤99999 and a sequence with period 3 in the interval 100000≤n≤999999. It is not difficult to verify that the above conditions are filled also in these intervals. This means that we have:

S(1008+9j)≡0 (mod 333667)	for j=01,2,...,999, i.e. $10^3 \le n \le 10^4-1$
S(10008+9j)≡0 (mod 333667)	for j=01,2,...,9999, i.e. $10^4 \le n \le 10^5-1$
S(100002+3j)≡0 (mod 333667)	for j=01,2,...,99999, i.e. $10^5 \le n \le 10^6-1$

It is one of the fascinations with large numbers to find such properties. This extraordinary property of the prime 333667 in relation to the Smarandache symmetric sequence probably holds for $n>10^6$. It easy to loose contact with reality when plying with numbers like this. We have S(999999)≡0 (mod 333667). What does this number S(999999) look like? Applying (1) we find that the number of digits D(999999) of S(999999) is

$$D(999999)=2\cdot 6\cdot 10^6-2\cdot(10^6-)/9=11777778$$

Let's write this number with 80 digits per line, 60 lines per page, using both sides of the paper. We will need 1226 sheets of paper – more that 2 reams!

Question 2. Why is there no sequence of S(n) divisible by 11 in the interval 100≤n≤999?

Condition1. We must have a sequence of the form 100100.. divisible by 11 to ensure the periodicity. As we can see from table 7 the sequence 100100 fills the condition and we would have a periodicity equal to 2 if the next condition is met.

Condition 2. There must exist a term S(n) with n≥100 divisible by 11 which would constitute the first term of the sequence. This time let's use a nice property of the prime 11:

$$10^s \equiv (-1)^s \pmod{11}$$

Let's deconcatenate the number a_b corresponding to the concatenation of the numbers a and b: We have:

$$a\_b = a\cdot 10^{1+[\log_{10} b]} + b = \begin{cases} -a+b \text{ if } 1+[\log_{10}b] \text{ is odd} \\ a+b \text{ if } 1+[\log_{10}b] \text{ is even} \end{cases}$$

Let's first consider a deconcatenated middle part of S(n) where the concatenation is done with three-digit integers. For convienience I have chosen a concrete example – the generalization should pose no problem

```
273274275275274273≡2-7+3-2+7-4+2-7+5-2+7-5+2-7+4-2+7-3≡0 (mod 11)
+-+-+-+-+-+-+-+-+-
```

It is easy to see that this property holds independent of the length of the sequence above and whether it start on + or -. It is also easy to understand that equivalent results are obtained for other primes although factors other than +1 and –1 will enter into the picture.

We now return to the question of finding the first term of the sequence. We must start from n=97 since S(97) it the last term for which we know that S(n)≡0 (mod 11). We form:

```
9899100101..n_n..1011009998≡2 (mod 11) independent of n<1000.
+-+-+-+-+-.. _ ..-+-+-+-+-
```

This means that S(n)≡2 (mod 11) for 100≤n≤999 and explains why there is no sequence divisible by 11 in this interval.

Question 3. Will there be a sequence divisible by 11 in the interval 1000≤n≤9999?

Condition 1. A sequence 10001000...1000 divisible by 11 exists and would provide a period of 11, se table 7.

Condition 2. We need to find one value n ≥1000 for which S(n)≡0 (mod 11). We have seen that S(999)≡2 (mod 11). We now look at the sequences following S(999). Since S(999)≡2 (mod 9) we need to insert a sequence 10001001..m_m...10011000≡9 (mod 11) so that S(m)≡0 (mod 11). Unfortunately m does not exist as we will see below

```
10001000≡2 (mod 11)
+-+-+-+-
1   1
1000100110011000≡2. (mod 11)
+-+-+-+-+-+-+-+-
1   1   1   1
        1   1
100010011002100210011000≡0 (mod 11)
+-+-+-+-+-+-+-+-+-+-+-+-
1   1   1   1   1   1
        1   2   2   1
10001001100210031003100210011000≡-4≡7 (mod 11)
+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+
1   1   1   1   1   1   1   1
        1   2   3   3   2   1
```

Continuing this way we find that the residues form the period 2,2,0,7,1,4,5,4,1,7,0. We needed a residue to be 9 in order to build sequences divisible by 9. We conclude that S(n) is not divisible by 11 in the interval 1000≤n≤9999.

Trying to do the above analysis with the computer programs used in the early part of this study causes overflow because the large integers involved. However, changing the approach and performing calculations modulus 11 posed no problems. The above method was preferred for clarity of presentation.

Epilog

There are many other questions that may be interesting to look into. This is left to the reader. The author's main interest in this has been to develop means by which it is possible to identify some properties of large numbers other than the so frequently asked question as to whether a big number is a prime or not. There are two important ways to generate large numbers that I found particularly interesting – iteration and concatenation. In this article the author has drawn on work done previously, references below. In both these areas very large numbers may be generated for which it may be impossible to find any practical use – the methods are often more important than the results.

References:

1. Tabirca, S. and T., *On Primality of the Smarandache Symmetic Sequences*, Smarandache Notions Journal, Vol. 12, No 1-3 Spring 2001, 114-121.
2. Smarandache F., *Only Problems, Not Solutions*, Xiquan Publ., Pheonix-Chicago, 1993.
3. Ibstedt H. *Surfing on the Ocean of Numbers*, Erhus University Press, Vail, 1997.
4. Ibstedt H, *Some Sequences of Large Integers*, Fibonacci Quarterly, 28(1990), 200-203.

Table 1. Prime factors of S(n) which are less than 10^8

n	Prime factors of S(n)	n	Prime factors of S(n)
1	11	51	3.37.1847.F180
2	3.11.37	52	F190
3	3.11.37.101	53	3^3.11.43.26539.17341993.F178
4	11.41.101.271	54	3^3.37.41.151.271.347.463.9091.333667.F174
5	3.7.11.13.37.41.271	55	67.F200
6	3.7.11.13.37.239.4649	56	3.11.F204
7	11.73.101.137.239.4649	57	3.31.37.F206
8	3^2.11.37.73.101.137.333667	58	227.9007.20903089.F200
9	3^2.11.37.41.271.9091.333667	59	3.41.97.271.9091.F207
10	F22	60	3.37.3368803.F213
11	3.43.97.548687.F16	61	91719497.F218
12	3.11.31.37.61.92869187.F15	62	3^2.1693.F225
13	109.3391.3631.F24	63	3^2.37.305603.333667.9136499.F213
14	3.41.271.9091.290971.F24	64	11.41.271.9091.F229
15	3.37.661.F37	65	3.839.F238
16	F46	66	3.37.43.F242
17	3.F49	67	11^2.109.467.3023.4755497.F233
18	3^2.37.1301.333667.6038161.87958883.F28	68	3.97.5843.F247
19	41.271.9091.F50	69	3.37.41.271.787.9091.716549.19208653.F232
20	3.11.97.128819.F53	70	F262
21	3.37.983.F61	71	3.F265
22	67.773.F65	72	3^2.31.37.61.163.333667.77696693.F248
23	3.11.7691.F68	73	379.323201.F266
24	3.37.41.43.271.9091.165857.F61	74	3.41^2.43^2.179.271.9091.8912921.F255
25	227.2287.33871.611999.F66	75	3.11.37.443.F276
26	3^3.163.5711.68432503.F70	76	1109.F283
27	3^3.31.37.333667.481549.F74	77	3.10034243.F282
28	146273.608521.F83	78	3.11.37.71.41549.F284
29	3.41.271.9091.F89	79	41.271.9091.F290
30	3.37.5167.F96	80	3.F300
31	11^3.4673.F99	81	3^5.37.333667.4274969.F289
32	3.43.1021.F104	82	F310
33	3.37.881.F109	83	3.20399.5433473.F302
34	11.41.271.9091.F109	84	3.37^2.41.271.9091.F306
35	3^2.3209.F117	85	1783.627041.F313
36	3^2.37.333667.68697367.F110	86	3.11.F324
37	F130	87	3.31.37.43.F324
38	3.1913.12007.58417.597269.63800419.F107	88	67.257.46229.F325
39	3.37.41.271.347.9091.23473.F121	89	3^2.11.41.271.9091.653659.76310887.F314
40	F142	90	3^2.37.244861.333667.F328
41	3.156841.F140	91	173.F343
42	3.11.31.37.61.20070529.F136	92	3.F349
43	71.5087.F148	93	3.37.1637.F348
44	3^2.41.271.9091.1553479.F142	94	41.271.9091.10671481.F343
45	3^2.11.37.43.333667.F151	95	3.43.2833.F356
46	F166	96	3.37.683.F361
47	3.F169	97	11.26974499.F361
48	3.37.173.60373.F165	98	3^2.1299169.F367
49	41.271.929.9091.34613.F162	99	3^2.37.41.271.2767.9091.263273.333667.481417.F347
50	3.167.1789.9923.F172	100	43.47.53.83.683.3533.4919.F367

Table 1 continued

n	Prime factors of S(n)	n	Prime factors of S(n)
101	3.F389	151	47.5783.405869.F679
102	3.149.21613.106949.333667.F378	152	3^2.53.F693
103	45823.F397	153	3^2.359.39623.333667.7192681.F681
104	3.41.271.28813.F399	154	41.73.271.487.14843.F695
105	3.47.333667.11046661.F399	155	3.14717.F709
106	73.167.F416	156	3.43.601.1289.14153.333667.1479589.11337023.F689
107	3^3.43.1447.1741.28649.161039.F406	157	F726
108	3^3.569.333667.F422	158	3.49055933.F723
109	41.271.367.9091.F427	159	3.37.41.271.347.9091.333667.F719
110	3.F443	160	97.179.1277.F736
111	3.313.333667.F441	161	3^4.3251.75193.496283.F734
112	F456	162	3^4.73.26881.28723.333667.3211357.F731
113	3.53.71.2617.52081.F449	163	43.1663.F757
114	3.41.43.73.271.333667.F454	164	3.41.271.136319.F758
115	2309.F470	165	3.53.83.919.184859.333667.3014983.F749
116	3.F479	166	1367.1454371.F770
117	3^2.333667.4975757.F472	167	3.F785
118	167.11243.13457.414367.F476	168	3.19913.333667.F781
119	3.41.271.9091.132059.182657.F479	169	41.271.2273.9091.F786
120	3.1511.7351.20431.167611.333667.57228299.F473	170	3^2.43.73.967.F796
121	43.501233.F502	171	3^2.333667.F803
122	3.37.73.2659.F508	172	643.96293.325681.7607669.F795
123	3.112207.333667.F511	173	3.37.F820
124	41.83.271.367.37441.F514	174	3.41.271.19423.333667.F813
125	3.F533	175	3607.20131291.F823
126	3^2.53.333667.395107.972347.F520	176	3.F839
127	F546	177	3.43.173.333667.F836
128	3.43.97.179.181.347.F540	178	53.73.11527.461317.F838
129	3.41.271.9091.333667.F544	179	3^2.41.271.1033.9091.F846
130	73.313.275083.F554	180	3^2.2861.26267.333667.1894601.F843
131	3.263.12511.210491.95558129.F549	181	F870
132	3.333667.F570	182	3.83.2417.F870
133	F582	183	3.71.1097.333667.F871
134	3^3.41.173.271.F580	184	41.43.271.F882
135	3^3.43.59.333667.F583	185	3.317371.F888
136	37.F598	186	3.73.333667.F892
137	3.F605	187	F906
138	3.73.28817.333667.F599	188	3^3.181.1129.5179.F901
139	41.53.271.9091.19433.F604	189	3^3.41.271.9091.13627.333667.F898
140	3.380623.F618	190	194087.F918
141	3.83.257.1091.333667.29618101.F609	191	3.43.53.401.F923
142	43.F634	192	3.47.97.333667.14445391.F919
143	3^2.8922281.F634	193	59.F940
144	3^2.41.59.271.1493.333667.F632	194	3.41.73.271.487.42643.F934
145	977.22811.5199703.F640	195	3.179533.333667.F942
146	3.47.73.F656	196	37.661.F955
147	3.1483.2341.333667.F653	197	3^2.47.18427.6309143.32954969.F944
148	71.14271083.47655077.F655	198	3^2.43^2.333667.F962
149	3.41.43.271.9091.F667	199	41.271.9091.10151.719779.F960
150	3.333667.F678	200	3.4409.F979

Table 3. Smarandache Symmetric Sequence of Second Order: The most frequently occurring prime factors.

#	11	diff	#	37	diff	#	41	diff	#	43	diff	#	271	diff	#	9091	diff	#	333667	diff
1	11		2	37		4	41		11	43		4	271		9	9091		8	333667	
2	11	1	3	37	1	5	41	1	24	43	13	5	271	1	14	9091	5	9	333667	1
3	11	1	5	37	2	9	41	4	32	43	8	9	271	4	19	9091	5	18	333667	9
4	11	1	6	37	1	14	41	5	45	43	13	14	271	5	24	9091	5	27	333667	9
5	11	1	8	37	2	19	41	5	53	43	8	19	271	5	29	9091	5	36	333667	9
6	11	1	9	37	1	24	41	5	66	43	13	24	271	5	34	9091	5	45	333667	9
7	11	1	12	37	3	29	41	5	74	43	8	29	271	5	39	9091	5	54	333667	9
8	11	1	15	37	3	34	41	5	87	43	13	34	271	5	44	9091	5	63	333667	9
9	11	1	18	37	3	39	41	5	95	43	8	39	271	5	49	9091	5	72	333667	9
12	11	3	21	37	3	44	41	5	100	43	5	44	271	5	54	9091	5	81	333667	9
20	11	8	24	37	3	49	41	5	107	43	7	49	271	5	59	9091	5	90	333667	9
23	11	3	27	37	3	54	41	5	114	43	7	54	271	5	64	9091	5	99	333667	9
31	11	8	30	37	3	59	41	5	121	43	7	59	271	5	69	9091	5	102	333667	3
34	11	3	33	37	3	64	41	5	128	43	7	64	271	5	74	9091	5	105	333667	3
42	11	8	36	37	3	69	41	5	135	43	7	69	271	5	79	9091	5	108	333667	3
45	11	3	39	37	3	74	41	5	142	43	7	74	271	5	84	9091	5	111	333667	3
53	11	8	42	37	3	79	41	5	149	43	7	79	271	5	89	9091	5	114	333667	3
56	11	3	45	37	3	84	41	5	156	43	7	84	271	5	94	9091	5	117	333667	3
64	11	8	48	37	3	89	41	5	163	43	7	89	271	5	99	9091	5	120	333667	3
67	11	3	51	37	3	94	41	5	170	43	7	94	271	5	109	9091	10	123	333667	3
75	11	8	54	37	3	99	41	5	177	43	7	99	271	5	119	9091	10	126	333667	3
78	11	3	57	37	3	104	41	5	184	43	7	104	271	5	129	9091	10	129	333667	3
86	11	8	60	37	3	109	41	5	191	43	7	109	271	5	139	9091	10	132	333667	3
89	11	3	63	37	3	114	41	5	198	43	7	114	271	5	149	9091	10	135	333667	3
97	11	8	66	37	3	119	41	5				119	271	5	159	9091	10	138	333667	3
			69	37	3	124	41	5				124	271	5	169	9091	10	141	333667	3
			72	37	3	129	41	5				129	271	5	179	9091	10	144	333667	3
			75	37	3	134	41	5				134	271	5	189	9091	10	147	333667	3
			78	37	3	139	41	5				139	271	5	199	9091	10	150	333667	3
			81	37	3	144	41	5				144	271	5				153	333667	3
			84	37	3	149	41	5				149	271	5				156	333667	3
			87	37	3	154	41	5				154	271	5				159	333667	3
			90	37	3	159	41	5				159	271	5				162	333667	3
			93	37	3	164	41	5				164	271	5				165	333667	3
			96	37	3	169	41	5				169	271	5				168	333667	3
			99	37	3	174	41	5				174	271	5				171	333667	3
			122	37	23	179	41	5				179	271	5				174	333667	3
			136	37	14	184	41	5				184	271	5				177	333667	3
			159	37	23	189	41	5				189	271	5				180	333667	3
			173	37	14	194	41	5				194	271	5				183	333667	3
			196	37	23	199	41	5				199	271	5				186	333667	3
																		189	333667	3
																		192	333667	3
																		195	333667	3
																		198	333667	3

Table 4. Smarandache Symmetric Sequence of Second Order: Less frequently occurring prime factors.

#	p	d	#	p	d	#	p	d	#	p	d	#	p	d	#	p	d	#	p
5	7		7	73		50	167		15	661		147	2341		154	14843		24	165857
6	7	1	8	73	1	106	167	56	196	661		182	2417		197	18427		120	167611
5	13		106	73	98	118	167	12	96	683		113	2617		174	19423		195	179533
6	13	1	114	73	8	48	173		100	683		122	2659		139	19433		119	182657
12	31		122	73	8	91	173	43	22	773		99	2767		168	19913		165	184859
27	31	15	130	73	8	134	173	43	69	787		95	2833		83	20399		190	194087
42	31	15	138	73	8	177	173	43	65	839		180	2861		120	20431		131	210491
57	31	15	146	73	8	74	179		33	881		67	3023		102	21613		90	244861
72	31	15	154	73	8	128	179	54	165	919		35	3209		145	22811		99	263273
87	31	15	162	73	8	160	179	32	49	929		161	3251		39	23473		130	275083
100	47		170	73	8	128	181		170	967		13	3391		180	26267		14	290971
105	47	5	178	73	8	188	181		145	977		100	3533		53	26539		63	305603
146	47	41	186	73	8	25	227		21	983		175	3607		162	26881		185	317371
151	47	5	194	73	8	58	227		32	1021		13	3631		107	28649		73	323201
192	47	41	100	83		6	239		179	1033		200	4409		162	28723		172	325681
197	47	5	124	83	24	7	239		141	1091		6	4649		104	28813		140	380623
100	53		141	83	17	88	257		183	1097		7	4649		138	28817		126	395107
113	53	13	165	83	24	141	257		76	1109		31	4673		25	33871		151	405869
126	53	13	182	83	17	131	263		188	1129		100	4919		49	34613		118	414367
139	53	13	11	97		111	313		160	1277		43	5087		124	37441		178	461317
152	53	13	20	97	9	130	313		156	1289		30	5167		153	39623		99	481417
165	53	13	59	97	39	39	347		18	1301		188	5179		78	41549		27	481549
178	53	13	68	97	9	54	347	15	166	1367		26	5711		194	42643		161	496283
191	53	13	128	97	60	128	347	74	107	1447		151	5783		103	45823		121	501233
135	59		160	97	32	159	347	31	147	1483		68	5843		88	46229		11	548687
144	59	9	192	97	32	153	359		144	1493		120	7351		113	52081		38	597269
193	59	49	3	101		109	367		120	1511		23	7691		38	58417		28	608521
12	61		4	101	1	124	367		93	1637		58	9007		48	60373		25	611999
42	61	30	7	101	3	73	379		163	1663		50	9923		161	75193		85	627041
72	61	30	8	101	1	191	401		62	1693		199	10151		172	96293		89	653659
22	67		13	109		75	443		107	1741		118	11243		102	106949		69	716549
55	67	33	67	109		54	463		85	1783		178	11527		123	112207		199	719779
88	67	33	7	137		67	467		50	1789		38	12007		20	128819		126	972347
43	71		8	137		154	487		51	1847		131	12511		119	132059			
78	71	35	102	149		194	487		38	1913		118	13457		164	136319			
113	71	35	54	151		108	569		169	2273		189	13627		28	146273			
148	71	35	26	163		156	601		25	2287		156	14153		41	156841			
183	71	35	72	163		172	643		115	2309		155	14717		107	161039			

Table 7. Prime factors of q(E) and occurrence of selected primes

q	E	Prime factors <350000	Selected primes
2	10	2.5.101	
3	10	2.3.5.7.13.37	37
4	10	2.5.73.101.137	
5	10	2.5.41.271.9091	41,271,9091
6	10	2.3.5.7.13.37.101.9901	37,9091
7	10	2.5.239.4649.	
8	10	2.5.17.73.101.137.	
9	10	$2.3^2.5.7.13.19.37.52579.333667$	333667
10	10	2.5.41.101.271.3541.9091.27961	41,271,9091
11	10	2.5.11.23.4093.8779.21649.	11
12	10	2.3.5.7.13.37.73.101.137.9901.	37
13	10	2.5.53.79.859.	
14	10	2.5.29.101.239.281.4649.	
15	10	2.3.5.7.13.31.37.41.211.241.271.2161.9091.	37,41,271,9091
16	10	2.5.17.73.101.137.353.449.641.1409.69857.	
2	100	$2^2.5^2.7.11.13$	11
3	100	$2^2.3.5^2.333667$	333667
4	100	$2^2.5^2.7.11.13.101.9901$	11
5	100	$2^2.5^2.31.41.271.$	41,271
6	100	$2^2.3.5^2.7.11.13.19.52579.333667$	11,333667
7	100	$2^2.5^2.43.239.1933.4649.$	43
8	100	$2^2.5^2.7.11.13.73.101.137.9901.$	11,73
9	100	$2^2.3^2.5^2.757.333667.$	333667
10	100	$2^2.5^2.7.11.13.31.41.211.241.271.2161.9091.$	11,41,271,9091
11	100	$2^2.5^2.67.21649.$	
12	100	$2^2.3.5^2.7.11.13.19.101.9901.52579.333667.$	11,333667
2	1000	$2^3.5^3.73.137$	
3	1000	$2^3.3.5^3.7.13.37.9901$	37
4	1000	$2^3.5^3.17.73.137.$	
5	1000	$2^3.5^3.41.271.3541.9091.27961$	41,271,9091
6	1000	$2^3.3.5^3.7.13.37.73.137.9901.$	37
7	1000	$2^3.5^3.29.239.281.4649.$	
8	1000	$2^3.5^3.17.73.137.353.449.641.1409.69857.$	
9	1000	$2^3.3^2.5^3.7.13.19.37.9901.52579.333667.$	37,333667
10	1000	$2^3.3.5^3.41.73.137.271.3541.9091.27961.$	41,271,9091
11	1000	$2^3.5^3.11.23.89.4093.8779.21649.$	11
2	10000	$2^4.5^4.11.9091$	11,9091
3	10000	$2^4.3.5^4.31.37.$	37
4	10000	$2^4.5^4.11.101.3541.9091.27961$	11,9091
5	10000	$2^4.5^4.21401.25601.$	
6	10000	$2^4.3.5^4.7.11.13.31.37.211.241.2161.9091.$	11,37,9091
7	10000	$2^4.5^4.71.239.4649.123551.$	
8	10000	$2^4.5^4.11.73.101.137.3541.9091.27961.$	11,9091
9	10000	$2^4.3.5^4.31.37.238681.333667.$	37,333667
2	100000	$2^5.5^5.101.9901$	
3	100000	$2^5.3.5^5.19.52579.333667$	333667
4	100000	$2^5.5^5.73.101.137.9901..$	
5	100000	$2^5.5^5.31.41.211.241.271.2161.9091..$	41,271,9091
6	100000	$2^5.3.5^5.19.101.9901.52579.333667..$	333667
7	100000	$2^5.5^5.7.43.127.239.1933.2689.4649..$	43
8	100000	$2^5.5^5.17.73.101.137.9901..$	
9	100000	$2^5.3^2.5^5.19.757.52579.333667..$	333667