On a problem concerning the Smarandache Unary sequence

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Abstract

In this paper a problem posed in [1] and concerning the number of primes in the Smarandache Unary sequence is analysed.

Introduction

In [1] the Smarandache Unary sequence is defined as the sequence obtained concatenating p_n digits of 1, where p_n is the n-th prime number:

11, 111, 11111, 1111111, 11111111111, 1111111111111, 11111111111111111, ...

In the same paper the following open question is reported:

How many terms in the Smarandache Unary sequence are prime numbers?

In this paper we analyse that question and a conjecture on the number of primes belonging to the Smarandache Unary sequence is formulated.

Results

A computer program with Ubasic software package has been written to check the first 311 terms of the Unary sequence; we have found only five prime numbers. If we indicate the n-th term of the unary sequence as:

$$u(n) = \frac{10^{p_n} - 1}{9} \qquad \text{where} \quad p_n \text{ is the n-th prime.}$$

those five primes have been found for p_n equal to 2, 19, 23, 317 and 1031.

This means a percentage of $\frac{5}{311} \approx 1.6\%$ prime numbers. According to this experimental evidence the following conjecture can be formulated:

Conjecture: *The number of primes in the Smarandache Unary sequence is upper limited.*

Unsolved question: *Find that upper limit.*

References.

[1] F. Iacobescu, *Smarandache Partition type and other sequences*, Smarandache Notions Journal, Vol. 11 N. 1-2-3 Spring 2000