

ON AN INEQUALITY CONCERNING THE SMARANDACHE FUNCTION

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Abstract. Let a, n be positive integers. In this paper we prove that $S(a)S(a^2)\dots S(a^n) \leq n!(S(a))^n$.

Key words Smarandache function, inequality.

For any positive integer a , let $S(a)$ be the Smarandache function. In [1], Bencze proposed the following problem.

Problem. For any positive integers a and n , prove the inequality.

$$(1) \quad \prod_{k=1}^n S(a^k) \leq n!(S(a))^n.$$

In this paper we completely solve this problem. We prove the following result.

Theorem. For any positive integers a and n , the inequality (1) holds.

Proof By [2, Theorem], we have

$$S(ab) \leq S(a) + S(b),$$

for any positive integers a and b . It implies that

$$(2) \quad S(a^k) \leq kS(a),$$

for any positive integers a and k . Therefore, by (2), we get

$$(3) \quad \prod_{k=1}^n S(a^k) \leq \prod_{k=1}^n (kS(a)) = n!(S(a))^n.$$

Thus, the inequality (1) is proved.

References

- [1] M.Bencze, PP.1388, Octogon Math. Mag.,7(1999),2:149.
- [2] M.-H.Le, An inequality concerning the Smarandache function, Smarandache Notions J. 9(1998),124-125.

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