On Numbers Where the Values of the Pseudo-Smarandache Function Of It and The Reversal Are Identical

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The Pseudo-Smarandache function was introduced by Kenichiro Kashihara in a book that is highly recommended[1].

Definition: For any n $\geq$ 1, the value of the Pseudo-Smarandache function is the smallest integer m such that n evenly divides

$$\sum_{k=1}^{m} k.$$

Definition: Let d = $a_1 a_2 \ldots a_k$ be a decimal integer. The **reversal** of d, Rev(d) is the number obtained by reversing the order of the digits of d.

$$\text{Rev}(d) = a_k a_{k-1} \ldots a_2 a_1.$$

If d contains trailing zeros, they are dropped when they become leading zeros.

In this paper, we will look for numbers n, such that Z(n) = Z(Rev(n)) and note some of the interesting properties of the solutions. If n is palindromic, then the above property is true by default. Therefore, we will restrict our set of interest to all non-palindromic numbers n such that Z(n) = Z(Rev(n)).

A computer program was written to search for all such n for 1 $\leq$ n $\leq$ 100,000 and the solutions are summarized below.

Z(180) = 80 = Z(81)
Z(990) = 44 = Z(99)
Z(1010) = 100 = Z(101)
Z(1089)=242 = Z(9801)
Z(1210) = 120 = Z(121)
Z(1313) = 403 = Z(3131)
Z(1572) = 392 = Z(2751)
Z(1810) = 180 = Z(181)
Z(1818) = 404 = Z(8181)
Z(2120) = 159 = Z(212)

Z(2178) = 1088 = Z(8712)
Z(2420) = 120 = Z(242)
Z(2626) = 403 = Z(6262)
Z(2720) = 255 = Z(272)
Z(2997) = 1295 = Z(7992)
Z(3630) = 120 = Z(363)
Z(3636) = 504 = Z(6363)
Z(4240) = 159 = Z(424)
Z(4284) = 1071 = Z(4842)
Z(4545) = 404 = Z(5454)
Z(4640) = 319 = Z(464)
Z(5050) = 100 = Z(505)
Z(6360) = 159 = Z(636)
Z(7170) = 239 = Z(717)
Z(8780) = 439 = Z(878)
Z(9090) = 404 = Z(909)
Z(9490) = 364 = Z(949)
Z(9890) = 344 = Z(989)
Z(13332) = 1616 = Z(23331)
Z(15015) = 714 = Z(51051)
Z(16610) = 604 = Z(1661)
Z(21296) = 6655 = Z(69212)
Z(25520) = 319 = Z(2552)
Z(26664) = 1616 = Z(46662)
Z(27027) = 2079 = Z(72072)
Z(29970) = 1295 = Z(7992)
Z(32230) = 879 = Z(3223)
Z(37730) = 1715 = Z(3773)
Z(39960) = 1295 = Z(6993)
Z(45045) = 2079 = Z(54054)
Z(46662) = 1616 = Z(26664)
Z(49940) = 1815 = Z(4994)
Z(56650) = 824 = Z(5665)
Z(57057) = 2925 = Z(75075)
Z(63630) = 504 = Z(3636)
Z(64460) = 879 = Z(6446)
Z(80080) = 2079 = Z(8008)
Z(80640) = 4095 = Z(4608)
Z(81810) = 404 = Z(1818)
Z(92290) = 3355 = Z(9229)
Z(93390) = 1980 = Z(9339)
Z(96690) = 879 = Z(9669)
Z(97790) = 2540 = Z(9779)

Several items to note from the previous list.

a) Of the 52 solutions discovered, 35 of the numbers have one trailing zero, where many of them are palindromes when the zeros are dropped. While no numbers with two trailing zeros were found, it seems likely that there are such numbers.

Unsolved Question: Given that Z(n) = Z(Rev(n)), what is the largest number of trailing zeros that n can have?

The previous question is directly related to the speed with which the Pseudo-Smarandache function grows.

d) Of the 17 remaining numbers, 9 exhibit the pattern $d_1d_2d_1d_2$ or $d_1d_20d_1d_2$.

Unsolved Question: Is this a pattern, in the sense that there is an infinite set of numbers n, such that $n = d_1d_20 \ldots 0d_1d_2$ and Z(n) = Z(Rev(n))?

e) Only three of the numbers contain unique nonzero digits and there are none with five digits.

Unsolved Question: What is the largest number of unique nonzero digits that a number n can have when Z(n) = Z(Rev(n)?

f) Three of the numbers exhibit the pattern $d_1d_2 \ldots d_2d_3$, with the largest interior pattern being three digits in length.

Unsolved Question: What is the largest interior pattern of repeating digits $d_2 \ldots d_2$ that can appear in a number $n = d_1d_2 \ldots d_2d_3$ such that Z(n) = Z(Rev(n))?

Reference

1. K. Kashihara, **Comments and Topics on Smarandache Notions and Problems**, Erhus University Press, Vail, AZ. 1996.