ON PRIMES IN THE SMARANDACHE PIERCED CHAIN

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Abstract. Let $C = \{c_n\}_{n=1}^{\infty}$ be the Smarandache pierced chain. In this paper we prove that if $n > 2$, then $c_n / 101$ is not a prime.

For any positive integer n, let

(1) $$c_n = 101 * \underbrace{100010001 \ldots 0001}_{n-1 \text{ times}}.$$

Then the sequence $C = \{c_n\}_{n=1}^{\infty}$ is colled the Smarandache percied chain (see[2, Notion 19]). In [3], Smarandache asked the following question:

Question. How many $c_n / 101$ are primes?

In this paper we give a complete anser as follows:

Theorem. If $n > 2$, then $c_n / 101$ is not a prime.

Proof. Let $\zeta_n = e^{2\pi\sqrt{-1}/n}$ be a primitive roof of unity with the degree n, and let

$$f_n(x) = \prod_{\substack{1 \le k \le n \\ gcd(k,n)=1}} (x - \zeta_n^{\,k}).$$

Then $f_n(x)$ is a polynomial with integer coefficients. Further, it is a well known fact that if $x > 2$, then $f_n(x) > 1$ (see [1]). This implies that if x is an integer with $x > 2$, then $f_n(x)$ is an integer with $f_n(x) > 1$. On the other hand, we have

(2) $$x^n - 1 = \prod_{d/n} f_d(x).$$

We see from (1) that if $n > 1$, then

(3) $$\frac{c_n}{101} = 1 + 10^4 + 10^8 + \ldots + 10^{4(n-1)} = \frac{10^{4n} - 1}{10^4 - 1}.$$

By the above definition, we find from (2) and (3) that

$$\frac{c_n}{101} = \left(\prod_{d|4n} f_d(10)\right) / \left(\prod_{d|n} f_d(10)\right).$$

Since $n > 2$, we get $2n > 4$ and $4n > 4$. It implies that both $2n$ and $4n$ are divisors of $4n$ but not of 4. Therefore, we get from (4) that

(5) $$\frac{c_n}{101} = f_{2n}(10)\ f_{4n}(10)t,$$

where t is not a positive integer. Notice that $f_{2n}(10) > 1$ and $f_{4n}(10) > 1$. We see from (5) that $c_n / 101$ is not a prime. The theorem is proved.

References

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