ON RUSSO'S CONJECTURE ABOUT PRIMES

Maohua Le

Abstract . Let n, k be positive integres with $k>2$, and let b be a positive number with $b\geq 1$. In this paper we prove that if $n>C(k)$, where $C(k)$ is an effectively computable constant depending on k, then we have $C(n,k)<2/k^b$.

Key words . Russo's conjecture, prime, gap, Smarandache constant.

For any positive integer n, let $P(n)$ be the n-th prime. Let k be a positive integer with $k>1$, and let

(1) $$C(n,k)=(P(n+1))^{1/k}-(P(n))^{1/k}.$$

In [2], Russo has been conjectured that

(2) $$C(n,k)<\frac{2}{k^{2a}},$$

where a=0.5671481302020177464684687 55... is the Smarandache constant. In this paper we prove a general result as follows.

Theorem. For any positive number b with $b\geq 1$, if $k>2$ and $n>C(k)$, where $C(k)$ is an effectively computable constant depending on k, then we have

(3) $$C(n,k)<\frac{2}{k^b}.$$

Proof. Since $k>2$, we get from (1) that

(4) $$C(n,k)<\frac{2}{k^b}\left(\frac{(P(n+1)-P(n))k^{b-1}}{2(P(n)^{2/3})}\right).$$

By the result of [1], we have

(5) $$P(n+1)-P(n)<C'(t)(P(n))^{11/20+t},$$

for any positive number t, where $C'(t)$ is an effectively computable constant depending on t. Put t=1/20. Since $k\geqslant 3$ and $(k-1)/k\geqslant 2/3$, we see from (4) and (5) that

(6) $$C(n,k)<\frac{2}{k^b}\left(\frac{C'(1/20)\,k^{b-1}}{2(P(n))^{1/15}}\right).$$

Notice that $C'(1/20)$ is an effectively computable absolute constant and $P(n)>n$ for any positive integer n. Therefore, if $n>C(k)$, then $2(P(n))^{1/15}>C'(1/20)k^{b-1}$. Thus, by (6), the inequaliy (3) holds. The theorem is proved.

References

[1] D.R. Heath-Brown and H. Iwaniec, On the difference between consecutive primes, Invent. Math. 55 (1979),49-69.

[2] F. Russo, An experimetal evidence on the validity of third Smarandache conjecture on primes, Smarandache Notions J. 11(2000), 38-41.

Department of Mathematics
Zhanjiang Normal College
Zhanjiang , Guangdong
P.R. CHINA