ON SMARANDACHE ALGEBRAIC STRUCURES. II: THE SMARANDACHE SEMIGROUP

Maohua Le

Abstract. In this paper we prove that $A(a,n)$ is a Smarandache semigroup.

Key words. Smarandache algorithm, Smarandache semigroup.

Let G be a semigroup. If G contains a proper subset which is a group under the same operation, then G is called a Smarandache semigroup (see [2]). For example, $G=\{18,18^2,18^3,18^4,18^5\}$ (mod 60) is a commutative multiplicative semigoup. Since the subset $\{18^2,18^3,18^4,18^5\}$ (mod 60) is a group, G is a Smarandache semigroup.

Let a,n be integers such that $a \neq 0$ and $n>1$. Further, let $A(a,n)$ be defined as in [1]. In this paper we prove the following result.

Theorem. $A(a,n)$ is a Smarandache semigroup.

Proof. Under the definitions and notations of [1], let $A'(a,n)=\{a^e,a^{e+1},\cdots e^{e+f-1}\}$ (mod n). Then $A'(n,a)$ is a proper subset of $A(a,n)$.

If $e=0$, then $a^e=1 \in A'(a,n)$. Clear, 1 is the unit of $A'(a,n)$. Moreover, for any $a^i \in A(a,n)$ with $i>0$, a^{f-i} is the inverse element of a^i in $A'(a,n)$.

If $e>0$, then $a^f \in A'(a,n)$. Since $a^f \equiv 1$ (mod m), a^f is the unit of $A'(a,n)$ and a^{ft-i} is the inverse element of a^i in $A'(a,n)$, where t is the integer satisfying $e<ft-i \leqslant e+f-1$. Thus, under the Smarandache algorithm, $A'(a,n)$ is a group. It implies that $A(a,n)$ is a Smarandache

semigroup. The theorem is proved.

References

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Department of Mathematics
Zhanjiang Normal College
Zhanjiang, Guangdong
P.R. CHINA