# ON SMARANDACHE ALGEBRAIC STRUCTURES. IV : THE COMMUTATIVE RING $C(a,n)$

Maohua Le

**Abstract**. In this paper we construct a new class of commutative rings under the Smarandache algorithm.

**Key words**. Smarandache algorithm, commutative ring.

Let $a,n$ be integers such that $a \neq 0$ and $n>1$. Let $d=\gcd(a,n)$, $b=a/d$ and $t=n/d$. Further, let

(1) $$C(a,n) = \{0, a, 2a, \ldots, (t-1)a\} \pmod{n}.$$

In this paper we prove the following result.

**Theorem**. $C(a,n)$ is a commutative ring under the Smarandache additive and multiplicative.

**Proof**. Let $u$, $v$ be two elements of $C(a,n)$. By (1), we have

(2) $$u=ia,\ v=ja, 0 \leqslant i,j \leqslant t-1.$$

Let $r$ be the least nonnegative residue of $i+j$ modulo $t$. Since $d=\gcd(a,n)$ and $n=dt$, we get from (2) that

(3) $$u+v \equiv (i+j)a \equiv ra \pmod{n}, 0 \leqslant r \leqslant t-1.$$

It implies that $u+v$ belongs to $C(a,n)$. Therefore, it is a commutative additive group under the Smarandache algorithm (see [1]).

On the other hand, let $r'$ be the least nonnegative residue of $ija$ modulo $t$. By (2), we get

(4) $$uv \equiv ija^2 \equiv r'a \pmod{n}, 0 \leqslant r' \leqslant t-1.$$

Hence, by (4), $C(a,n)$ is a commutative multiplicative semigroup. Thus, $C(a,n)$ is a commutative ring. The theorem is proved.

## Reference

[1] R. Padilla, Smarandache algebraic structures, Smarandache Notions J. 9(1998), 36-38.

Department of Mathematics
Zhanjiang Normal College
Zhanjiang, Guangdong
P.R. CHINA