ON SMARANDACHE DIVISOR PRODUCTS

Maohua Le

Department of Mathematics, Zhanjiang Normal College
Zhanjiang, Guangdong, P.R.China.

Abstract. In this paper we give a formula for Smarandache divisor products.

Let n be a positive integer. In [1, Notion 20], the product of all positive divisors of n is called the Smarandache divisor product of n and denoted by $P_d(n)$. In this paper we give a formula of $P_d(n)$ as follows:

Theorem. Let $n = p_1^{r_1} \dots p_k^{r_k}$ be the factorization of n, and let

$$(1) \quad r(n) = \begin{cases} \frac{1}{2}(r_1+1)\dots(r_k+1), & \text{if n is not a square,} \\ \frac{1}{2}((r_1+1)\dots(r_k+1)-1), & \text{if n is a square.} \end{cases}$$

Then we have $P_d(n) = n^{r(n)}$.

Proof. Let f(n) denote the number of distinct positive divisors of n. It is a well known fact that

$$(2) \qquad f(n) = (r_1+1)\dots(r_k+1),$$

(See [2, Theorem 273]). If n is not a square and d is a positive divisor of n, then n/d is also a positive divisor of n with $n / d \neq d$. It implies that

$$(3) \qquad P_d(n) = n^{f(n)/2}.$$

Hence, by (1), (2) and (3), we get $P_d(n) = n^{r(n)}$.

If n is a square and d is a positive divisor of n with $d \neq \sqrt{n}$, then n/d is also a positive divisor of n with $n / d \neq d$. So we have

$$(4) \qquad P_d(n) = \frac{n^{f(n)/2}}{\sqrt{n}} = n^{(f(n-1)/2}.$$

Therefore, by (1), (2) and (4), we get $P_d(n) = n^{r(n)}$ too. The theorem is proved.

References.

1. Dumitrescu and V. Seleacu, Some Notions and Questions In Number Theory, Erhus Univ. Press, Glendale, 1994.
2. G.H.Hardy and e.M.Wright, An Introduction to the Theory of numbers, Oxford Univ. Press, Oxford, 1938.