On Smarandache sequences and subsequences

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Abstract A Smarandache sequence partial perfect additive sequence is studied completely in the first paragraph. In the second paragraph both Smarandache square-digital subsequence and square-partial-digital subsequence are studied.

Key words Smarandache partial perfect additive sequence, Smarandache square-digital subsequence , Smarandache square-partial-digital subsequence.

§ 1 Smarandache partial perfect additive sequence

The Smarandache partial perfect additive sequence is defined to be a sequence: 1, 1, 0, 2, −1, 1, 1, 3, −2, 0, 0, 2, 0, 2, 2, 4, −3, −1, −1, 1, −1, 1, 1, 3, −1, 1, ⋯.

This sequence has the property that:

$$\sum_{i=1}^{p} a_i = \sum_{j=p+1}^{2p} a_j, \quad \text{for all} \quad p \geq 1.$$

It is constructed in the following way:

$$a_1 = a_2 = 1,$$

$$a_{2p+1} = a_{p+1} - 1,$$

and $$a_{2p+2} = a_{p+1} + 1 \quad \text{for all} \quad p \geq 1.$$

In [1] M. Bencze raised the following two questions:

(a) Can you, readers, find a general expression of a_n (as function of n)?

Is it periodical or convergent or bounded?

(b) Please design (invent) yourselves other Smarandache perfect (or partial perfect) f-sequences.

In this paper we solved the question (a) completely.

Suppose the binary notation of $n(n \geq 2)$ as $n = (\varepsilon_k \varepsilon_{k-1} \cdots \varepsilon_1 \varepsilon_0)_2$, among which $\varepsilon_k = 1, \varepsilon_i = 0$ or 1 $(i = 0,1,\cdots,k-1)$. Define $f(n)$ are the numbers of $\varepsilon_i = 1 (i = 0,1,\cdots,k)$, $g(n)$ is the minimum of i that makes $\varepsilon_i = 1$.

Thus we may prove the expression of $a(n)$ (i.e. a_n) as the following:

$$a(n) = \begin{cases} k, & \text{if } \varepsilon_0 = \varepsilon_1 = \cdots = \varepsilon_{k-1} = 0, \\ -k + 2f(n) + 2g(n) - 3 & \text{otherwise} \end{cases}$$

We may use mathematical induction to prove it.

$a(2) = 1, a(3) = 0 = -1 + 2 \times 2 + 2 \times 0 - 3 = -1 + 2f(3) + 2g(3) - 3.$

So the conclusion is valid for $n = 2,3,$

Suppose that the conclusion is also valid for $2,3,\cdots,n-1 (n \geq 3)$. Let's consider the cases of n.

1 When $\varepsilon_0 = \varepsilon_1 = \cdots = \varepsilon_{k-1} = 0$.

$a(n) = a((\varepsilon_k \varepsilon_{k-1} \cdots \varepsilon_1 \varepsilon_0)_2) = a((\varepsilon_k \varepsilon_{k-1} \cdots \varepsilon_1) + 1$

$= k - 1 + 1 = k$.

2 When not all the $\varepsilon_0, \varepsilon_1, \cdots, \varepsilon_{k-1}$ are zeroes, two kinds of cases should be discussed.

(1) If $\varepsilon_0 = 0$.

Then $f(n) = f((\varepsilon_k\varepsilon_{k-1}\cdots\varepsilon_1\varepsilon_0)_2) = f((\varepsilon_k\varepsilon_{k-1}\cdots\varepsilon_1)_2) = f(\frac{n}{2})$,

$$g(n) = g((\varepsilon_k\varepsilon_{k-1}\cdots\varepsilon_1\varepsilon_0)_2) = g((\varepsilon_k\varepsilon_{k-1}\cdots\varepsilon_1)_2) + 1 = g(\frac{n}{2}) + 1.$$

According to inductive hypothesis, we have

$$a(n) = a(\frac{n}{2}) + 1 = -(k-1) + 2f(\frac{n}{2}) + 2g(\frac{n}{2}) - 3 + 1$$

$$= -k + 2f(n) + 2(g(n) - 1) - 1$$

$$= -k + 2f(n) + 2g(n) - 3.$$

(2) If $\varepsilon_0 = 1$, three subcases exist

(i) If $\varepsilon_1 = 0$.

Then $f(n) = f((\varepsilon_k\varepsilon_{k-1}\cdots\varepsilon_1\varepsilon_0)_2) = f((\varepsilon_k\varepsilon_{k-1}\cdots\varepsilon_2 1)_2)$

$= f(\left[\frac{n}{2}\right] + 1)$, the notation $[x]$ denotes the greatest integer not more than x.

$$g(n) = g((\varepsilon_k\varepsilon_{k-1}\cdots\varepsilon_1\varepsilon_0)_2) = g((\varepsilon_k\varepsilon_{k-1}\cdots\varepsilon_3\varepsilon_2 1)_2) = g(\left[\frac{n}{2}\right] + 1) = 0.$$

so, it's easily known from inductive hypothesis

$$a(n) = a(\left[\frac{n}{2}\right] + 1) - 1 = -(k-1) + 2f(\left[\frac{n}{2}\right] + 1) + 2g(\left[\frac{n}{2}\right] + 1) - 3 - 1$$

$$= -k + 2f(n) + 2g(n) - 3.$$

(ii) If $\varepsilon_1 = \varepsilon_2 = \cdots = \varepsilon_i = 1, \varepsilon_{i+1} = 0, 1 \le i \le k-2$.

$$n = (\varepsilon_k\varepsilon_{k-1}\cdots\varepsilon_1\varepsilon_0)_2, \quad \left[\frac{n}{2}\right] + 1 = (\varepsilon_k\varepsilon_{k-1}\cdots\varepsilon_{i+2}\varepsilon_{i+1}\varepsilon_i\cdots\varepsilon_2\varepsilon_1)_2 + (1)_2$$

$$=(\varepsilon_k\varepsilon_{k-1}\cdots\varepsilon_{i+2}1\underbrace{00\cdots0}_{i\ \text{times}})_2.$$

So, $f(\left[\frac{n}{2}\right]+1)=f(n)-i,\ \ g(\left[\frac{n}{2}\right]+1)=i=i+g(n)$.

Then, According to inductive hypothesis, we have

$$a(n)=a(\left[\frac{n}{2}\right]+1)-1=-(k-1)+2f(\left[\frac{n}{2}\right]+1)+2g(\left[\frac{n}{2}\right]+1)-3-1$$

$$=-k+2(f(n)-i)+2(i+g(n))-3$$

$$=-k+2f(n)+2g(n)-3.$$

(iii) If $\varepsilon_1=\varepsilon_2=\cdots=\varepsilon_{k-1}=\varepsilon_k=1$, then

$f(n)=k+1,\ \ g(n)=0.\ \ \left[\frac{n}{2}\right]+1=(\varepsilon_k\varepsilon_{k-1}\cdots\varepsilon_2\varepsilon_1)_2+(1)_2$

$=(1\underbrace{00\cdots0}_{k\ \text{times}})_2$, so from 1

$$a(\left[\frac{n}{2}\right]+1)=k.$$

Then $$a(n)=a(\left[\frac{n}{2}\right]+1)-1=k-1$$

$$=-k+2(k+1)+2\times0-3=-k+2f(n)+2g(n)-3.$$

From the above, the conclusion is true for all the natural numbers $n(n\geq2)$.

Having proved above fact, the remaining problem in question (a) can be solved easily. For if $n=2^k$, we have $a(n)=k$, so sequence $\{a(n)\}$ is unbounded, therefore cannot be periodical and convergent.

§ 2 Smaranche square-digital subsequence and Smaranche square-partial-digital subsequence

The Smaranche square-digital subsequence is defined to be a subsequence:

0, 1, 4, 9, 49, 100, 144, 400, 441, $\cdots$

i.e. from 0, 1, 4, 9, 16, 25, 36, $\cdots$, n^2, $\cdots$ we choose only the terms those digits are all perfect squares (Therefore only 0, 1, 4 and 9)

In [1] M.Bencze questioned: Disregarding the square numbers of the form $\overline{N\underbrace{0\cdots0}_{2k\ times}}$, where N is also a perfect square, how many other numbers belong to this sequence?

We find that 1444, 11449, 491401, also belong to the sequence by calculating.

In fact, we may find infinitely many numbers that belong to the sequence.

$$(2\cdot10^k+1)^2 = 4\cdot10^{2k}+4\cdot10^k+1,$$

$$(10^k+2)^2 = 10^{2k}+4\cdot10^k+4 \quad \text{for all} \quad k\geq 1.$$

Smarandache square-partial-digital subsequence is defined to be a sequence:

49, 100, 144, 169, 400, 441, $\cdots$

i.e. the square numbers that can be partitioned into groups of digits which are also perfect squares (169 can be partitioned as $16=4^2$ and $9=3^2$, etc.).

In the same way it is questioned: Disregarding the square numbers of the form $\overline{N\underbrace{0\cdots0}_{2k\ times}}$, where N is also a perfect square, how many other numbers belong to this sequence?

We may find 22 numbers in the form $\overline{a0b}^2$ (here neither a or b is zero).

10404, 11025, 11449, 11664, 40401, 41616, 42025, 43681, 93025, 93636, 161604, 164025, 166464, 251001, 254016, 259081, 363609, 491401, 641601, 646416, 813604, 819025.

We may construct infinitely many numbers by adding zero in the middle of there numbers like 102^2, 105^2, 107^2, 108^2, 201^2, 204^2, 205^2, 209^2, 305^2, 306^2, 402^2, 405^2, 408^2, 501^2, 504^2, 509^2, 603^2, 701^2, 801^2, 804^2, 902^2, 905^2 as well . we may find some other numbers as the following:

$\underline{324}\,\underline{3601}$, $\underline{1024}\,\underline{6401}$, $\underline{256}\,\underline{6404}$, $\underline{1}\,\underline{036}\,\underline{324}$, $\underline{4}\,\underline{064}\,\underline{256}$, $\underline{36}\,\underline{144}\,\underline{144}$, $\underline{49}\,\underline{196}\,\underline{196}$, $\underline{81}\,\underline{324}\,\underline{324}$, $\underline{64}\,\underline{256}\,\underline{256}$, $\underline{121}\,\underline{484}\,\underline{484}$, $\underline{169}\,\underline{676}\,\underline{676}$, $\underline{196}\,\underline{784}\,\underline{784}$, $\underline{484}\,\underline{484}\,\underline{121}$, $\underline{576}\,\underline{576}\,\underline{144}$, $\underline{676}\,\underline{676}\,\underline{169}$, $\underline{784}\,\underline{784}\,\underline{196}$ $\underline{900}\,\underline{900}\,\underline{225}$, $\underline{144}\,\underline{2401}$, $\underline{324}\,\underline{3601}$, $\underline{400}\,\underline{4001}$, $\underline{484}\,\underline{4401}$, $\underline{1024}\,\underline{6401}$, $\underline{2025}\,\underline{9001}$, $\underline{2401}\,\underline{9801}$, $\underline{2501}\,\underline{0001}$, $\underline{4901}\,\underline{4001}$, $\underline{6401}\,\underline{6001}$.

§ 3 Smaranche cube-partial-digital subsequence

1000, 8000, 10648, 27000, $\cdots$

i.e. the cube numbers that can be partitioned into groups of digits which are also perfect cubes (10648 can be partitioned as $1 = 1^3, 0 = 0^3, 64 = 4^3$, and $8 = 2^3$)

Same question: disregarding the cube numbers of the form: $\overline{M\underbrace{0\cdots0}_{3k\ times}}$, where M is also a perfect cube, how many other numbers belong to this sequence?

As the above said, we may find infinitely many numbers that belong to the sequence as well

$$(3\cdot 10^{k+2}+3)^3, (6\cdot 10^{k+3}+1)^3, (6\cdot 10^{k+3}+6)^3, (10^{k+3}+6)^3 \quad \text{for all} \quad k \geq 0$$

for example $\underline{27818127} = 303^3$, $\underline{216648648216} = 6006^3$,

$\underline{216108018001} = 6001^3$, $\underline{1018108216} = 1006^3$.

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