On Smarandache's Periodic Sequences

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Abstract:
This paper is based on an article in Mathematical Spectrum,Vol. 29, No 1. It concerns what happens when an operation applied to an n-digit integer results in an n digit integer. Since the number of n-digit integers is finite a repetition must occur after applying the operation a finite number of times. It was assumed in the above article that this would lead to a periodic sequence which is not always true because the process may lead to an invariant. The second problem with the initial article is that, say, 7 is considered as 07 or 007 as the case may be in order make its reverse to be 70 or 700. However, the reverse of 7 is 7. In order not to loose the beauty of these sequences the author has introduced stringent definitions to prevent the sequences from collapse when the reversal process is carried out.

Four different operations on n-digit integers is considered.

The Smarandache n-digit periodic sequence. Definition: Let N_k be an integer of at most n digits and let R_k be its reverse. N_k' is defined through

$$N_k^{'} = R_k \cdot 10^{n-1-[\log_{10} N_k]}$$

The element N_{k+1} of the sequence through

$N_{k+1} = | N_k - N_k' |$

where the sequence is initiated by an arbitrary n-digit integer N_1 in the domain $10^n \leq N_1 < 10^{n+1}$.

The Smarandache Subtraction Periodic Sequence: Definition: Let N_k be a positive integer of at most n digits and let R_k be its digital reverse. N_k' is defined through

$$N_k^{'} = R_k \cdot 10^{n-1-[\log_{10} N_k]}$$

The element N_{k+1} of the sequence through

$N_{k+1} = | N_k' - c |$

where c is a positive integer. The sequence is initiated by an arbitrary positive n-digit integer N_1. It is obvious from the definition that $0 \leq N_k < 10^{n+1}$, which is the range of the iterating function.

The Smarandache Multiplication Periodic Sequence: Definition: Let c>1 be a fixed integer and N_0 and arbitrary positive integer. N_{k+1} is derived from N_k by multiplying each digit x of N_k by c retaining only the last digit of the product cx to become the corresponding digit of N_{k+1}.

The Smarandache Mixed Composition Periodic Sequence: Definition. Let N_0 be a two-digit integer $a_1 \cdot 10 + a_0$. If $a_1 + a_0 < 10$ then $b_1 = a_1 + a_0$ otherwise $b_1 = a_1 + a_0 + 1$. $b_0 = |a_1 - a_0|$. We define $N_1 = b_1 \cdot 10 + b_0$. N_{k+1} is derived from N_k in the same way

Starting points for loops (periodic sequences), loop length and the number of loops of each kind has been calculated and displayed in tabular form in all four cases. The occurrence of invariants has also been included.

Introduction

In *Mathematical Spectrum*, vol 29 No 1 [1], is an article on Smarandache's periodic sequences which terminates with the statement:

> *"There will always be a periodic sequence whenever we have a function $f: S \to S$, where S is a finite set of positive integers and we repeat the function f."*

We must adjust the above statement by a counterexample before we look at this interesting set of sequences. Consider the following trivial function $f(x_k): S \to S$, where S is an ascending set of integers $\{a_1, a_2, \ldots a_r, \ldots a_n\}$:

$$f(x_k) = \begin{cases} x_{k-1} \text{ if } x_k > a_r \\ x_k \text{ if } x_k = a_r \\ x_{k+1} \text{ if } x_k < a_r \end{cases}$$

As we can see the iteration of the function f in this case converges to an invariant a_r, which we may of course consider as a sequence (or loop) of only one member. We will however make a distinction between a sequence and an invariant in this paper.

There is one more snag to overcome. In the Smarandache sequences 05 is considered as a two-digit integer. The consequence of this is that 00056 is considered as as a five digit integer while 056 is considered as a three-digit integer. We will abolish this ambiguity, 05 is a one-digit integer and 00200 is a three-digit integer.

With these two remarks in mind let's look at these sequences. There are in all four different ones reported in the above mentioned article in Mathematical Spectrum. The study of the first one will be carried out in much detail in view of the above remarks.

1a. The Two-Digit Smarandache Periodic Sequence

It has been assumed that the definition given below leads to a repetition according to Dirichlet's box principle (or the statement made above). However, as we will see, this definition leads to a collapse of the sequence.

Preliminary definition. Let N_k be an integer of at most two digits and let N_k' be its digital reverse. We define the element N_{k+1} of the sequence through

$$N_{k+1} = |N_k - N_k'|$$

where the sequence is initiated by an arbitrary two digit integer N_1.

Let's write N_1 in the form $N_1 = 10a+b$ where a and b are digits. We then have

$$N_2 = |10a+b-10b-a| = 9 \cdot |a-b|$$

The $|a-b|$ can only assume 10 different values $0,1,2, \ldots ,9$. This means that N_3 is generated from only 10 different values of N_2. Let's first find out which two digit integers result in $|a-b| = 0,1,2, \ldots$ and 9 respectively.

|a-b|	Corresponding two digit integers																
0	11	22	33	44	55	66	77	88	99								
1	10	12	21	23	32	34	43	45	54	56	65	67	76	78	87	89	98
2	13	20	24	31	35	42	46	53	57	64	68	75	79	86	97		
3	14	25	30	36	41	47	52	58	63	69	74	85	96				
4	15	26	37	40	48	51	59	62	73	84	95						
5	16	27	38	49	50	61	72	83	94								
6	17	28	39	60	71	82	93										
7	19	29	70	81	92												
8	19	80	91														
9	90																

It is now easy to follow the iteration of the sequence which invariably terminates in 0, table 1.

Table 1. Iteration of sequence according to the preliminary definition

|a-b|	N_2	N_3	N_4	N_5	N_5	N_6
0	0					
1	9	0				
2	18	63	27	45	9	0
3	27	45	9	0		
4	36	27	45	9	0	
5	45	9	0			
6	54	9	0			
7	63	27	45	9	0	
8	72	45	9	0		
9	81	63	27	45	9	0

The termination of the sequence is preceded by the one digit element 9 whose reverse is 9. The following definition is therefore proposed.

Definition of Smarandache's two-digit periodic sequence. Let N_k be an integer of at most two digits. N_k' is defined through

$$N_k' = \begin{cases} \text{the reverse of } N_k \text{ if } N_k \text{ is a two digit integer} \\ N_k \cdot 10 \text{ if } N_k \text{ is a one digit integer} \end{cases}$$

We define the element N_{k+1} of the sequence through

$$N_{k+1} = |N_k - N_k'|$$

where the sequence is initiated by an arbitrary two digit integer N_1 with unequal digits.

Modifying table 1 according to the above definition results in table 2.

Table 2. Iteration of the Smarandache two digit sequence

|a-b|	N_2	N_3	N_4	N_5	N_5	N_6	N_7
1	9	81	63	27	45	9	
2	18	63	27	45	9	81	63
3	27	45	9	81	63	27	
4	36	27	45	9	81	63	27
5	45	9	81	63	27	45	
6	54	9	81	63	27	45	9
7	63	27	45	9	81	63	
8	72	45	9	81	63	27	45
9	81	63	27	45	9	81	

Conclusion: The iteration always produces a loop of length 5 which starts on the second or the third term of the sequence. The period is 9, 81, 63, 27, 45 or a cyclic permutation thereof.

1b. Smarandache's n-digit periodic sequence.

Let's extend the definition of the two-digit periodic sequence in the following way.

Definition of Smarandache's n-digit periodic sequence.

Let N_k be an integer of at most n digits and let R_k be its reverse. N_k' is defined through

$$N_k' = R_k \cdot 10^{n-1-[\log_{10} N_k]}$$

We define the element N_{k+1} of the sequence through

$$N_{k+1} = |N_k - N_k'|$$

where the sequence is initiated by an arbitrary n-digit integer N_1 in the domain $10^n \le N_1 < 10^{n+1}$. It is obvious from the definition that $0 \le N_k < 10^{n+1}$, which is the range of the iterating function.

Let's consider the cases n=3, n=4, n=5 and n=6.

n=3.

Domain $100 \le N_1 \le 999$. . The iteration will lead to an invariant or a loop (periodic sequence)[1]. There are 90 symmetric integers in the domain, 101, 111, 121, ...202, 212, ..., for which $N_2=0$ (invariant). All other initial integers iterate into various entry points of the same periodic sequence. The number of numbers in the domain resulting in each entry of the loop is denoted s in table 3.

Table 3. Smarandache 3-digit periodic sequence

s	239	11	200	240	120
Loop	99	891	693	297	495

It is easy to explain the relation between this loop and the loop found for n=2. Consider $N=a_0+10a_1+100a_2$. From this we have $|N-N'|=99|a_2-a_0|=11\cdot 9|a_2-a_0|$ which is 11 times the corresponding expression for n=2 and as we can see this produces a 9 as middle (or first) digit in the sequence for n=3.

n=4.

Domain $1000 \le N_1 \le 9999$. The largest number of iterations carried out in order to reach the first member of the loop is 18 and it happened for $N_1=1019$. The iteration process ended up in the invariant 0 for 182 values of N_1, 90 of these are simply the symmetric integers in the domain like $N_1=4334$, 1881, 7777, etc., the other 92 are due to symmetric integers obtained after a couple of iterations. Iterations of the other 8818 integers in the domain result in one of the following 4 loops or a cyclic permutation of one of these. The number of numbers in the domain resulting in each entry of the loops is denoted s in table 4.

[1] This is elaborated in detail in *Surfing on the Ocean of Numbers* by the author, Vail Univ. Press 1997.

Table 4. Smarandache 4-digit periodic sequences

s	378	259			
Loop	2178	6534			
s	324	18	288	2430	310
Loop	90	810	630	270	450
s	446	2	449	333	208
Loop	909	8181	6363	2727	4545
s	329	11	290	2432	311
Loop	999	8991	6993	2997	4995

n=5.

Domain $10000 \leq N_1 \leq 99999$. There are 900 symmetric integers in the domain. 920 integers in the domain iterate into the invariant 0 due to symmetries.

Table 5. Smarandache 5-digit periodic sequences

s	3780	2590			
Loop	21978	65934			
s	3240	180	2880	24300	3100
Loop	990	8910	6930	2970	4950
s	4469	11	4490	3330	2080
Loop	9009	81081	63063	27027	45045
s	3299	101	2900	24320	3110
Loop	9999	89991	69993	29997	49995

n=6.

Domain $100000 \leq N_1 \leq 999999$. There are 900 symmetric integers in the domain. 12767 integers in the domain iterate into the invariant 0 due to symmetries. The longest sequence of iterations before arriving at the first loop member is 53 for N=100720. The last column in table 6 shows the number of integers iterating into each loop.

Table 6. Smarandache 6-digit periodic sequences

s	13667																		13667
L1	0																		
s	13542	12551																	26093
L2	13586	65340																	
s	12685	12685																	26271
L3	219978	659934																	
s	19107	2711	7127	123320	12446														164711
L4	900	8100	6300	2700	4500														
s	25057	18	12259	20993	4449														62776
L5	9090	81810	63630	27270	45450														
s	47931	14799	42603	222941	29995														358269
L6	9990	89910	69930	29970	49950														
s	25375	11	12375	21266	4409														63436
L7	90009	810081	630063	270027	450045														
s	1488	2	1005	1033	237														3765
L8	90909	818181	636363	272727	454545														
s	1809	11	1350	1570	510														5250
L9	99099	891891	693693	297297	495495														
s	19139	2648	7292	123673	12472														165224
L10	99999	899991	699993	299997	499995														
s	152	4	1254	972	492	111	826	485	429										4725
L11	10989	978021	857142	615384	131868	736263	373626	252747	494505										
s	623	64	156	796	377	36	525	140	194	596	117	156	793	327	65	530	139	179	5813
L12	43659	912681	726462	461835	76329	847341	703593	308286	374517	340956	318087	462726	164538	670923	341847	406296	286308	517374	

2. The Smarandache Subtraction Periodic Sequence

Definition:

Let N_k be a positive integer of at most n digits and let R_k be its digital reverse. N_k' is defined through

$$N_k' = R_k \cdot 10^{n-1-[\log_{10} N_k]}$$

We define the element N_{k+1} of the sequence through

$$N_{k+1} = | N_k' - c |$$

where c is a positive integer. The sequence is initiated by an arbitrary positive n-digit integer N_1. It is obvious from the definition that $0 \le N_k < 10^{n+1}$, which is the range of the iterating function.

c=1, n=2, $10 \le N_1 \le 99$

When N_1 is of the form $11 \cdot k$ or $11 \cdot k-1$ then the iteration process results in 0, see figure 1a.

Every other member of the interval $10 \le N_1 \le 99$ is a entry point into one of five different cyclic periodic sequences. Four of these are of length 18 and one of length 9 as shown in table 7 and illustrated in figures 1b and 1c, where important features of the iteration chains are shown.

Table 7. The subtraction periodic sequence, $10 \le N_1 \le 99$

Seq. No 1	12	20	1	9	89	97	78	86	67	75	56	64	45	53	34	42	23	31
Seq. No 2	13	30	2	19	90	8	79	96	68	85	57	74	46	63	35	52	24	41
Seq. No 3	14	40	3	29	91	18	80	7	69	95	58	84	47	73	35	62	25	51
Seq. No 4	15	50	4	39	92	28	81	17	70	6	59	94	48	83	37	72	26	61
Seq. No 5	16	60	5	49	93	38	82	27	71									

	99	
	98	
	88	←
←(=)	87	
	77	
→	76	
	66	→(-1)
	65	
	55	
	54	
	44	
	43	
	33	
	32	
	22	
	21	
	11	
	10	
	0	

Fig. 1a

	37	
	72	
	26	
	61	
	15	
	50	
→	04	←
←(-1)	39	→(+9)
	92	
	28	
	81	
	17	
	70	
→	06	←
(-1)←	59	→(+9)
	94	
	48	
	83	
	37	

Fig 1b

	38	
	82	
	27	
	71	
	16	
	60	
→	05	←
(-1)←	49	→(+9)
	93	
	38	

Fig 1c

$1 \le c \le 9$, n=2, $100 \le N_1 \le 999$

A computer analysis revealed a number of interesting facts concerning the application of the iterative function.

There are no periodic sequences for c=1, c=2 and c=5. All iterations result in the invariant 0 after, sometimes, a large number of iterations.

For the other values of c there are always some values of N_1 which do not produce periodic sequences but terminate on 0 instead. Those values of N_1 which produce periodic sequences will either have N_1 as the first term of the sequence or one of the values f determined by $1 \leq f \leq c-1$ as first term. There are only eight different possible value for the length of the loops, namely 11, 22, 33, 50, 100, 167, 189, 200. Table 8 shows how many of the 900 initiating integers in the interval $100 \leq N_1 \leq 999$ result in each type of loop or invariant 0 for each value of c.

Table 8. Loop statistics, L=length of loop, f=first term of loop

c	f↓ / L→	0	11	22	33	50	100	167	189	200
1	N_1	900								
2	N_1	900								
3	N_1	241			59			150		
	1							240		
	2							210		
4	N_1	494				42				
	1					364				
5	N_1	900								
6	N_1	300			59		84			
	1						288			
	2						169			
7	N_1	109								535
	1									101
	2									101
	3									14
	4									14
	5									13
	6									13
8	N_1	203				43	85			
	1						252			
	2					305				
	3						12			
9	N_1	21	79	237					170	
	4								20	
	5								10	
	6		161							
	7			121						
	8			81						

A few examples:
For c=2 and N_1=202 the sequence ends in the invariant 0 after only 2 iterations:
202 200 **0**
For c=9 and N_1=208 a loop is closed after only 11 iterations:
208 793 388 874 469 955 550 46 631 127 712 **208**
For c=7 and N_1=109 we have an example of the longest loop obtained. It has 200 elements and the loop is closed after 286 iterations:

109 894 491 187 774 470 67 753 350 46 633 329 916 612 209 895 591 188 874 471
167 754 450 47 733 330 26 613 309 896 691 189 974 472 267 755 550 48 833 331
126 614 409 897 791 190 84 473 367 756 650 49 933 332 226 615 509 898 891 191
184 474 467 757 750 50 43 333 326 616 609 899 991 192 284 475 567 758 850 51
143 334 426 617 709 900 2 193 384 476 667 759 950 52 243 335 526 618 809 901
102 194 484 477 767 760 60 53 343 336 626 619 909 902 202 195 584 478 867 761
160 54 443 337 726 620 19 903 302 196 684 479 967 762 260 55 543 338 826 621
119 904 402 197 784 480 77 763 360 56 643 339 926 622 219 905 502 198 884 481
177 764 460 57 743 340 36 623 319 906 602 199 984 482 277 765 560 58 843 341
136 624 419 907 702 200 5 493 387 776 670 69 953 352 246 635 529 918 812 211
105 494 487 777 770 70 63 353 346 636 629 919 912 212 205 495 587 778 870 71
163 354 446 637 729 920 22 213 305 496 687 779 970 72 263 355 546 638 829 921
122 214 405 497 787 780 80 73 363 356 646 639 929 922 222 215 505 498 887 781
180 74 463 357 746 640 39 923 322 216 605 499 987 782 280 75 563 358 846 641
139 924 422 217 705 500 2

3. The Smarandache Multiplication Periodic Sequence

Definition:
Let c>1 be a fixed integer and N_0 and arbitrary positive integer. N_{k+1} is derived from N_k by multiplying each digit x of N_k by c retaining only the last digit of the product cx to become the corresponding digit of N_{k+1}.

In this case each digit position goes through a separate development without interference with the surrounding digits. Let's as an example consider the third digit of a 6-digit integer for c=3. The iteration of the third digit follows the schema:

xx7yyy ----- the third digit has been arbitrarily chosen to be 7.
xx1yyy
xx3yyy
xx9yyy
xx7yyy ----- which closes the loop for the third digit.

Let's now consider all the digits of a six-digit integer 237456:

237456
691258
873654
419852
237456 ----- which closes the loop.

The digits 5 and 0 are invariant under this iteration. All other digits have a period of 4 for c=3.

Conclusion: Integers whose digits are all equal to 5 are invariant under the given operation. All other integers iterate into a loop of length 4.

We have seen that the iteration process for each digit for a given value of c completely determines the iteration process for any n-digit integer. It is therefore of interest to see these single digit iteration sequences:

Table 9. One-digit multiplication sequences

c=2	c=3	c=4	c=5
1 2 4 8 6 2	1 3 9 7 1	1 4 6 4	1 5 5
2 4 8 6 2	2 6 8 4 2	2 8 2	2 0 0
3 6 2 4 8 6	3 9 7 1 3	3 2 8 2	3 5 5
4 8 6 2 4	4 2 6 8 4	4 6 4	4 0 0
5 0 0	5 5	5 0 0	5 5
6 2 4 8 6	6 8 4 2 6	6 4 6	6 0 0
7 4 8 6 2 4	7 1 3 9 7	7 8 2 8	7 5 5
8 6 2 4 8	8 4 2 6 8	8 2 8	8 0 0
9 8 6 2 4 8	9 7 1 3 9	9 6 4 6	9 5 5

c=6	c=7	c=8	c=9
1 6 6	1 7 9 3 1	1 8 4 2 6 8	1 9 1
2 2	2 4 8 6 2	2 6 8 4 2	2 8 2
3 8 8	3 1 7 9 3	3 4 2 6 8 4	3 7 3
4 4	4 8 6 2 4	4 2 6 8 4	4 6 4
5 0 0	5 5	5 0 0	5 5
6 6	6 2 4 8 6	6 8 4 2 6	6 4 6
7 2 2	7 9 3 1 7	7 6 8 4 2 6	7 3 7
8 8	8 6 2 4 8	8 4 2 6 8	8 2 8
9 4 4	9 3 1 7 9	9 2 6 8 4 2	9 1 9

With the help of table 9 it is now easy to characterize the iteration process for each value of c.

Integers composed of the digit 5 result in an invariant after one iteration. Apart form this we have for:

c=2. Four term loops starting on the first or second term.

c=3. Four term loops starting with the first term.

c=4. Two term loops starting on the first or second term (could be called a switch or pendulum).

c=5. Invariant after one iteration.

c=6. Invariant after one iteration.

c=7. Four term loop starting with the first term.

c=8. Four term loop starting with the second term.

c=9. Two term loops starting with the first term (pendulum).

4. The Smarandache Mixed Composition Periodic Sequence

Definition. Let N_0 be a two-digit integer $a_1 \cdot 10+a_0$. If $a_1+a_0<10$ then $b_1= a_1+a_0$ otherwise $b_1= a_1+a_0+1$. $b_0=|a_1-a_0|$. We define $N_1=b_1 \cdot 10+b_0$. N_{k+1} is derived from N_k in the same way.[2]

There are no invariants in this case. 36, 90, 93 and 99 produce two-element loops. The longest loops have 18 elements. A complete list of these periodic sequences is presented below.

10 11 20 22 40 44 80 88 70 77 50 55 10
11 20 22 40 44 80 88 70 77 50 55 10 11
12 31 42 62 84 34 71 86 52 73 14 53 82 16 75 32 51 64 12
13 42 62 84 34 71 86 52 73 14 53 82 16 75 32 51 64 12 31 42
14 53 82 16 75 32 51 64 12 31 42 62 84 34 71 86 52 73 14
15 64 12 31 42 62 84 34 71 86 52 73 14 53 82 16 75 32 51 64
16 75 32 51 64 12 31 42 62 84 34 71 86 52 73 14 53 82 16
17 86 52 73 14 53 82 16 75 32 51 64 12 31 42 62 84 34 71 86
18 97 72 95 54 91 18
19 18 97 72 95 54 91 18
20 22 40 44 80 88 70 77 50 55 10 11 20
21 31 42 62 84 34 71 86 52 73 14 53 82 16 75 32 51 64 12 31
22 40 44 80 88 70 77 50 55 10 11 20 22
23 51 64 12 31 42 62 84 34 71 86 52 73 14 53 82 16 75 32 51
24 62 84 34 71 86 52 73 14 53 82 16 75 32 51 64 12 31 42 62
25 73 14 53 82 16 75 32 51 64 12 31 42 62 84 34 71 86 52 73
26 84 34 71 86 52 73 14 53 82 16 75 32 51 64 12 31 42 62 84
27 95 54 91 18 97 72 95
28 16 75 32 51 64 12 31 42 62 84 34 71 86 52 73 14 53 82 16
29 27 95 54 91 18 97 72 95
30 33 60 66 30
31 42 62 84 34 71 86 52 73 14 53 82 16 75 32 51 64 12 31
32 51 64 12 31 42 62 84 34 71 86 52 73 14 53 82 16 75 32
33 60 66 30 33
34 71 86 52 73 14 53 82 16 75 32 51 64 12 31 42 62 84 34
35 82 16 75 32 51 64 12 31 42 62 84 34 71 86 52 73 14 53 82
36 93 36
37 14 53 82 16 75 32 51 64 12 31 42 62 84 34 71 86 52 73 14
38 25 73 14 53 82 16 75 32 51 64 12 31 42 62 84 34 71 86 52 73
39 36 93 36

[2] Formulation conveyed to the author: "Let N be a two-digit number. Add the digits, and add them again if the sum is greater than 10. Also take the absolute value of their difference. These are the first and second digits of N_1."

40 44 80 88 70 77 50 55 10 11 20 22 40
41 53 82 16 75 32 51 64 12 31 42 62 84 34 71 86 52 73 14 53
42 62 84 34 71 86 52 73 14 53 82 16 75 32 51 64 12 31 42
43 71 86 52 73 14 53 82 16 75 32 51 64 12 31 42 62 84 34 71
44 80 88 70 77 50 55 10 11 20 22 40 44
45 91 18 97 72 95 54 91
46 12 31 42 62 84 34 71 86 52 73 14 53 82 16 75 32 51 64 12
47 23 51 64 12 31 42 62 84 34 71 86 52 73 14 53 82 16 75 32 51
48 34 71 86 52 73 14 53 82 16 75 32 51 64 12 31 42 62 84 34
49 45 91 18 97 72 95 54 91
50 55 10 11 20 22 40 44 80 88 70 77 50
51 64 12 31 42 62 84 34 71 86 52 73 14 53 82 16 75 32 51
52 73 14 53 82 16 75 32 51 64 12 31 42 62 84 34 71 86 52
53 82 16 75 32 51 64 12 31 42 62 84 34 71 86 52 73 14 53
54 91 18 97 72 95 54
55 10 11 20 22 40 44 80 88 70 77 50 55
56 21 31 42 62 84 34 71 86 52 73 14 53 82 16 75 32 51 64 12 31
57 32 51 64 12 31 42 62 84 34 71 86 52 73 14 53 82 16 75 32
58 43 71 86 52 73 14 53 82 16 75 32 51 64 12 31 42 62 84 34 71
59 54 91 18 97 72 95 54
60 66 30 33 60
61 75 32 51 64 12 31 42 62 84 34 71 86 52 73 14 53 82 16 75
62 84 34 71 86 52 73 14 53 82 16 75 32 51 64 12 31 42 62
63 93 36 93
64 12 31 42 62 84 34 71 86 52 73 14 53 82 16 75 32 51 64
65 21 31 42 62 84 34 71 86 52 73 14 53 82 16 75 32 51 64 12 31
66 30 33 60 66
67 41 53 82 16 75 32 51 64 12 31 42 62 84 34 71 86 52 73 14 53
68 52 73 14 53 82 16 75 32 51 64 12 31 42 62 84 34 71 86 52
69 63 93 36 93
70 77 50 55 10 11 20 22 40 44 80 88 70
71 86 52 73 14 53 82 16 75 32 51 64 12 31 42 62 84 34 71
72 95 54 91 18 97 72
73 14 53 82 16 75 32 51 64 12 31 42 62 84 34 71 86 52 73
74 23 51 64 12 31 42 62 84 34 71 86 52 73 14 53 82 16 75 32 51
75 32 51 64 12 31 42 62 84 34 71 86 52 73 14 53 82 16 75
76 41 53 82 16 75 32 51 64 12 31 42 62 84 34 71 86 52 73 14 53
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